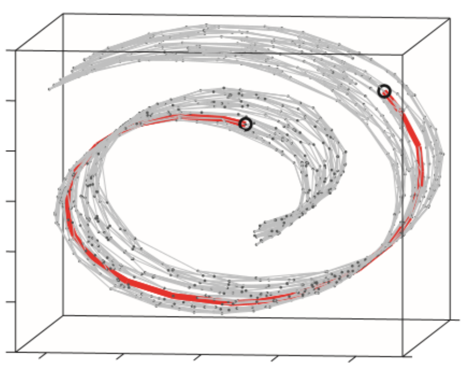
CPSC 340: Machine Learning and Data Mining

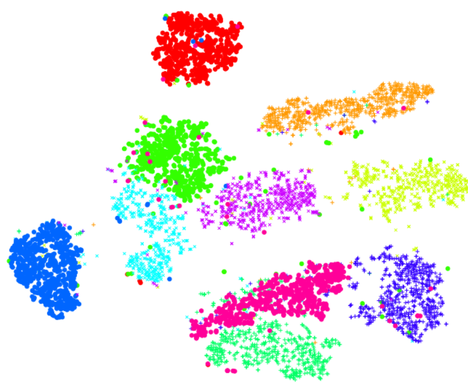
Deep Learning

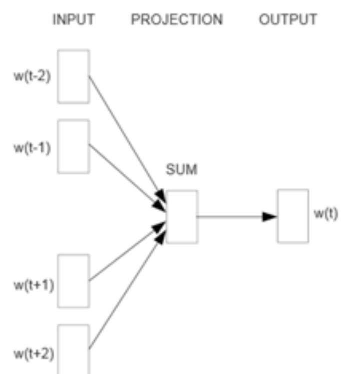
Fall 2020

Last Time: Multi-Dimensional Scaling

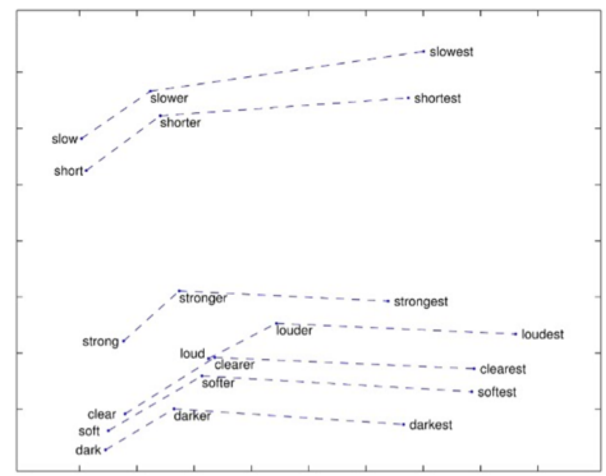
- Modern multi-dimensional scaling (MDS) methods:
  - ISOMAP uses geodesic distance in data manifold.
  - T-SNE tends to reveal clusters and manifold structures.
  - Word2vec gives continuous alternative to bag of words.







CBOW



http://wearables.cc.gatech.edu/paper_of_week/isomap.pdf
http://lvdmaaten.github.io/publications/papers/JMLR_2008.pdf
http://sebastianruder.com/secret-word2vec
http://sebastianruder.com/secret-word2vec

Word2Vec

- Subtracting word vectors to find related vectors.

Table 8: *Examples of the word pair relationships, using the best word vectors from Table 4 (Skip-gram model trained on 783M words with 300 dimensionality).*

Relationship	Example 1	Example 2	Example 3
France - Paris	Italy: Rome	Japan: Tokyo	Florida: Tallahassee
big - bigger	small: larger	cold: colder	quick: quicker
Miami - Florida	Baltimore: Maryland	Dallas: Texas	Kona: Hawaii
Einstein - scientist	Messi: midfielder	Mozart: violinist	Picasso: painter
Sarkozy - France	Berlusconi: Italy	Merkel: Germany	Koizumi: Japan
copper - Cu	zinc: Zn	gold: Au	uranium: plutonium
Berlusconi - Silvio	Sarkozy: Nicolas	Putin: Medvedev	Obama: Barack
Microsoft - Windows	Google: Android	IBM: Linux	Apple: iPhone
Microsoft - Ballmer	Google: Yahoo	IBM: McNealy	Apple: Jobs
Japan - sushi	Germany: bratwurst	France: tapas	USA: pizza

Table 8 shows words that follow various relationships. We follow the approach described above: the relationship is defined by subtracting two word vectors, and the result is added to another word. Thus for example, *Paris - France + Italy = Rome*. As it can be seen, accuracy is quite good, although

- Word vectors for 157 languages [here](#).

https://arxiv.org/pdf/1301.3781.pdf

End of Part 4: Key Concepts

- We discussed linear latent-factor models:

$$f(W,Z) = \sum_{i=1}^{n}\sum_{j=1}^{d}(\langle w^j, z_i\rangle - x_{ij})^2$$
$$= \sum_{i=1}^{n} \|W^T z_i - x_i\|^2$$
$$= \|ZW - X\|_F^2$$

- Represent 'X' as linear combination of latent factors 'w_c'.
  - Latent features 'z_i' give a lower-dimensional version of each 'x_i'.
  - When k=1, finds direction that minimizes squared orthogonal distance.
- Applications:
  - Outlier detection, dimensionality reduction, data compression, features for linear models, visualization, factor discovery, filling in missing entries.

End of Part 4: Key Concepts

- We discussed linear latent-factor models:

$$f(W,Z) = \sum_{i=1}^{n} \sum_{j=1}^{d} (\langle w^j, z_i \rangle - x_{ij})^2$$

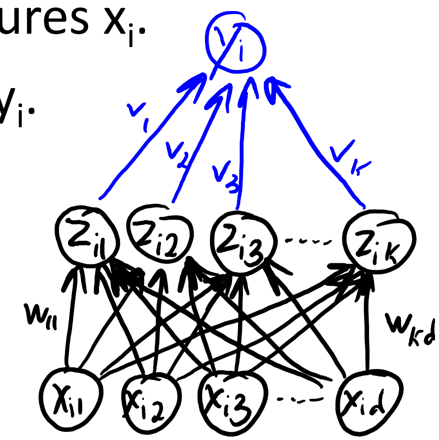
- Principal component analysis (PCA):
  - Often uses orthogonal factors and fits them sequentially (via SVD).
- Non-negative matrix factorization:
  - Uses non-negative factors giving sparsity.
  - Can be minimized with projected gradient.
- Many variations are possible:
  - Different regularizers (sparse coding) or loss functions (robust/binary PCA).
  - Missing values (recommender systems) or change of basis (kernel PCA).

End of Part 4: Key Concepts

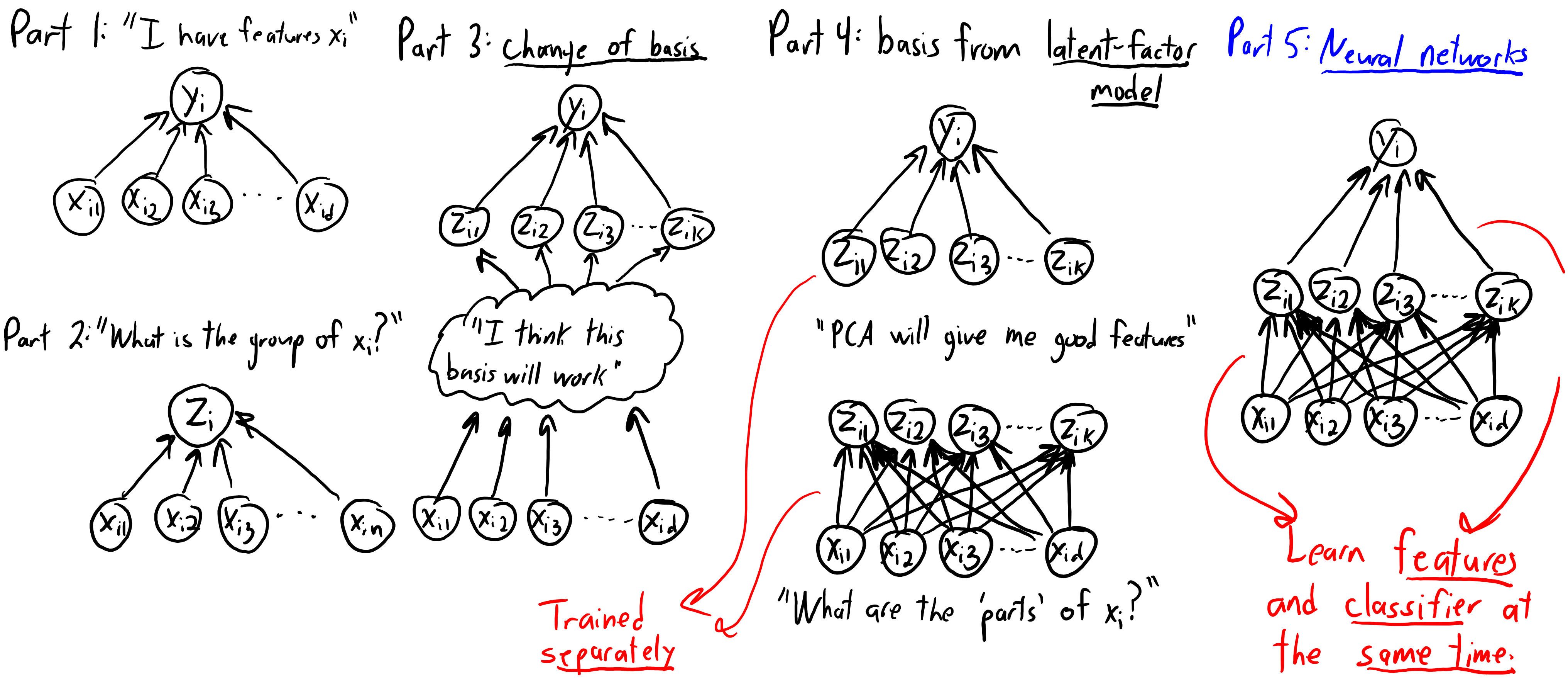
- We discussed multi-dimensional scaling (MDS):
  - Non-parametric method for high-dimensional data visualization.
  - Tries to match distance/similarity in high-/low-dimensions.
    - “Gradient descent on scatterplot points”.
- Main challenge in MDS methods is “crowding” effect:
  - Methods focus on large distances and lose local structure.
- Common solutions:
  - Sammon mapping: use weighted cost function.
  - ISOMAP: approximate geodesic distance using via shortest paths in graph.
  - T-SNE: give up on large distances and focus on neighbour distances.
- Word2vec is a recent MDS method giving better “word features”.

