CS 554m

controlled experiments I

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learning goals

be able to answer the following:

what is the experimental method?
what is an experimental hypothesis?
how do I plan an experiment?
why are statistics used?
within- & between-subject comparisons: how do they differ?
how do I compute a t-test?
what are the different types of t-tests?

Acknowledgement: Some of the material in this lecture is based on material prepared for similar courses by Saul Greenberg (University of Calgary)

a good portion of the material in these lectures on experimental design should be familiar from ugrad stats class, although perhaps presented here from a slightly different perspective

also, most of this material is well covered in today's readings:

Newman & Lamming, Ch 10

Lazar, Feng, & Hochheiser, Ch 2 - 4

Who has run an experiment?

material I assume you already know and will not be covered (some additional slides at end)

types of variables

samples & populations

normal distribution

variance and standard deviation

quantitative methods

1. user performance data collection
   - data is collected on system use

descriptive statistics:

– frequency of request for on-line assistance
  – what did people ask for help with?

– frequency of use of different parts of the system
  – why are parts of system unused?

– number of errors and where they occurred
  – why does an error occur repeatedly?

– time it takes to complete some operation
  – what tasks take longer than expected?

- collect heaps of data in the hope that something interesting shows up
- often difficult to sift through data unless specific aspects are targeted (as in list above)

quantitative methods

2. controlled experiments

the traditional scientific method

- reductionist
  – clear convincing result on specific issues
- in HCI
  – insights into cognitive process, human performance limitations, ...
  – allows comparison of systems, fine-tuning of details ...

strives for

- lucid and testable hypothesis (usually a causal inference)
- quantitative measurement
- measure of confidence in results obtained (inferencial statistics)
- replicability of experiment
- control of variables and conditions
- removal of experimenter bias

desired outcome of a controlled experiment

statistical inference of an event or situation's probability:

"Design A is better <*in some specific sense*> than Design B"

or, Design A meets a target:

"90% of incoming students who have web experience can complete course registration within 30 minutes"

steps in the experimental method

step 1: begin with a lucid, testable hypothesis

Example 1:

H_0: there is no difference in the number of cavities in children and teenagers using crest and no-teeth toothpaste

H_1: children and teenagers using crest toothpaste have fewer cavities than those who use no-teeth toothpaste

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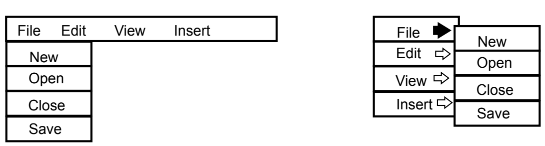
step 1: begin with a lucid, testable hypothesis

Example 2:

H_0: there is no difference in user performance (time and error rate) when selecting a single item from a pop-up or a pull down menu, regardless of the subject's previous expertise in using a mouse or using the different menu types

H_1: selecting from a pop-up menu will be faster and less error prone than selecting from a pull down menu



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general: hypothesis testing

hypothesis = **prediction** of the outcome of an experiment.

framed in terms of **independent** and **dependent** variables:

a variation in the independent variable will cause a difference in the dependent variable.

aim of the experiment: prove this prediction

do by: *disproving* the "null hypothesis"

H_0: experimental conditions **have no effect** on performance (to some degree of **significance)** → **null hypothesis**

H_1: experimental conditions **have an effect** on performance (to some degree of **significance)** → **alternate hypothesis**

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step 2: explicitly state the independent variables

Independent variables
- things you control/manipulate (independent of how a subject behaves) to produce different conditions for comparison
- two different kinds:
  - treatment manipulated (can establish cause/effect, true experiment)
  - subject individual differences (can never fully establish cause/effect)

in toothpaste experiment
- toothpaste type: Crest or No-teeth toothpaste *(treatment)*
- age: <= 12 years *or* > 12 years *(subject)*

in menu experiment
- menu type: pop-up or pull-down *(treatment)*
- menu length: 3, 6, 9, 12, 15 *(treatment)*
- expertise: expert or novice *(often subject, but can train an expert)*

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step 3: carefully choose the dependent variables

Dependent variables
- things that are measured
- expectation that they depend on the subject's behaviour / reaction to the independent variable (but unaffected by other factors)

in toothpaste experiment:

in menu experiment:

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step 4: consider possible nuisance variables & determine mitigation approach

- undesired variations in experiment conditions which **cannot be eliminated**, but which **may affect** dependent variable
  - critical to know about them
- experiment design & analysis must generally accommodate them:
  - treat as an additional experiment independent variable (if they can be controlled)
  - randomization (if they cannot be controlled)
- common nuisance variable: *subject* (individual differences)

in toothpaste experiment:

in menu experiment:

how to manage?

