LINEAR REGRESSION MODELS W4315 HOMEWORK 5 QUESTIONS November 4, 2010

Instructor: Frank Wood

1. (20 points) ¹ Bonferroni inequality (4.2a) which is given as

 $P(\bar{A}_1 \cap \bar{A}_2) \ge 1 - \alpha - \alpha = 1 - 2\alpha$

deals with the case of two statements, A_1 and A_2 . Extend the inequality to the case of n statements, namely, A_1, A_2, \ldots, A_n , each with statement confidence coefficient $1 - \alpha$.

2. (40 points)³ In a small-scale regression study, the following data were obtained: Assume

i:	1	2	3	4	5	6
X_{i1}	7	4	16	3	21	8
X_{i2}	33	41	$\overline{7}$	49	5	31
Y_i	42	33	75	28	91	55

that regression model (1) which is:

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta X_{i2} + \epsilon_i \tag{1}$$

with independent normal error terms is appropriate. Using matrix methods, obtain (a) **b**; (b) **e**; (c) **H**; (d) SSR; (e) s^2 {**b**}; (f) \hat{Y}_h when $X_{h1} = 10$, $X_{h2} = 30$; (g) s^2 { \hat{Y}_h } when $X_{h1} = 10$, $X_{h2} = 30$. For the notations, please refer to section 6.4.

3. (40 points) Consider the classical regression setup

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$$

We want to find the maximum likelihood estimate of the parameters. a. if $\epsilon \sim \mathbf{N}(\mathbf{0}, \sigma^2 \mathbf{I})$. Derive the maximum likelihood estimate of β and σ^2 . b. if $\epsilon \sim \mathbf{N}(\mathbf{0}, \boldsymbol{\Sigma})$ and $\boldsymbol{\Sigma}$ is known. Derive the maximum likelihood estimate of β .

¹This is problem 4.22 in 'Applied Linear Regression Models(4th edition)' by Kutner etc.

³This is problem 6.27 in 'Applied Linear Regression Models(4th edition)' by Kutner etc.