Supervised Learning Roadmap

- Part 1: "Direct" Supervised Learning.
  - We learned parameters 'w' based on the original features x_i and target y_i.
- Part 3: Change of Basis.
  - We learned parameters 'v' based on a change of basis z_i and target y_i.
- Part 4: Latent-Factor Models.
  - We learned parameters 'W' for basis z_i based on only on features x_i.
  - You can then learn 'v' based on change of basis z_i and target y_i.
- Part 5: Neural Networks.
  - Jointly learn 'W' and 'v' based on x_i and y_i.
  - **Learn basis z_i that is good for supervised learning.**



A Graphical Summary of CPSC 340 Parts 1-5



Notation for Neural Networks

We have our usual supervised learning notation:

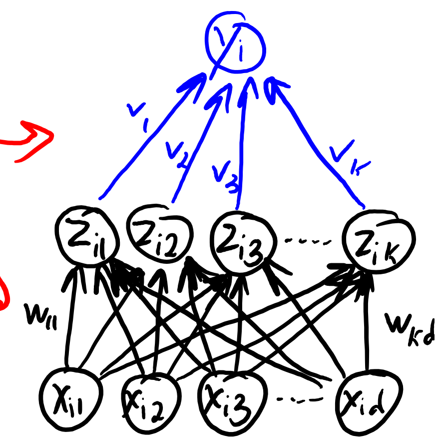
$$X = \begin{bmatrix} — & x_1^T & — \\ — & x_2^T & — \\ & \vdots & \\ — & x_n & — \end{bmatrix}_{n \times d} \qquad y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}_{n \times 1}$$

We have our latent features:

$$Z = \begin{bmatrix} — & z_1^T & — \\ — & z_2^T & — \\ & \vdots & \\ — & z_n^T & — \end{bmatrix}_{n \times k}$$

We have two sets of parameters:

$$v = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_k \end{bmatrix}_{k \times 1} \qquad W = \begin{bmatrix} — & w_1 & — \\ — & w_2 & — \\ & \vdots & \\ — & w_k & — \end{bmatrix}_{k \times d}$$



Linear-Linear Model

- Obvious choice: linear latent-factor model with linear regression.

Use features from latent-factor model: $z_i = Wx_i$

Make predictions using a linear model: $y_i = v^T z_i$

- We want to train 'W' and 'v' jointly, so we could minimize:

$$f(W, v) = \frac{1}{2}\sum_{i=1}^{n}(\underbrace{v^T z_i - y_i}_{\text{linear regression with } z_i \text{ as features}})^2 = \frac{1}{2}\sum_{i=1}^{n}(v^T(\underbrace{Wx_i}_{z_i \text{ come from latent-factor model}}) - y_i)^2$$

- But this is just a linear model: $\hat{y}_i = v^T z_i = v^T(Wx_i) = \underbrace{(\overbrace{v^T W}^{1 \times d})}_{\text{some vector 'w'}} x_i = w^T x_i$

Introducing Non-Linearity

- To increase flexibility, something needs to be non-linear.
- Typical choice: transform z_i by non-linear function 'h'.

$$z_i = Wx_i \qquad y_i = v^T h(z_i)$$

  - Here the function 'h' transforms 'k' inputs to 'k' outputs.
- Common choice for 'h': applying sigmoid function element-wise:

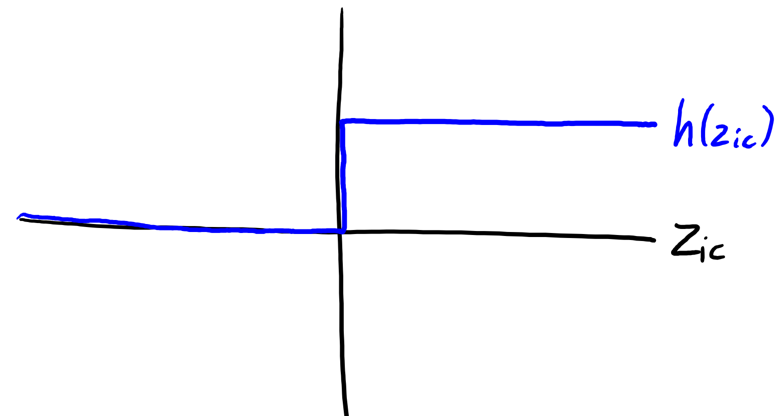
$$h(z_{ic}) = \frac{1}{1 + \exp(-z_{ic})}$$

- So this takes the z_{ic} in $(-\infty,\infty)$ and maps it to $(0,1)$.
- This is called a "multi-layer perceptron" or a "neural network".

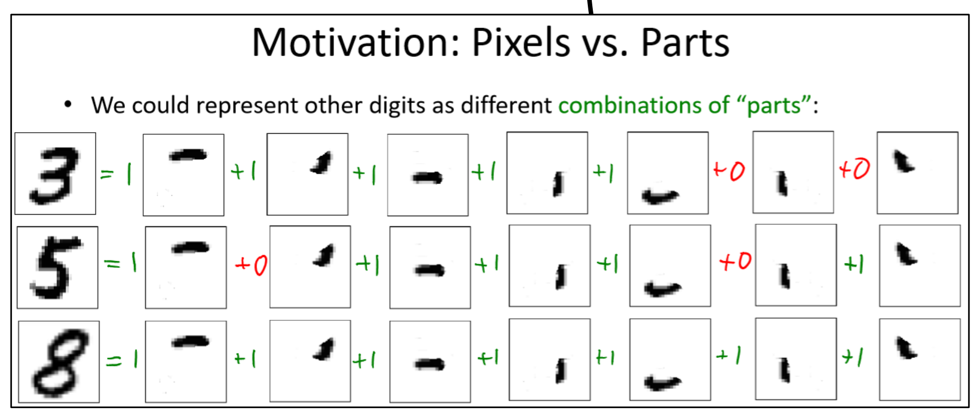
Why Sigmoid?

- Consider setting 'h' to define binary features z_i using:

$$h(z_{ic}) = \begin{cases} 1 & \text{if } z_{ic} \geq 0 \\ 0 & \text{if } z_{ic} < 0 \end{cases}$$



  - Each h(zi) can be viewed as binary feature.
    - "You either have this 'part' or you don't have it."
  - We can make 2^k objects by all the possible "part combinations".



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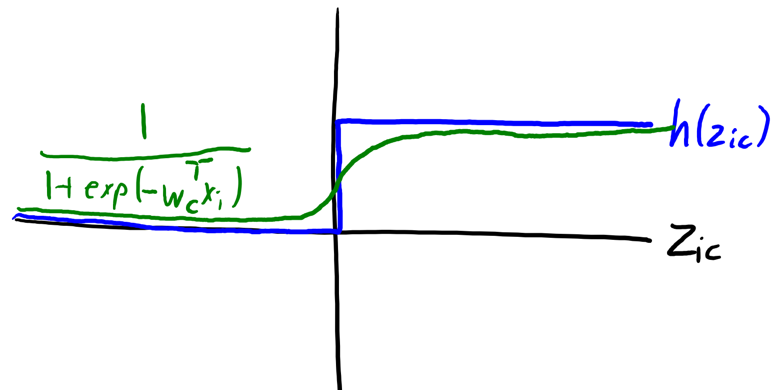
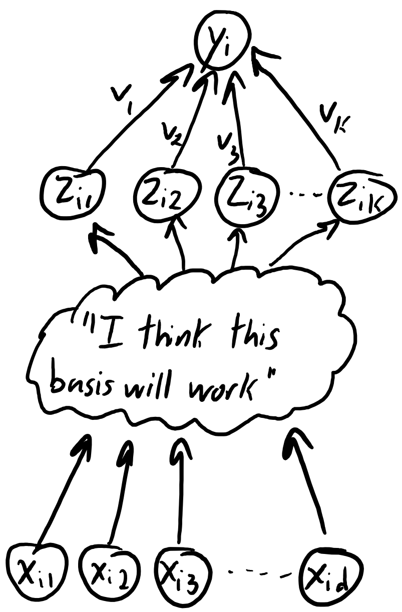


  – Each h(zi) can be viewed as binary feature.
    - "You either have this 'part' or you don't have it."
  – But this is hard to optimize (non-differentiable/discontinuous).
- Sigmoid is a smooth approximation to these binary features.
  – Non-parametric version is a universal approximator:
    - If 'k' grows appropriately with 'n', can model any continuous function.

