Boundary Layer Preconditioners for a Finite-Element Method

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We consider the iterative solution of linear systems of equations arising from the discretization of singularly perturbed reaction-diffusion differential equations by a finite-element method. The equations feature a perturbation parameter, which may be arbitrarily small, and, correspondingly, their solutions feature layers: regions where the solution changes rapidly. Therefore, numerical solutions are computed on specially designed, highly anisotropic layer-adapted meshes. Usually, the resulting linear systems are ill-conditioned, and, so, careful design of suitable preconditioners is necessary in order to solve them in a way that is robust, with respect to the perturbation parameter, and efficient. We propose a *boundary layer preconditioner* and prove the optimality of this preconditioner, and establish a suitable stopping criterion for one-dimensional problems. Numerical results are presented which demonstrate that the ideas extend to problems in two dimensions.

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