The Merits of Keeping It Smooth: Implementing a Smooth Exact Penalty Function for Nonlinear Optimization

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We consider constrained nonlinear programs of the form

$$\underset{x\in\mathbb{R}^n}{\text{minimize}} \quad f(x) \quad \text{subject to} \quad c(x) = 0, \tag{NP}$$

where f and c are second-order smooth functions. Such models are ubiquitous in the computational sciences, used in applications such as optimal control, seismic imaging, and systems biology. We discuss a penalty function approach originally proposed by [1], where instead of minimizing (NP), we instead minimize the smooth, unconstrained penalty function:

$$\begin{aligned} \underset{x\in\mathbb{R}^n}{\text{minimize}} \quad & \phi_\sigma(x) := f(x) - y_\sigma(x)^T c(x) \\ & y_\sigma(x) := \arg\min_y \tfrac{1}{2}\|\nabla f(x) - \nabla c(x)^T y\|_2^2 + \sigma c(x)^T y. \end{aligned}$$

This penalty function is *exact* in the sense that minimizers of (NP) are minimizers of ϕ_σ for a sufficiently large (but finite) penalty parameter.

Fletcher originally envisioned that this penalty function would be applied to small, dense problems. We challenge this notion by demonstrating how to compute the quantities necessary for most off-the-shelf optimization solvers with computational cost comparable to widely accepted methods for nonlinear optimization, such as sequential quadratic programming. In particular, we also demonstrate how to combine the penalty function with matrix-free optimization solvers, in order to target large-scale problems, particularly in the case of PDE-constrained optimization.

We discuss further extensions of the penalty function, including regularization for stability, problems with inequality constraints, and the use of inexact evaluations. We also provide some preliminary numerical results on some standard optimization test problems and PDE-constrained problems.

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References

[1] R. Fletcher, *A class of methods for nonlinear programming with termination and convergence properties*, In J. Abadie, editor. Integer and nonlinear programming pp. 157–175. North-Holland, Amsterdam, 1970.