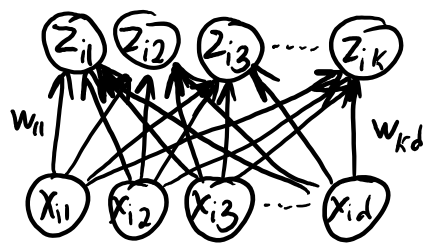
Supervised Learning Roadmap

Hand-engineered features:

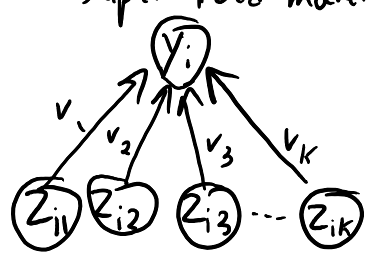


Requires domain knowledge and can be time-consuming

Learn a latent-factor model:

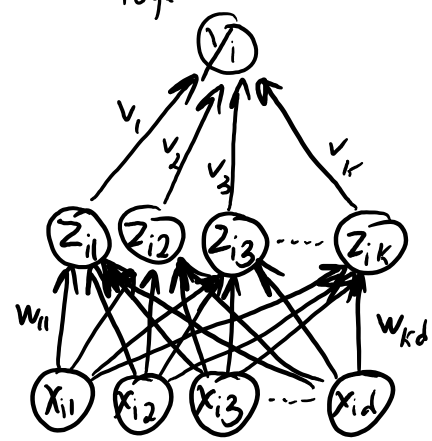


Use latent features in supervised model:



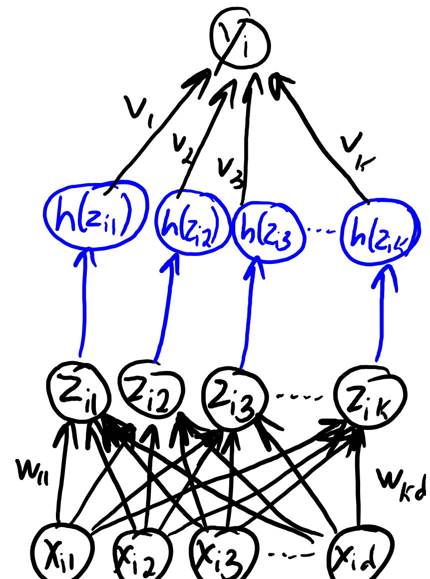
Good representation of x_i might be bad for predicting y_i

Learn 'w' and 'W' together:



But still gives a linear model.

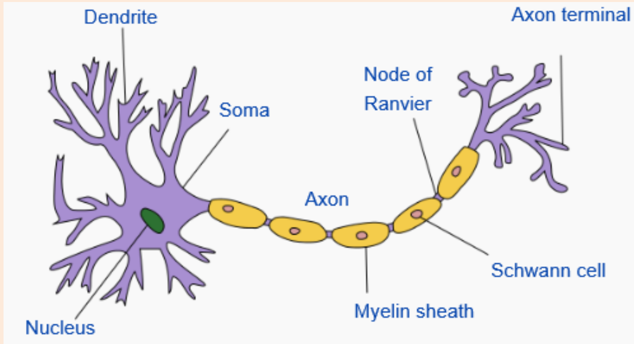
Neural network:



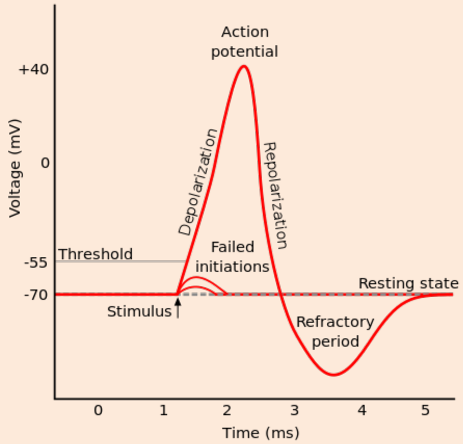
Extra non-linear transformation 'h'.

Why “Neural Network”?

- Cartoon of “typical” neuron:

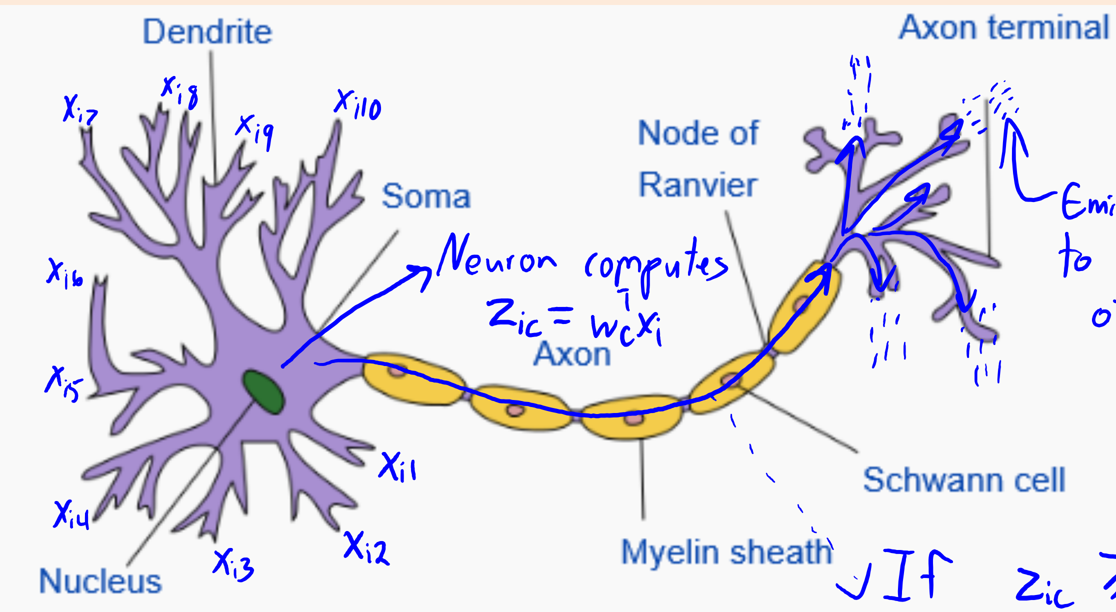


- Neuron has many “dendrites”, which take an input signal.
- Neuron has a single “axon”, which sends an output signal.
- With the right input to dendrites:
  - “Action potential” along axon (like a binary signal):



https://en.wikipedia.org/wiki/Neuron
https://en.wikipedia.org/wiki/Action_potential

Why "Neural Network"?



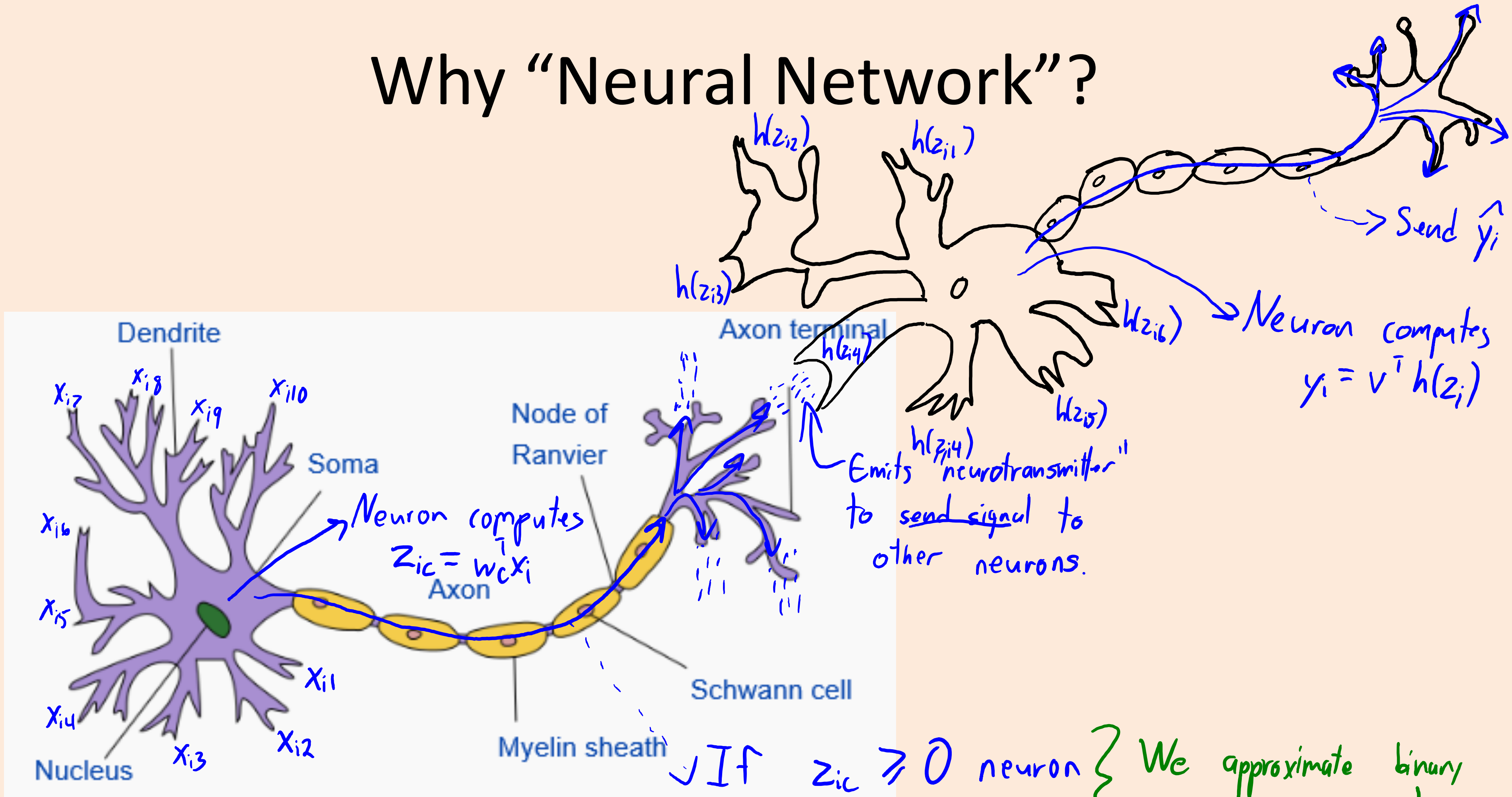
Emits "neurotransmitter" to send signal to other neurons.

