Formalizing Preferences Over Runtime Distributions

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Motivation

Which algorithm do you prefer?

Algorithm 1

Solves 99 problems in 1 second, runs the 100th problem for 10 days without solving.

Algorithm 2

Runs all 100 problems for 10 days each without solving any.

By what criteria?

(Average runtime cannot distinguish between these...)

Axiomatic Approach

Axioms:

- Von Neumann-Morgenstern axioms [1]
- solving faster is better
- solving is better than running out of time

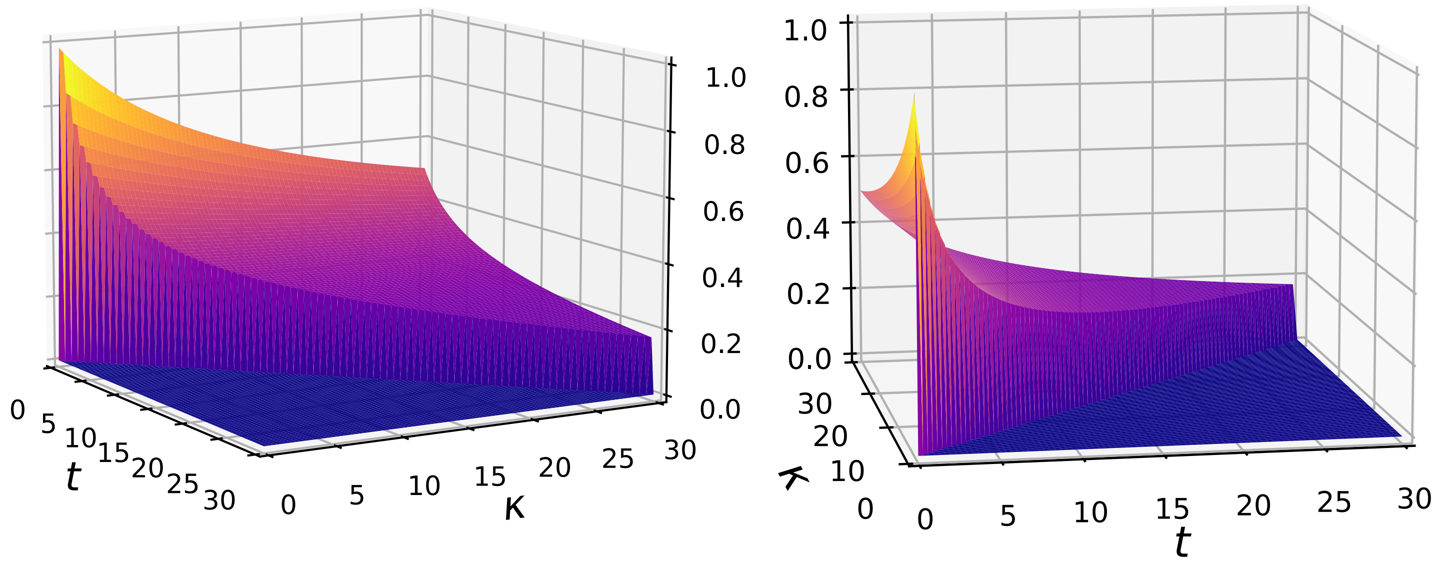
Theorem:

Given time budget κ,
Algorithm A is preferred to **Algorithm B**
if and only if

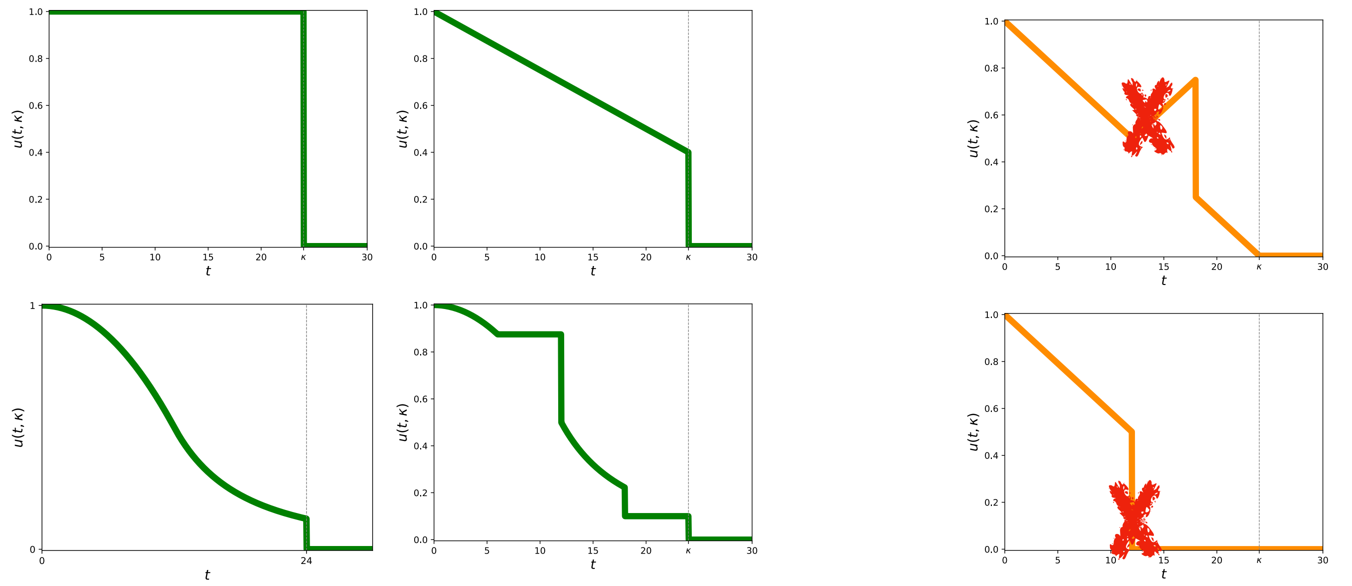
$$\mathbb{E}[u(t_A, \kappa)] \geq \mathbb{E}[u(t_B, \kappa)]$$

Utility Functions

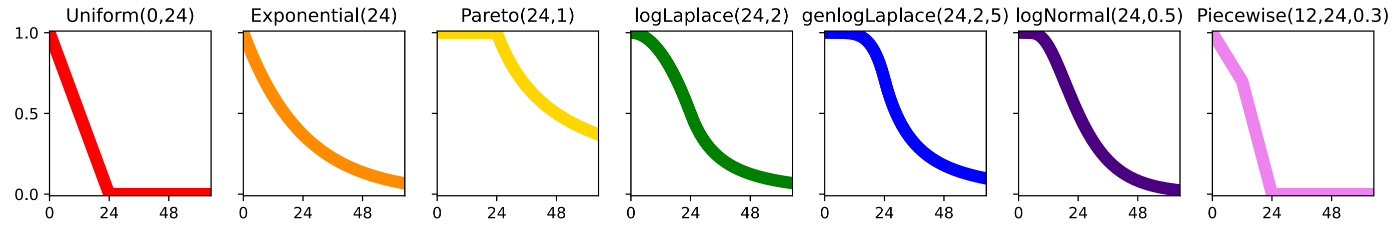
$u(t, \kappa)$: utility from solving in t seconds when given κ seconds



For fixed κ:



Incorporate information about κ using Method of Maximum Entropy [2]:



It Matters

Algorithm Configuration [3]:

True utility function \ Optimized utility function	Uniform(5)	Exp(0.1)	Pareto(1, 5)	LogLaplace(0.1, 5)
Uniform(5)	1.000	0.662	0.976	0.705
Exp(0.1)	0.732	1.000	0.691	0.995
Pareto(1, 5)	0.998	0.835	1.000	0.856
LogLaplace(0.1, 5)	0.649	0.994	0.630	1.000

"Optimizing the right utility function gives a higher-quality solution."

International SAT Competition [4]:

Ranking \ Utility Function	Uniform(20)	Uniform(500)	Pareto(5, 3)	Pareto(5, 1)	Exp(1)
1st	pakis	mallobparallel	pakis	mallobparallel	mallobparallel
2nd	mallobparallel	pmcomsps	mallobparallel	pakis	plingeling
3rd	pmcomspsstrsc	pakis	plingeling	pmcomspsstrsc	pakis
	pmcomsps	pmcomspscom	pmcomspsstrsc	pmcomsps	pmcomspscom
	pmcomspscom	pmcomspsstrsc	pmcomspscom	pmcomspscom	pmcomsps
	plingeling	mergehordesatparallel	pmcomsps	plingeling	painlessmaple
	mergehordesatparallel	painlessmaple	painlessmaple	mergehordesatparallel	pmcomspsstrsc
	painlessmaple	plingeling	mergehordesatparallel	painlessmaple	mergehordesatparallel
	abcdparascavel	abcdparascavel	abcdparascavel	abcdparascavel	abcdparascavel

"Different utility functions lead to different rankings."

[1] Von Neumann, Morgenstern. Theory of Games and Economic Behaviour. 1947.
[2] Jaynes. Information Theory and Statistical Mechanics. 1957.
[3] Weisz, György, Szepesvári. LeapsAndBounds: A Method for Approximately Optimal Algorithm Configuration. 2018.
[4] 2021 International SAT Competition