# Conservative methods for dynamical systems

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Many interesting dynamical systems possess geometric invariants or conserved quantities that are important for understanding their long term behaviours. Conservative methods are numerical methods which preserve these invariants on their numerical solutions. Unfortunately, classical methods, such as linear multistep methods and Runge-Kutta methods, are not conservative for general types of invariants [1]. While projection methods are in general conservative, they can lead to instability over long-term integration if their numerical solutions are projected on to a different connected component. Discrete gradient method [2] is another class of conservative methods, which expresses the differential equations in a skew-gradient tensor form. However, for large dynamical systems with multiple invariants, sparse representations of such tensor form are not known at this time.

In this talk, we introduce a new class of conservative methods, called the multiplier method [3], which can be applied to general dynamical systems with arbitrary forms of invariants, without the need for projection or finding sparse tensor representations. We illustrate this method on a variety of examples, such as non-Hamiltonian systems and large dynamical systems. Also, we will discuss an important connection between conservative methods and long-term stability. Specifically, under appropriate conditions, for conservative methods with a uniformly bounded displacement property, their global error is bounded for all time [4].

This is joint work with Alexander Bihlo (Memorial University of Newfoundland) and Jean-Christophe Nave (McGill University).

## References

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