If $z_{ic} \geq 0$ neuron sends signal along axon.

We approximate binary signal with $\frac{1}{1+\exp(-z_{ic})}$

https://en.wikipedia.org/wiki/Neuron

Why "Neural Network"?



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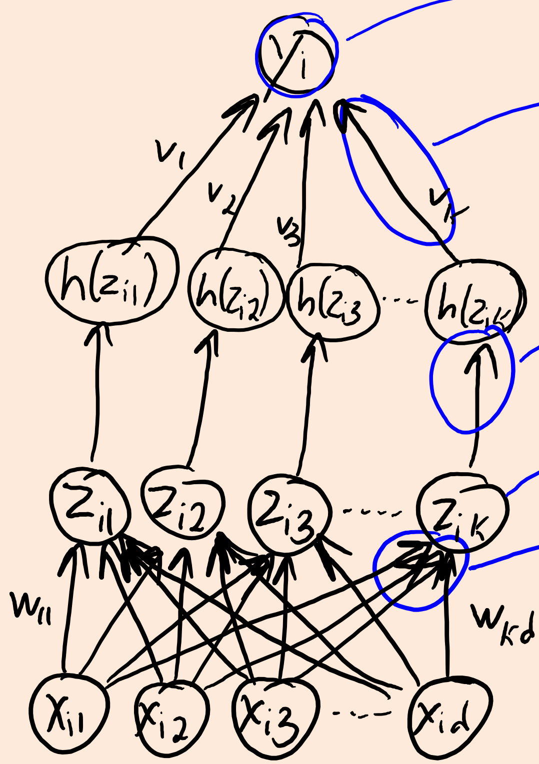
Why "Neural Network"?



Predictions based on aggregation $v^T h(Wx_i)$ at y_i "neuron"

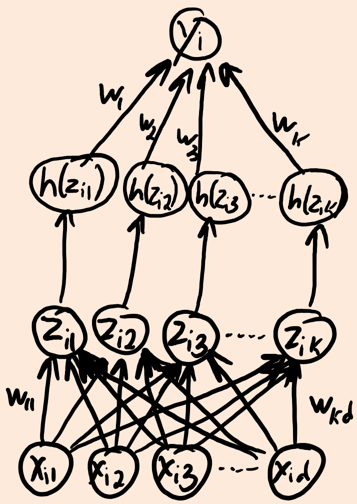
Synapse between z_{ik} and y_i "neuron"

binary signal $h(w_c^T x_i)$ sent along "axon"

Neuron aggregates signals: $w_c^T x_i$

"dendrites" for z_{ik} "neuron" are reciving x_{ij} values

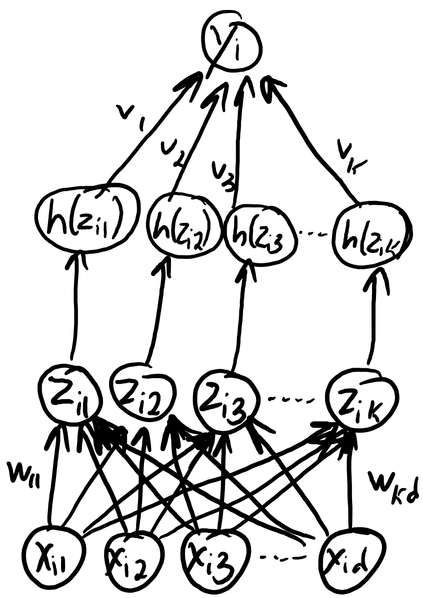
"Artificial" Neural Nets vs. "Real" Networks Nets



- Artificial neural network:
  - x_i is measurement of the world.
  - z_i is internal representation of world.
  - y_i is output of neuron for classification/regression.
- Real neural networks are more complicated:
  - Timing of action potentials seems to be important.
    - "Rate coding": frequency of action potentials simulates continuous output.
  - Neural networks don't reflect sparsity of action potentials.
  - How much computation is done inside neuron?
  - Brain is highly organized (e.g., substructures and cortical columns).
  - Connection structure changes.
  - Different types of neurotransmitters.

Deep Learning

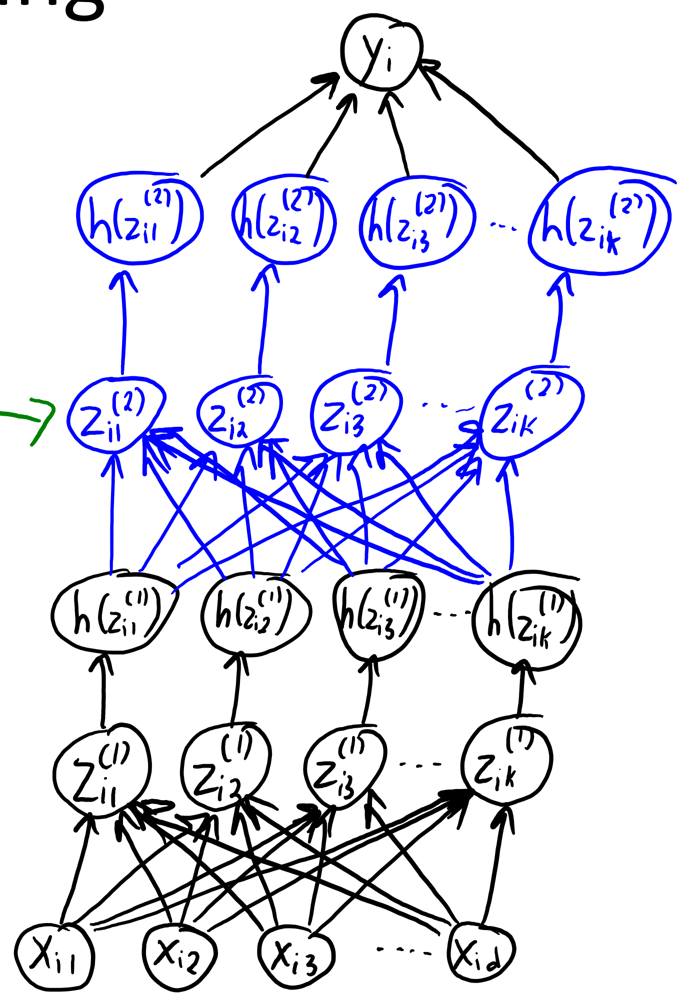
Neural network:



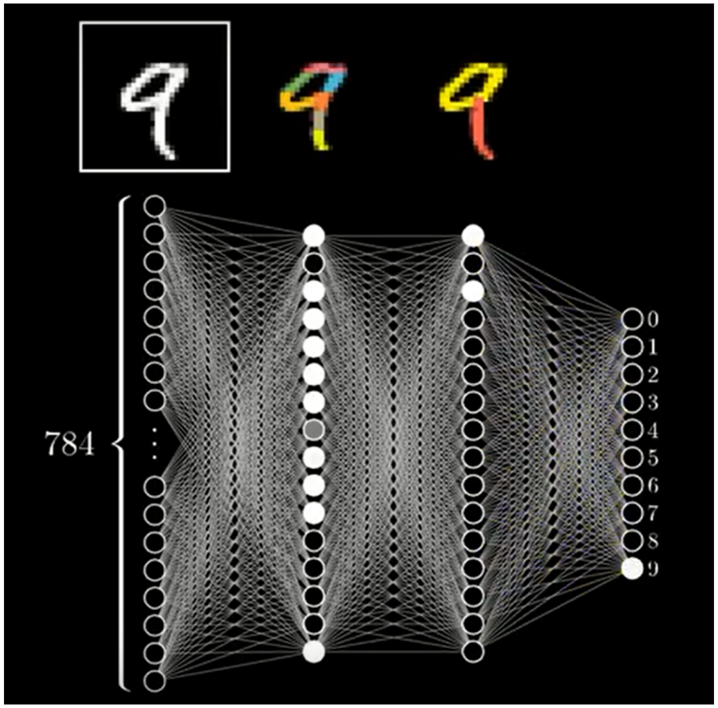
Deep learning:

Second "layer" of latent features

You can add more "layers" to go "deeper"



“Hierarchies of Parts” Motivation for Deep Learning



- Each “neuron” might recognize a “part” of a digit.
  - “Deeper” neurons might recognize combinations of parts.
  - Represent complex objects as hierarchical combinations of re-useable parts (a simple “grammar”).
- Watch the full video here:
  - https://www.youtube.com/watch?v=aircAruvnKk
- Theory:
  - 1 big-enough hidden layer already gives universal approximation.
  - But some functions require exponentially-fewer parameters to approximate with more layers (can fight curse of dimensionality).

