Multi Space Reduced Basis Preconditioners for Large-Scale Parametrized PDEs with Applications to Blood Flow Simulations

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In this talk present a two-level preconditioner for the efficient solution of large-scale linear systems arising from the (finite element) discretization of parametrized partial differential equations (PDEs). Our preconditioner combines multiplicatively a reduced basis (RB) coarse correction and a nonsingular fine grid preconditioner. The proposed technique hinges upon the construction of a new Multi Space Reduced Basis (MSRB) method, where a RB space is built through proper orthogonal decomposition (POD) at each step of the iterative method used to solve the linear system. As a matter of fact, each RB space is tailored to solve a particular iteration and aims at fixing the scales that have not been treated by previous iterations and by the fine preconditioner yet. The MSRB preconditioner allows to tune the error decay by properly choosing the accuracies of the RB spaces. Since they affect the overall convergence of the iterative method in a multiplicative way, a very accurate solution of the large-scale system can be computed in very few (order of 10) iterations.

In the context of linear affine elliptic problems, the MSRB preconditioner has given remarkable results in terms of iteration count and computational times required by the iterative solution of Richardson and flexible GMRES methods, and it has proven to be strongly reliable also in the case of complex parametric dependence, e.g. with nonaffine problems. In this presentation, this methodology is extended to saddle point problems. Numerical tests are carried out to evaluate the performance of the MSRB preconditioner in different large-scale modeling settings related to parametrized saddle point PDEs, up to millions of degrees of freedom, and compared with the current state-of-art preconditioners. As relevant application, we employ the aforementioned technique to deal with cardiovascular blood flow simulations, where the problem is nonaffinely parametrized with respect to the deformation of the computational domain. This is a joint work with A. Quarteroni.

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