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step 5: design the task to be performed

tasks must:

be externally valid

external validity = do the results generalize?

… will they be an accurate predictor of how well users can perform tasks as they would in real life?

for a large interactive system, can probably only test a small subset of all possible tasks.

exercise the designs, bringing out any differences in their support for the task

e.g., if a design supports website **navigation**, test task should **not** require subject to work within a **single page**

be feasible - supported by the design/prototype, and executable within experiment time scale

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step 5: design the task to be performed

in toothpaste experiment:

in menu experiment:

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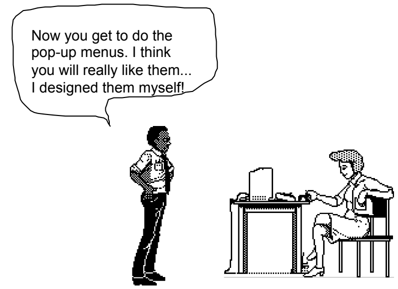
step 6: design experiment protocol

- steps for executing experiment are prepared well ahead of time
- includes unbiased instructions + instruments (questionnaire, interview script, observation sheet)
- double-blind experiments, ...



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step 7: make formal experiment design explicit

simplest: 2-sample (2-condition) experiment

based on comparison of **two sample means**:

- performance data from using Design A & Design B
  - e.g., new design & status quo design
  - e.g., 2 new designs

or, comparison of **one sample mean with a constant**:

- performance data from using Design A, compared to performance requirement
  - determine whether single new design meets key design requirement

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step 7: make formal experiment design explicit

more complex: factorial design

in toothpaste experiment:

2 toothpaste types (crest, no-teeth)
x 2 age groups (<= 12 years *or* > 12 years)

in menu experiment:

2 menu types (pop-up, pull down)
x 5 menu lengths (3, 6, 9, 12, 15)
x 2 levels of expertise (novice, expert)

(more on this later)

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step 8: judiciously select/recruit and assign subjects to groups

subject pool: *similar issues as for informal studies*

- match expected user population as closely as possible
- age, physical attributes, level of education
- general experience with systems similar to those being tested
- experience and knowledge of task domain

sample size: *more critical in experiments than informal studies*

- going for "statistical significance"
- should be large enough to be "representative" of population
- guidelines exist based on statistical methods used & required significance of results
- pragmatic concerns may dictate actual numbers
- "10" is often a good place to start

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step 8: judiciously select/recruit and assign subjects to groups

- if there is too much variability in the data collected, you will not be able to achieve statistical significance (more later)
- you can reduce variability by controlling subject variability

how?

- recognize classes and make them an independent variable
  - e.g., older users vs. younger users
  - e.g., superstars versus poor performers
- use reasonable number of subjects and random assignment

Novice

Expert

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step 9: apply statistical methods to data analysis

examples: t-tests, ANOVA, correlation, regression (more on these later)

confidence limits: the confidence that your conclusion is correct

- "The hypothesis that mouse experience makes no difference is rejected at the .05 level" (i.e., null hypothesis rejected)
- this means:
  - a 95% chance that your finding is correct
  - a 5% chance you are wrong

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step 10: interpret your results

what *you* believe the results mean, and their implications

yes, there can be a subjective component to quantitative analysis

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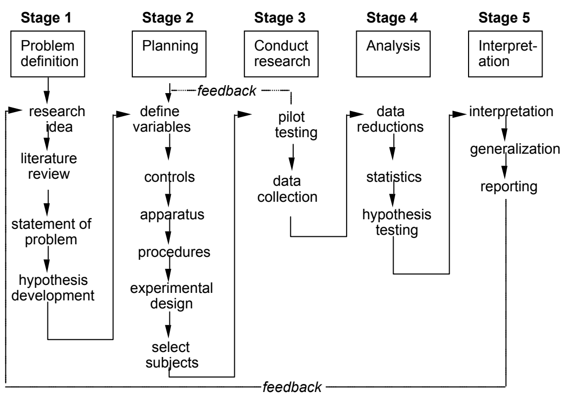
the planning flowchart