Deep Learning

Linear model:

$$\hat{y}_i = w^T x_i$$

Neural network with 1 hidden layer:

$$\hat{y}_i = v^T h(\underbrace{W x_i}_{z_i})$$

Neural network with 2 hidden layers:

$$\hat{y}_i = v^T h(\underbrace{W^{(2)} h(\underbrace{W^{(1)} x_i}_{z_i^{(1)}})}_{z_i^{(2)}})$$

Neural network with 3 hidden layers

$$\hat{y}_i = v^T h(\underbrace{W^{(3)} h(\underbrace{W^{(2)} h(\underbrace{W^{(1)} x_i}_{z_i^{(1)}})}_{z_i^{(2)}})}_{z_i^{(3)}})$$

Deep learning:

Second "layer" of latent features

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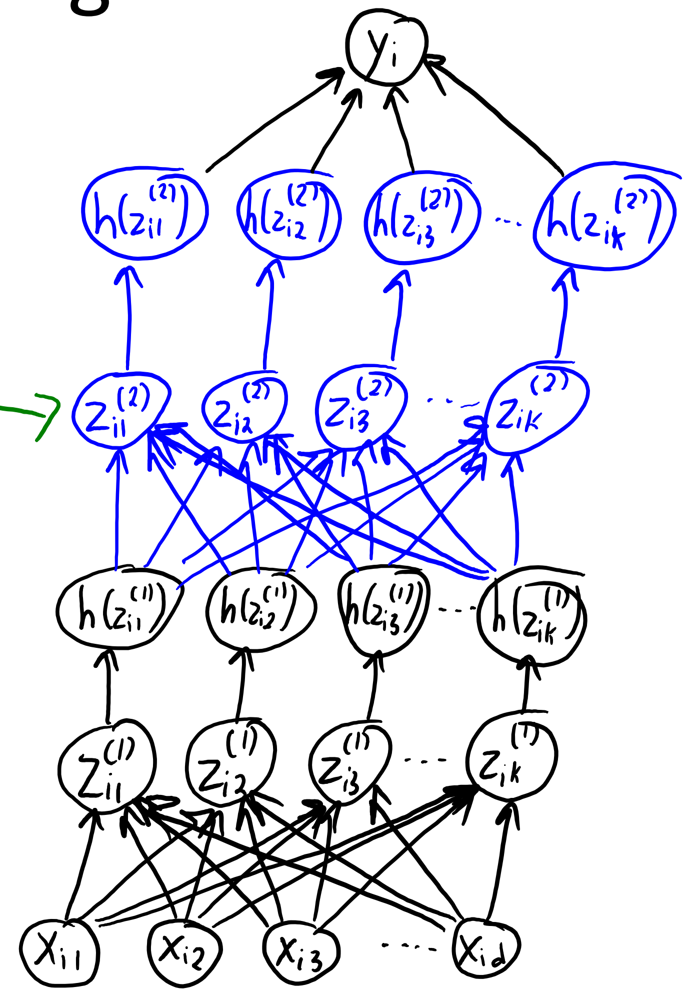
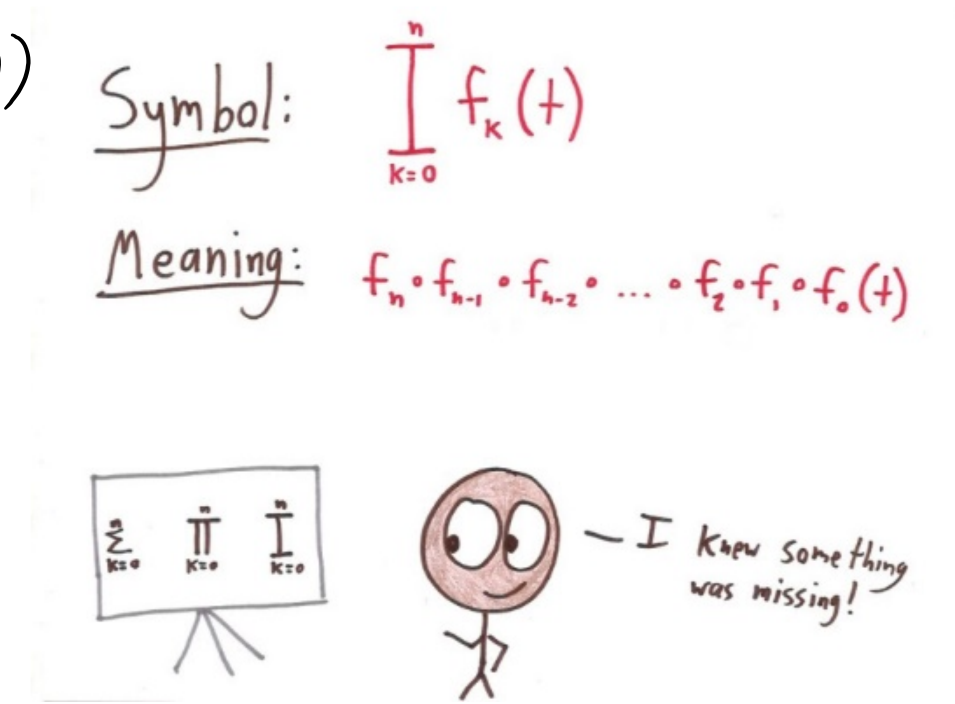


Deep Learning

- For 4 layers, we could write the prediction as:

$$\hat{y}_i = v^T h(W^{(4)} h(W^{(3)} h(W^{(2)} h(W^{(1)} x_i))))$$



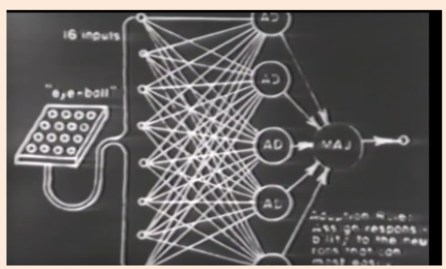
- For ‘m’ layers, we could use:

$$\hat{y}_i = v^T \left(\coprod_{\ell=1}^{m} h(W^{(\ell)} x_i) \right)$$

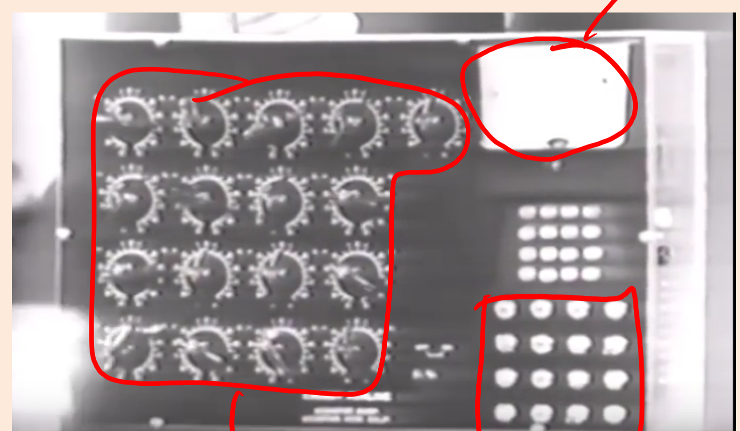
https://mathwithbaddrawings.com/2016/04/27/symbols-that-math-urgently-needs-to-adopt

ML and Deep Learning History

- 1950 and 1960s: Initial excitement.
  - Perceptron: linear classifier and stochastic gradient (roughly).
  - "the embryo of an electronic computer that [the Navy] expects will be able to walk, talk, see, write, reproduce itself and be conscious of its existence." New York Times (1958).
    - https://www.youtube.com/watch?v=IEFRtz68m-8
  - Object recognition assigned to students as a summer project



$w^T x_i$



W

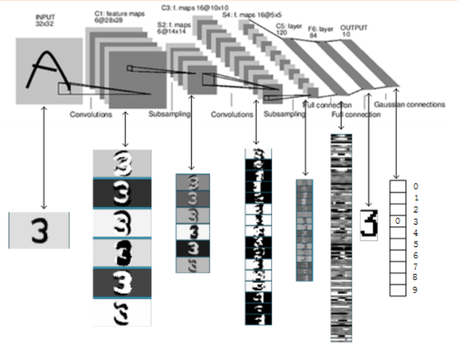
x_i

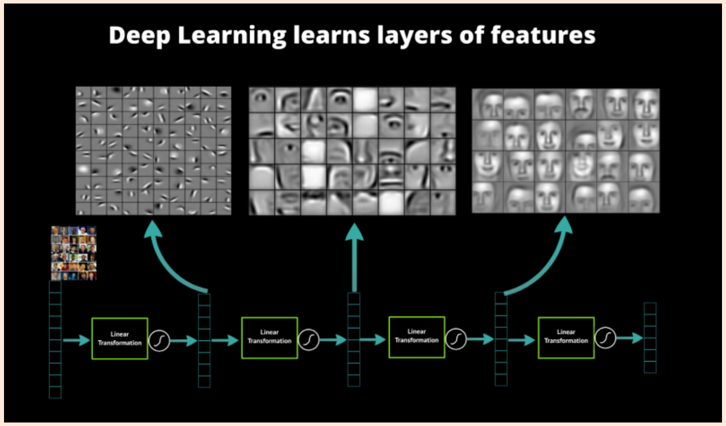
- Then drop in popularity:
  - Quickly realized limitations of linear models.

