Preconditioning for Radial Basis Function Partition of Unity Methods

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Meshfree methods based on localized radial basis function (RBF) approximation have been successfully applied to various types of PDE problems. We consider RBF partition of unity methods (RBF-PUM), where global PDE solutions are formed through a weighted sum of local RBF approximations. The local RBF matrices are notoriously ill-conditioned for certain regimes of the shape parameter of the basis functions. By employing stable evaluation methods such as the RBF-QR method, the conditioning is improved, while the resulting differentiation matrices still are not well conditioned. We are interested in elliptic or parabolic PDEs where the solution process involves the solution of a global linear system. We want to construct preconditioned iterative solution methods, both to reduce the computational cost and the memory requirements, compared with a direct solution method. The sparse coefficient matrices are locally unstructured, but with a global structure due to the partitioning of the domain. We have implemented and evaluated different algebraic preconditioners in combination with incomplete factorisations. For the construction of the preconditioner we organise the unknowns such that locality is maximized, concentrating strong relationships to the central band of the matrix. A no-fill incomplete factorisation of the central band is shown to provide a robust and efficient preconditioner.

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