LINEAR REGRESSION MODELS W4315

HOMEWORK 5 QUESTIONS

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1. (10 points) [1] Bonferroni inequality (4.2a) which is given as

$$P(\bar{A}_1 \bigcap \bar{A}_2) \geq 1 - \alpha - \alpha = 1 - 2\alpha$$

deals with the case of two statements, A_1 and A_2. Extend the inequality to the case of n statements, namely, $A_1, A_2 \ldots, A_n$, each with statement confidence coefficient $1 - \alpha$.

2. (30 points) [3] In a small-scale regression study, the following data were obtained: Assume

i:	**1**	**2**	**3**	**4**	**5**	**6**
X_{i1}	7	4	16	3	21	8
X_{i2}	33	41	7	49	5	31
Y_i	42	33	75	28	91	55

that regression model (1) which is:

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta X_{i2} + \epsilon_i \tag{1}$$

with independent normal error terms is appropriate. Using matrix methods, obtain (a) $\mathbf{b}$; (b) $\mathbf{e}$; (c) $\mathbf{H}$; (d) SSR; (e) $s^2\{\mathbf{b}\}$; (f) $\hat{Y}_h$ when $X_{h1} = 10$, $X_{h2} = 30$; (g) $s^2\{\hat{Y}_h\}$ when $X_{h1} = 10$, $X_{h2} = 30$. For the notations, please refer to section 6.4.

3. (30 points) Consider the classical regression setup

$$\mathbf{y} = \mathbf{X}\beta + \epsilon$$

We want to find the maximum likelihood estimate of the parameters.
a. if $\epsilon \sim \mathbf{N}(\mathbf{0}, \sigma^2\mathbf{I})$. Derive the maximum likelihood estimate of β and σ^2.
b. if $\epsilon \sim \mathbf{N}(\mathbf{0}, \boldsymbol{\Sigma})$ and Σ is known. Derive the maximum likelihood estimate of β.

[1] This is problem 4.22 in 'Applied Linear Regression Models(4th edition)' by Kutner etc.
[3] This is problem 6.27 in 'Applied Linear Regression Models(4th edition)' by Kutner etc.

4. (30 points) Suppose $X_1, ..., X_n$ are i.i.d. samples from $N(0, \sigma^2)$. Denote $\bar{X}$ as the sample mean. Prove $S = \sum_{i=1}^{n}(X_i - \bar{X})^2 \sim \sigma^2\chi^2(n-1)$ following the steps below using Cochran's theorem:

a. Remember that we have the decomposition

$$\sum_{i=1}^{n} X_i^2 = \sum_{i=1}^{n}(X_i - \bar{X})^2 + n\bar{X}^2$$

Show the matrices corresponding to all the three quadratic terms in (3).

b. Derive the rank of each matrix above.

c. Use Cochran's theorem to prove $S \sim \sigma^2\chi^2(n-1)$.