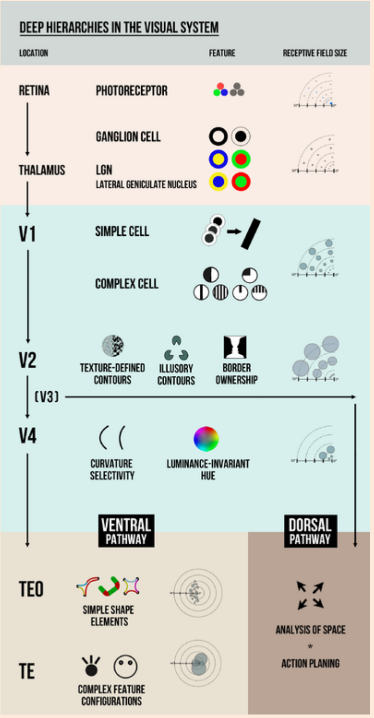
https://mitpress.mit.edu/books/perceptrons/

ML and Deep Learning History

- 1970 and 1980s: Connectionism (brain-inspired ML)
  - Want "connected networks of simple units".
    - Use parallel computation and distributed representations.
  - Adding hidden layers z_i increases expressive power.
    - With 1 layer and enough sigmoid units, a universal approximator.
  - Success in optical character recognition.







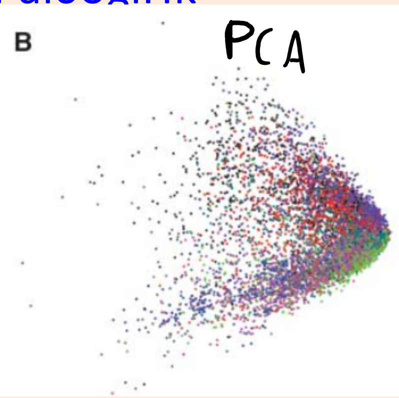
https://en.wikibooks.org/wiki/Sensory_Systems/Visual_Signal_Processing
http://www.datarobot.com/blog/a-primer-on-deep-learning/
http://blog.csdn.net/strint/article/details/44163869

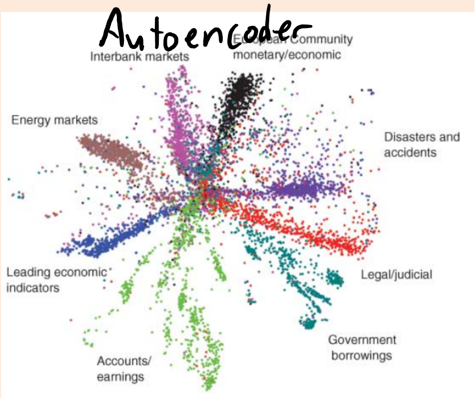
ML and Deep Learning History

- 1990s and early-2000s: drop in popularity.
  - It proved really difficult to get multi-layer models working robustly.
  - We obtained similar performance with simpler models:
    - Rise in popularity of logistic regression and SVMs with regularization and kernels.
  - Lots of internet successes (spam filtering, web search, recommendation).
  - ML moved closer to other fields like numerical optimization and statistics.

ML and Deep Learning History

- Late 2000s: push to revive connectionism as "deep learning".
  - Canadian Institute For Advanced Research (CIFAR) NCAP program:
    - "Neural Computation and Adaptive Perception".
    - Led by Geoff Hinton, Yann LeCun, and Yoshua Bengio ("Canadian mafia").
  - Unsupervised successes: "deep belief networks" and "autoencoders".
    - Could be used to initialize deep neural networks.
    - https://www.youtube.com/watch?v=KuPai0ogiHk

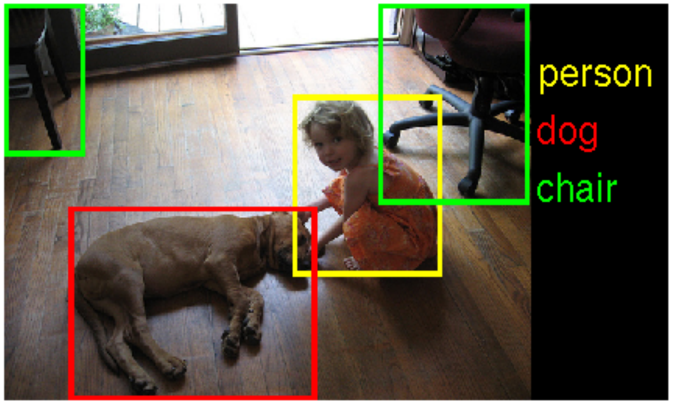




https://www.cs.toronto.edu/~hinton/science.pdf

2010s: DEEP LEARNING!!!

- Bigger datasets, bigger models, parallel computing (GPUs/clusters).
  - And some tweaks to the models from the 1980s.
- Huge improvements in automatic speech recognition (2009).
  - All phones now have deep learning.
- Huge improvements in computer vision (2012).
  - Changed computer vision field almost instantly.
  - This is now finding its way into products.



http://www.image-net.org/challenges/LSVRC/2014/

2010s: DEEP LEARNING!!!

- Media hype:
  - "How many computers to identify a cat? 16,000"
    New York Times (2012).
  - "Why Facebook is teaching its machines to think like humans"
    Wired (2013).
  - "What is 'deep learning' and why should businesses care?"
    Forbes (2013).
  - "Computer eyesight gets a lot more accurate"
    New York Times (2014).

- 2015: huge improvement in language understanding.

Summary

- Neural networks learn features z_i for supervised learning.
- Sigmoid function avoids degeneracy by introducing non-linearity.
  - Universal approximator with large-enough 'k'.
- Biological motivation for (deep) neural networks.
- Deep learning considers neural networks with many hidden layers.
  - Can more-efficiently represent some functions.
- Unprecedented performance on difficult pattern recognition tasks.

- Next time:
  - Training deep networks.

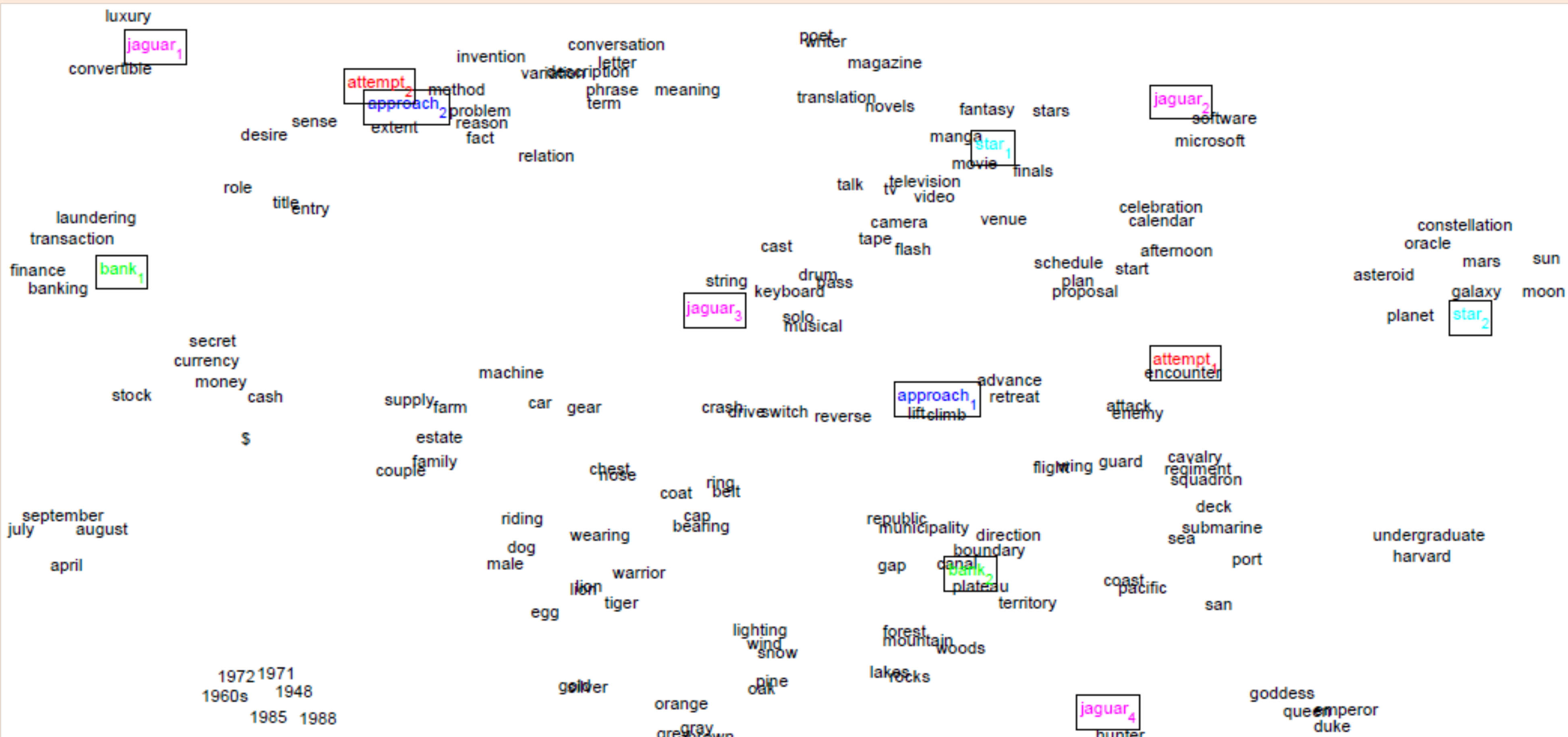
Multiple Word Prototypes

- What about homonyms and polysemy?
  - The word vectors would need to account for all meanings.

