Preconditioning for Accurate Solutions of Linear Systems and Eigenvalue Problems

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This paper develops the preconditioning technique as a method to address the accuracy issue caused by ill-conditioning. Given a preconditioner M for an ill-conditioned linear system $Ax = b$, we show that, if the inverse of the preconditioner M^{-1} can be applied to a vector *accurately*, then the linear system can be solved *accurately*. A stability concept called *inverse-equivalent accuracy* is introduced to describe the higher accuracy that may be achieved and an error analysis will be presented. As an application, we shall also use the preconditioning approach to accurately compute a few smallest eigenvalues of certain ill-conditioned matrices through accurate inverses.

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