Formalizing Preferences Over Runtime Distributions

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Motivation

Which algorithm do you prefer?

Algorithm 1

Solves 99 problems in 1 second, runs the 100th problem for 10 days without solving.

Algorithm 2

Runs all 100 problems for 10 days each without solving any.

By what criteria?

(Average runtime cannot distinguish between these...)

Axiomatic Approach

Axioms:

- Von Neumann-Morgenstern axioms \cite{1}
- solving faster is better
- solving is better than running out of time

Theorem:

Given time budget $\kappa$, Algorithm A is preferred to Algorithm B if and only if

$$E[u(t_A, \kappa)] \geq E[u(t_B, \kappa)]$$

Utility Functions

$u(t, \kappa) :$ utility from solving in $t$ seconds when given $\kappa$ seconds

For fixed $\kappa$:

- Allowed:
  - Incorporate information about $\kappa$ using Method of Maximum Entropy \cite{2}

- Not allowed:
  - $\times$

It Matters

Algorithm Configuration \cite{3}:

<table>
<thead>
<tr>
<th>Method</th>
<th>Uniform(1)</th>
<th>Optimized utility function (Exp(0.1))</th>
<th>Pareto(1)</th>
<th>Laplace(0.1, 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform(1)</td>
<td>1.000</td>
<td>0.662</td>
<td>0.976</td>
<td>0.705</td>
</tr>
<tr>
<td>Exp(0.1)</td>
<td>0.732</td>
<td>1.000</td>
<td>0.691</td>
<td>0.995</td>
</tr>
<tr>
<td>Pareto(1)</td>
<td>0.998</td>
<td>0.835</td>
<td>1.000</td>
<td>0.856</td>
</tr>
<tr>
<td>Laplace(0.1, 1)</td>
<td>0.649</td>
<td>0.994</td>
<td>0.630</td>
<td>1.000</td>
</tr>
</tbody>
</table>

"Optimizing the right utility function gives a higher-quality solution."

International SAT Competition \cite{4}:

| Method            | Uniform(20) | Uniform(500) | Pareto(2) | Pareto(0.1) | Exp(0.1) |

"Different utility functions lead to different rankings."

\textsuperscript{1} Von Neumann, Morgenstern. Theory of Games and Economic Behaviour. 1947.
\textsuperscript{2} Jaynes. Information Theory and Statistical Mechanics. 1957.
\textsuperscript{4} 2021 International SAT Competition