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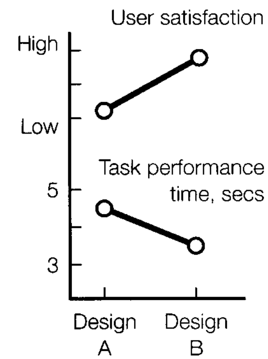
goal of experiment design



guard against ambiguous or misleading results

← a good (definitive) result

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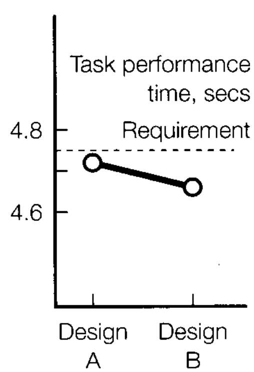
poor experiment design or results

less distinguishable results:

perhaps task was poorly chosen – or there's really no difference



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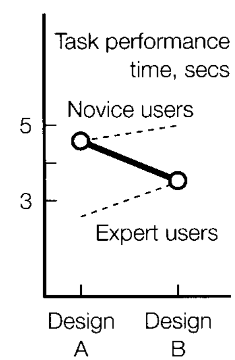
poor experiment design



misleading results

e.g. subject assignment not controlled: one design tested on novices, other on experts, disguising actual trend

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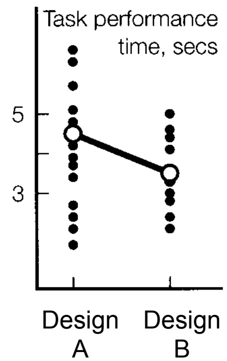
poor experiment design or results

large spread in values

perhaps conditions were not well controlled



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as we have seen

individual (subject) differences may pose a **nuisance variable:**

variation in individual abilities can mask real differences in test conditions, if not analyzed properly

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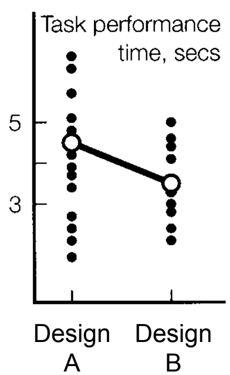

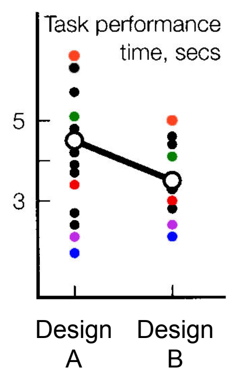

most common way to deal with:

account for each individual's performance in the two conditions (paired t-test, we'll cover more)

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within/between subject comparisons

within-subject comparisons:

- **subjects exposed to multiple treatment conditions**
- → primary comparison internal to each subject
- allows control over subject variable
- greater statistical power, fewer subjects required
- not always possible (exposure to one condition might "contaminate" subject for another condition; or session too long)

between-subject comparisons:

- **subjects only exposed to one condition**
- → primary comparison is from subject to subject
- less statistical power, more subjects required
- why? because greater variability due to more individual differences

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within/between subject comparisons

in toothpaste experiment

2 toothpaste types (crest, no-teeth) *between or within*

x 2 age groups (<= 12 years *or* > 12 years) *must be between*

in menu experiment :

2 menu types *(pop-up, pull down) between or within*

*x 5 menu lengths (*3, 6, 9, 12, 15*) should be within*

x 2 levels of expertise (novice, expert) *must be between*

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to summarize so far:

how a controlled experiment works

1. formulate an **alternate** and a **null** hypothesis:

   H_1: experimental conditions **have an effect** on performance

   H_0: experimental conditions **have no effect** on performance

2. through **experiment task**, try to demonstrate that the **null hypothesis is false** (reject it),

   for a particular level of **significance**

3. if successful, we can **accept** the alternate hypothesis,

   and state the probability ***p*** that we are wrong (the null hypothesis is true after all) → this is the result's **confidence level**

   e.g., selection speed is significantly faster in menus of length 5 than of length 10 ($p<.05$)

   → 5% chance we've made a mistake, 95% confident

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statistical analysis

what is a statistic?

- a number that describes a sample
- sample is a subset (hopefully representative) of the population we are interested in understanding

statistics are calculations that tell us

- mathematical attributes about our data sets (sample)
  - mean, amount of variance, ...
- how data sets relate to each other
  - whether we are "sampling" from the same or different populations
- the probability that our claims are correct
  - "statistical significance"

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example: differences between means

given: two data sets measuring a condition

- e.g., height difference of males and females,
  time to select an item from different menu styles ...

question:

- is the difference between the means of the data statistically significant?

null hypothesis:

- there is no difference between the two means
- statistical analysis can only reject the hypothesis at a certain level of confidence
- *note: we never actually prove the null hypothesis true*

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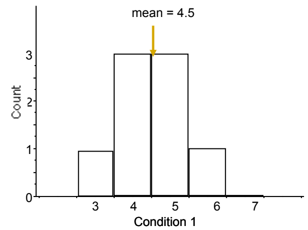
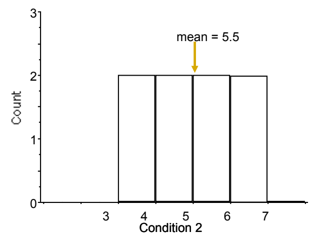
example:

Is there a *significant* difference between the means?

