# LINEAR REGRESSION MODELS W4315 HOMEWORK 5 QUESTIONS 

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1. (20 points) ${ }^{1}$ Bonferroni inequality (4.2a) which is given as

$$
P\left(\bar{A}_{1} \bigcap \bar{A}_{2}\right) \geq 1-\alpha-\alpha=1-2 \alpha
$$

deals with the case of two statements, $A_{1}$ and $A_{2}$. Extend the inequality to the case of $n$ statements, namely, $A_{1}, A_{2} \ldots, A_{n}$, each with statement confidence coefficient $1-\alpha$.
2. (40 points) ${ }^{3}$ In a small-scale regression study, the following data were obtained: Assume

| i: | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $X_{i 1}$ | 7 | 4 | 16 | 3 | 21 | 8 |
| $X_{i 2}$ | 33 | 41 | 7 | 49 | 5 | 31 |
| $Y_{i}$ | 42 | 33 | 75 | 28 | 91 | 55 |

that regression model (1) which is:

$$
\begin{equation*}
Y_{i}=\beta_{0}+\beta_{1} X_{i 1}+\beta X_{i 2}+\epsilon_{i} \tag{1}
\end{equation*}
$$

with independent normal error terms is appropriate. Using matrix methods, obtain (a) $\mathbf{b}$; (b) $\mathbf{e} ;(\mathrm{c}) \mathbf{H}$; (d) SSR; (e) $s^{2}\{\mathbf{b}\} ;(\mathrm{f}) \hat{Y}_{h}$ when $X_{h 1}=10, X_{h 2}=30 ;(\mathrm{g}) s^{2}\left\{\hat{Y}_{h}\right\}$ when $X_{h 1}=10$, $X_{h 2}=30$. For the notations, please refer to section 6.4.
3. (40 points) Consider the classical regression setup

$$
\mathbf{y}=\mathbf{X} \beta+\epsilon
$$

We want to find the maximum likelihood estimate of the parameters.
a. if $\epsilon \sim \mathbf{N}\left(\mathbf{0}, \sigma^{\mathbf{2}} \mathbf{I}\right)$. Derive the maximum likelihood estimate of $\beta$ and $\sigma^{2}$.
b. if $\epsilon \sim \mathbf{N}(\mathbf{0}, \boldsymbol{\Sigma})$ and $\Sigma$ is known. Derive the maximum likelihood estimate of $\beta$.

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[^0]:    ${ }^{1}$ This is problem 4.22 in 'Applied Linear Regression Models(4th edition)' by Kutner etc.
    ${ }^{3}$ This is problem 6.27 in 'Applied Linear Regression Models(4th edition)' by Kutner etc.

