$$H = X(X'X)^{-1}X'$$

$$H_0 = \beta_0, \dots, \beta_{k} = 0$$

$$F^* = SSR / SSE$$

SSE:

$$Y'(I - H)Y = (X\beta + \varepsilon)'(I - H)(X\beta + \varepsilon)$$

$$\left((X\beta)' - (X\beta)'H + \varepsilon' - \varepsilon'H\right)(X\beta + \varepsilon)$$

$$= (X\beta)'X\beta + (X\beta)'\varepsilon - (X\beta)'HX\beta - (X\beta)'H\varepsilon + \varepsilon'X\beta + \varepsilon'\varepsilon$$
$$- \varepsilon'HX\beta - \varepsilon'H\varepsilon$$

$$= \beta'X'X\beta + \beta'X\varepsilon - \beta'X'HX\beta - \beta'X'H\varepsilon + \varepsilon'X\beta + \varepsilon'\varepsilon$$
$$- \varepsilon'HX\beta - \varepsilon'H\varepsilon$$

$$= \beta'X'X\beta + \beta'X\varepsilon - \beta'X'X\cancel{(X'X)^{-1}X'X}\beta - \beta'X'X(X'X)^{-1}X'\varepsilon + \varepsilon'X\beta + \varepsilon'\varepsilon$$
$$- \varepsilon'X(X'X)^{-1}X'X\beta - \varepsilon'H\varepsilon$$

$$= \cancel{\beta'X'X\beta} + \cancel{\beta'X\varepsilon} - \beta'\cancel{X'X}\beta - \cancel{\beta'X}\varepsilon + \cancel{\varepsilon'X\beta} + \varepsilon'\varepsilon$$
$$- \cancel{\varepsilon'X}\beta - \varepsilon'H\varepsilon$$

$$= \varepsilon'\varepsilon - \varepsilon'H\varepsilon$$

$$= \varepsilon'(I - H)\varepsilon$$

SSR :

$$Y'\left[H - \frac{1}{n}J\right]Y =$$

$$(X\beta + \varepsilon)'\left(H - \frac{1}{n}J\right)(X\beta + \varepsilon)$$

$$= \varepsilon'\left(H - \frac{1}{n}J\right)\varepsilon \quad \text{if} \quad \beta = 0$$

Under the null Hypothesis

$$H_0 : \beta_1 = 0, \ \dots, \ \beta_k = 0$$

then by Cochran's theorem

$$\left.\begin{array}{l} \frac{SSR}{\sigma^2} \sim \chi^2(p-1) \\ \text{and} \quad \frac{SSE}{\sigma^2} \sim \chi^2(n-p) \end{array}\right\} \text{independent}$$

$$\text{So} \quad F^* = \frac{\frac{SSR}{\cancel{\sigma^2}(p-1)}}{\frac{SSE}{\cancel{\sigma^2}(n-p)}} \sim F(p-1, n-p)$$

SSR can be decomposed into many "smaller pieces"