Condition one: 3, 4, 4, 4, 5, 5, 5, 6



Condition two: 4, 4, 5, 5, 6, 6, 7, 7



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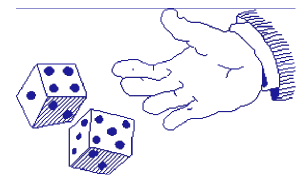
the problem with visual inspection of data

there is almost always variation in the collected data

differences between data sets may be due to:

- normal variation
  - e.g., two sets of ten tosses with different but fair dice
    - differences between data and means are accountable by expected variation
- real differences between data
  - e.g., two sets of ten tosses with loaded dice and fair dice
    - differences between data and means are not accountable by expected variation



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t-test

a statistical test

allows one to say something about differences between two means at a certain confidence level

null hypothesis of the t-test:
no difference exists between the means

possible results:

- I am 95% sure that null hypothesis is rejected
  - there is probably a true difference between the means
- I cannot reject the null hypothesis
  - the means are likely the same

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different types of t-tests

comparing two sets of independent observations ***(between subjects)***

usually different subjects in each group (number may differ as well)

Condition 1	Condition 2
S1–S20	S21–S43

paired observations ***(within subjects)***

usually single group studied under separate experimental conditions

data points of one subject are treated as a pair

Condition 1	Condition 2
S1–S20	S1–S20

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different types of t-tests

non-directional vs directional alternatives

non-directional (two-tailed)
- no expectation that the direction of difference matters

directional (one-tailed)
- only interested if the mean of a given condition is greater than the other

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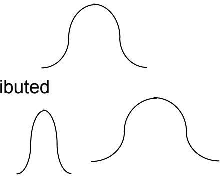
t-tests



Assumptions of t-tests
- data points of each sample are normally distributed
  – but t-test very robust in practice
- sample variances are equal
  – t-test reasonably robust for differing variances
  – deserves consideration
- individual observations of data points in sample are independent
  – must be adhered to *(can you think of examples where they are not?)*

Significance level
- decide upon the level before you do the test!
- typically stated at the .05 or .01 level
- .10 can be considered a trend, but is controversial

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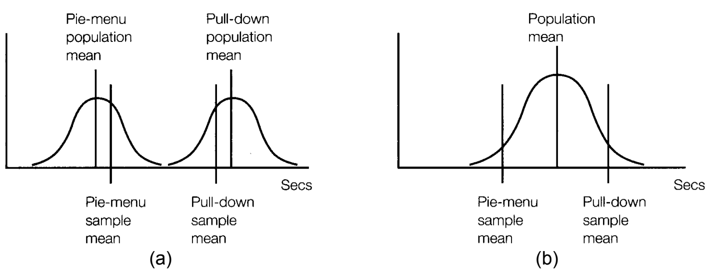
what the t-test is testing

(a) the two samples come from two different populations;
(b) the two samples are part of the same population.



Which represents H_0 and which represents H_1?

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two-tailed unpaired t-test

n: number of data points in the one sample ($N = n_1 + n_2$)
ΣX: sum of all data points in one sample
$\bar{X}$: mean of data points in sample
$\Sigma(X^2)$: sum of squares of data points in sample
s^2: unbiased estimate of population variation
t: t ratio
df = degrees of freedom = $n_1 + n_2 - 2$

N&L shows derivation of formula

How to maximize t?