- More recent approaches:
  - Try to cluster the different contexts where words appear.
  - Use different vectors for different contexts.

$$X_{jaguar} = \begin{array}{cc} \begin{bmatrix} \cdots \\ \cdots \\ \end{bmatrix} & \begin{array}{c} z_{j1} \\ z_{j2} \\ z_{j3} \end{array} \end{array}$$

Multiple Word Prototypes



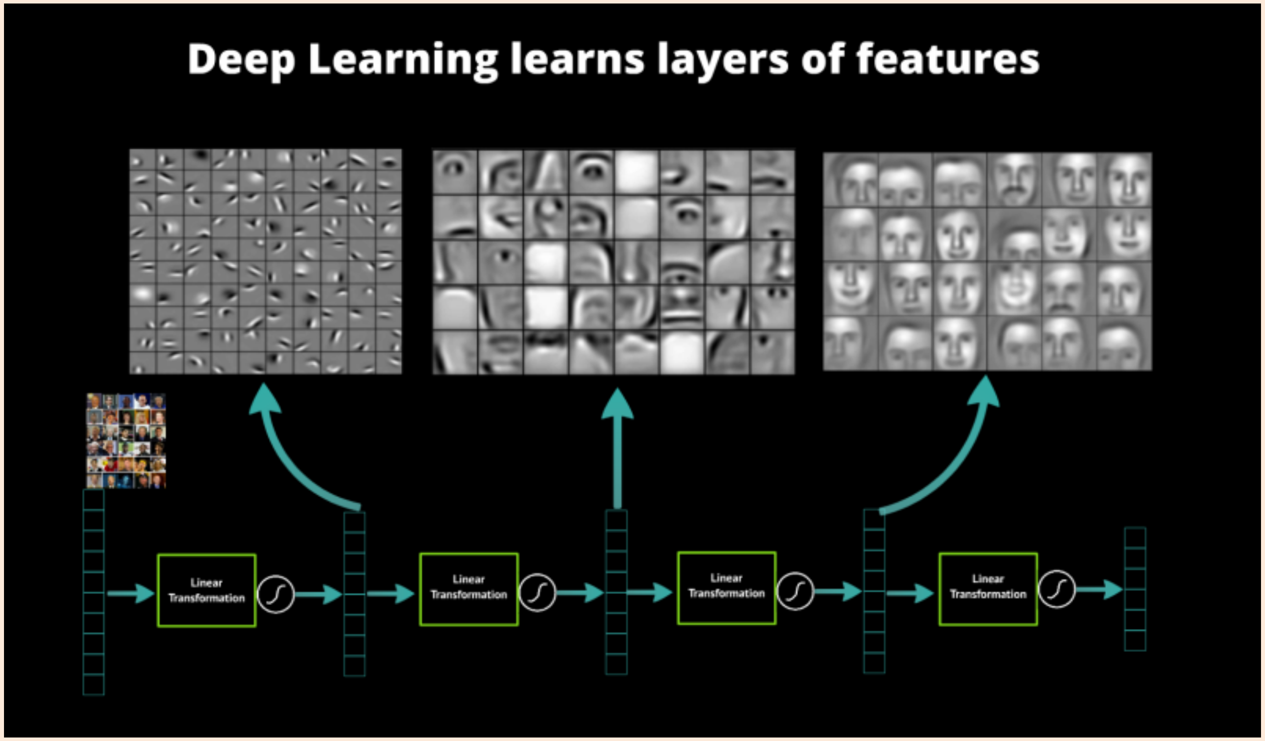
http://www.socher.org/index.php/Main/ImprovingWordRepresentationsViaGlobalContextAndMultipleWordPrototypes

Why $z_i = Wx_i$?

- In PCA we had that the optimal $Z = XW^T(WW^T)^{-1}$.
- If W had normalized+orthogonal rows, $Z = XW^T$ (since $WW^T = I$).
  - So $z_i = Wx_i$ in this normalized+orthogonal case.
- Why we would use $z_i = Wx_i$ in neural networks?
  - We didn't enforce normalization or orthogonality.
- Well, the value $W^T(WW^T)^{-1}$ is just "some matrix".
  - You can think of neural networks as just directly learning this matrix.

Cool Picture Motivation for Deep Learning

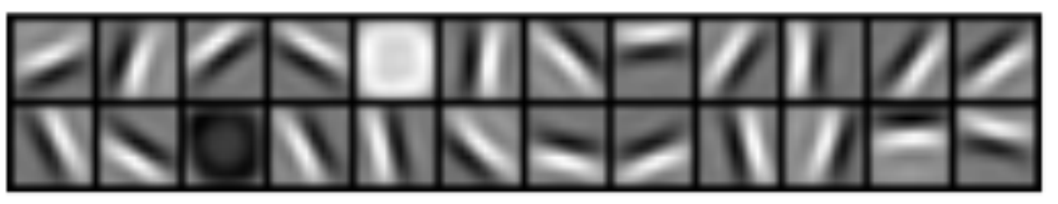
- Faces might be composed of different "parts":



http://www.datarobot.com/blog/a-primer-on-deep-learning/

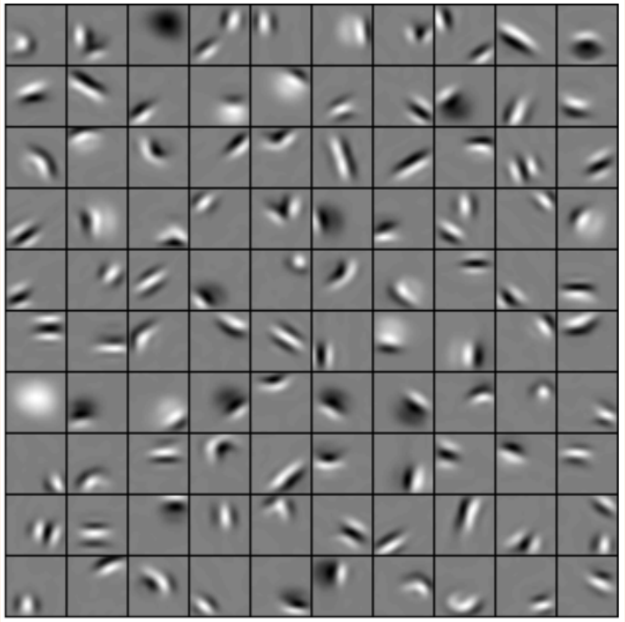
Cool Picture Motivation for Deep Learning

- First layer of z_i trained on 10 by 10 image patches:



"Gabor filters"

- Attempt to visualize second layer:
  - Corners, angles, surface boundaries?



- Models require many tricks to work.
  - We'll discuss these next time.

http://www.cs.toronto.edu/~rgrosse/icml09-cdbn.pdf

Cool Picture Motivation for Deep Learning

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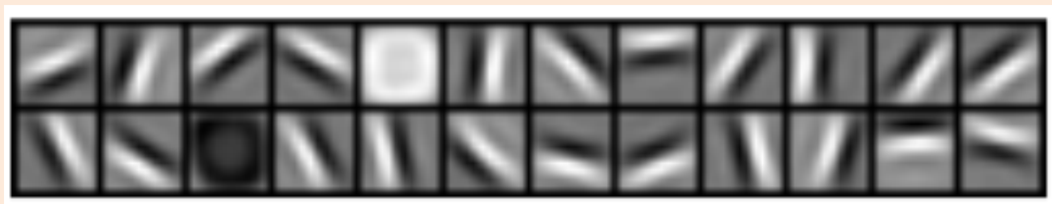
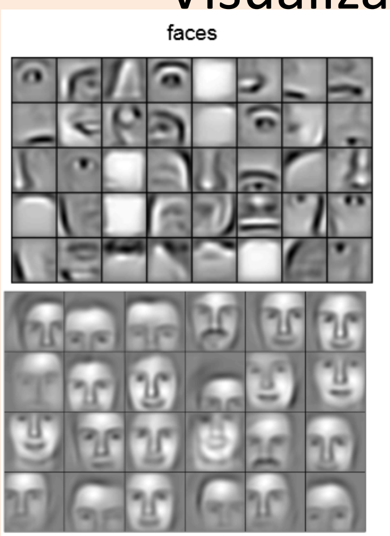


"Gabor filters"

- Visualization of second and third layers trained on specific objects:



http://www.cs.toronto.edu/~rgross

Cool Picture Motivation for Deep Learning

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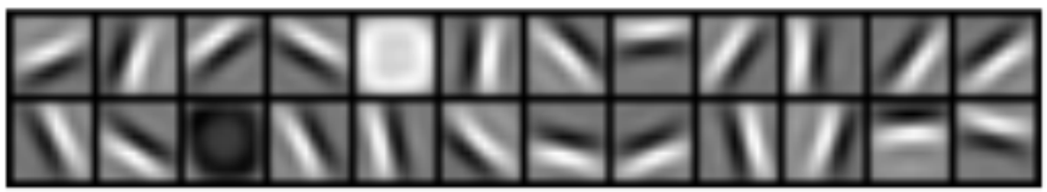

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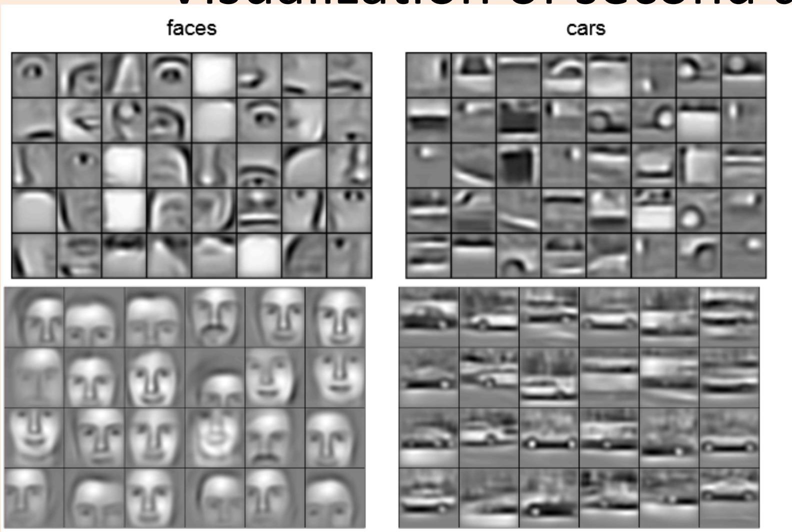

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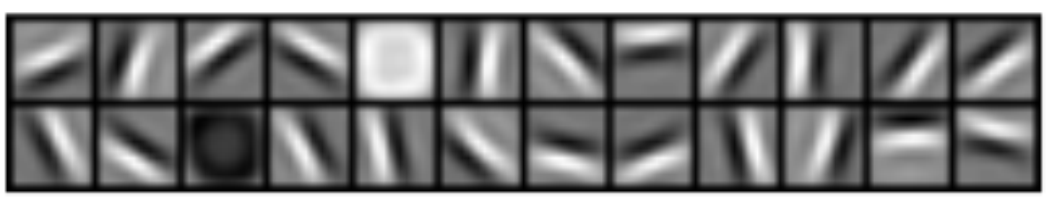

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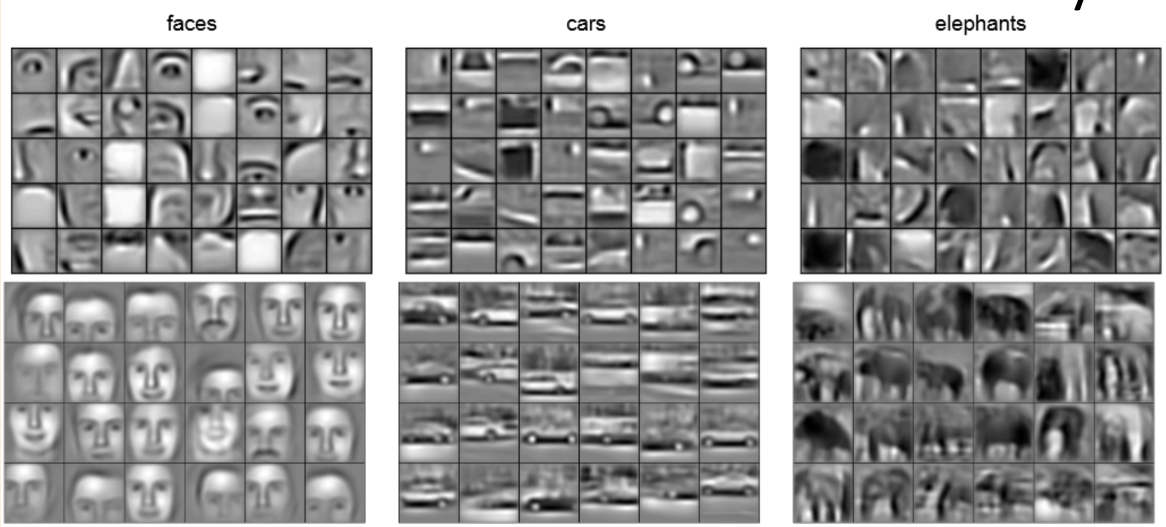

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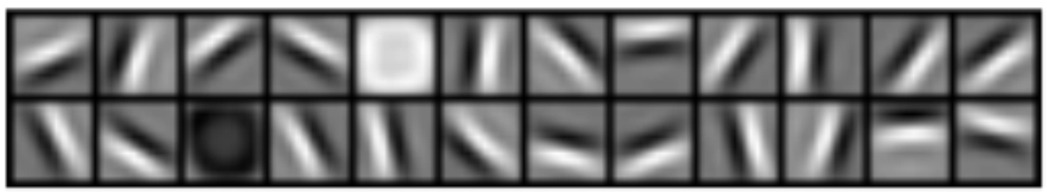

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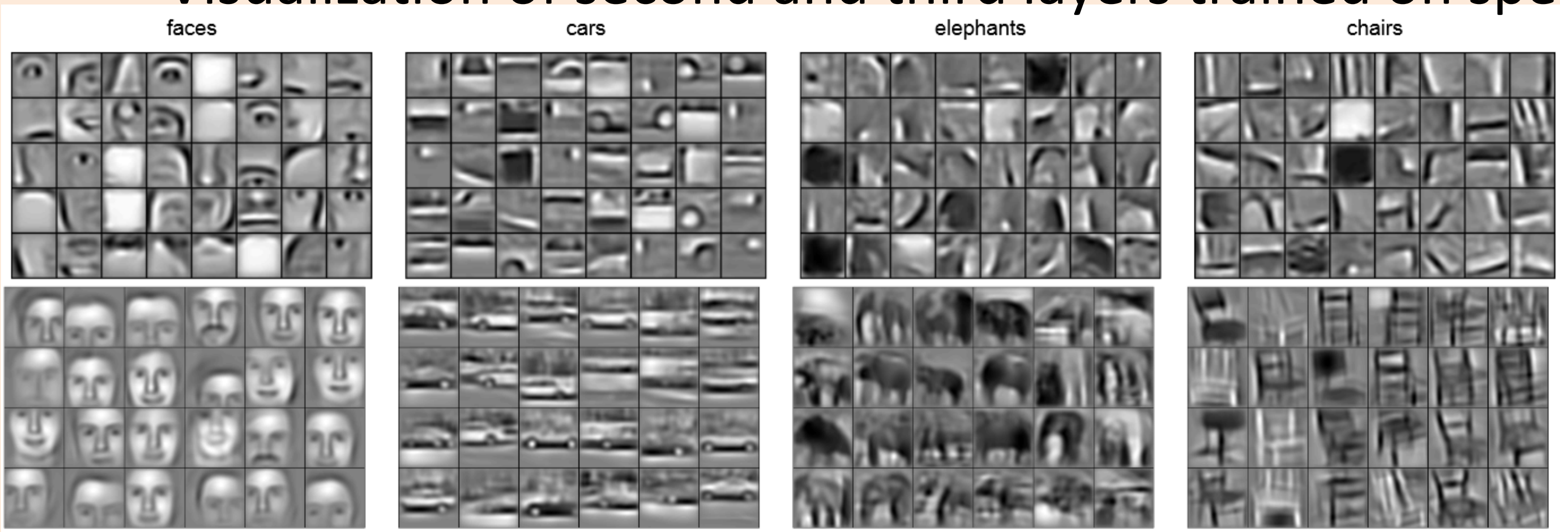

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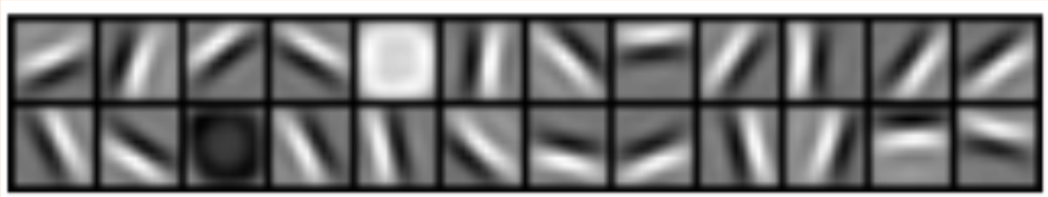

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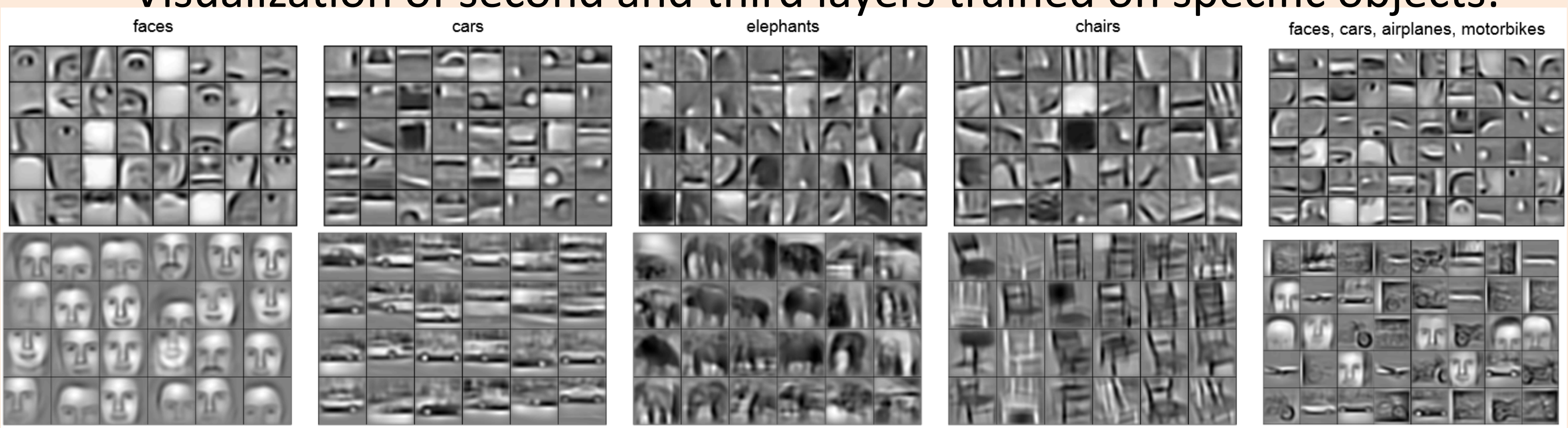

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