Formulas

$$s^2 = \frac{\Sigma(X_1^2) - \frac{(\Sigma x_1)^2}{n_1} + \Sigma(X_2^2) - \frac{(\Sigma x_2)^2}{n_2}}{n_1 + n_2 - 2}$$

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s^2}{n_1} + \frac{s^2}{n_2}}}$$

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<N&L derivation>

mean & sum of squares

mean $= \bar{X} = \frac{\sum X_i}{N}$

sum of squares $= SS = \sum (X_i - \bar{X})^2$

(same, faster) $= \sum X_i^2 - \frac{(\sum X_i)^2}{N}$

error in N&L pg. 231

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degrees of freedom (df)

freedom of a set of values to vary independently of one another:

$X = \{21, 20, 24\}$ N=3

$\bar{X} = \frac{65}{3} = 21.6$: ← $\bar{X}$ has N-1=2 df

once you know the mean of N values, only N-1 can vary independently

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sample variance & standard deviation

sample variance $= s^2 = \frac{SS}{N-1}$

standard deviation $= sd = \sqrt{s^2}$

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</N&L derivation>

calculating *t*

compute **combined variance** for the two samples:

$$s^2 = \frac{SS_1 + SS_2}{n_1 + n_2 - 2}$$

← *note df computation*

compute **standard error of difference**, s_{ed} :

$$s_{ed} = \sqrt{s^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

compute *t*:

$$t = \frac{|\bar{X}_1 - \bar{X}_2|}{s_{ed}}$$

no, you won't have to memorize the formula for exams. but you *should* know how / when to use it.

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Level of significance for two-tailed test

df	.05	.01	df	.05	.01
1	12.706	63.657	16	2.120	2.921
2	4.303	9.925	18	2.101	2.878
3	3.182	5.841	20	2.086	2.845
4	2.776	4.604	22	2.074	2.819
5	2.571	4.032	24	2.064	2.797
6	2.447	3.707			
7	2.365	3.499			
8	2.306	3.355			
9	2.262	3.250			
10	2.228	3.169			
11	2.201	3.106			
12	2.179	3.055			
13	2.160	3.012			
14	2.145	2.977			
15	2.131	2.947			

Critical value (threshold) that t statistic much reach to achieve significance.

How does critical value change based on *df* and confidence level?

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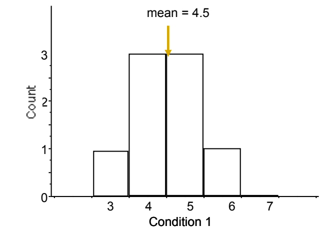
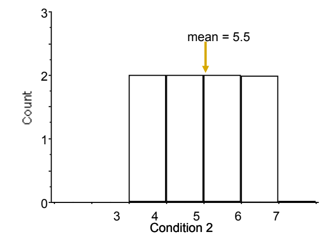
back to example:

Is there a *significant* difference between the means?

Condition one: 3, 4, 4, 4, 5, 5, 5, 6



Condition two: 4, 4, 5, 5, 6, 6, 7, 7



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example calculation

x_1 = 3 4 4 4 5 5 5 6
x_2 = 4 4 5 5 6 6 7 7

hypothesis: there is no significant difference between the means at the .05 level

Step 1. Calculating s^2

	1	2
N	8	8
Σx	36	44
$\bar{x}$	4.5	5.5
$\Sigma(x^2)$	168	252
$(\Sigma x)^2$	1296	1936

$df = 14$

$$s^2 = \frac{\Sigma x_1^2 - (\Sigma x_1)^2/N_1 + \Sigma x_2^2 - (\Sigma x_2)^2/N}{N_1 + N_2 - 2}$$

$$= \frac{168 - 1296/8 + 252 - 1936/8}{8+8-2}$$

$$= 1.1429$$

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example calculation

Step 2. Calculating *t*

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2/N_1 + s^2/N_2}}$$

$$= \frac{4.5 - 5.5}{\sqrt{2 \cdot (1.1429/8)}}$$

$$= \frac{-1}{.5345}$$

$$= -1.871$$

Step 3: Looking up critical value of *t*
- Use table for two-tailed *t*-test, at *p*=.05, *df*=14
- critical value = 2.145
- because *t*=1.871 < 2.145, there is no significant difference
- therefore, we cannot reject the null hypothesis
  i.e., there is no significant difference between the means

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two-tailed unpaired t-test

Condition one: 3, 4, 4, 4, 5, 5, 5, 6

Condition two: 4, 4, 5, 5, 6, 6, 7, 7

What the results would look like in stats software.

Unpaired t-test

DF:	Unpaired t Value:	Prob. (2-tail):
14	-1.871	.0824 hint

probability that means are from the same underlying population

Group:	Count:	Mean:	Std. Dev.:	Std. Error:
one	8	4.5	.926	.327
two	8	5.5	1.195	.423

How does the outcome change for a confidence level of 0.10?

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summary of the t-test

the point: establish a confidence level in the difference we've found between 2 sample means.

the process:

1. compute df
2. choose desired **significance, *p*** (aka α)
3. calculate value of the ***t* statistic**
4. compare it to the **critical value** of t given p, df: $t_{(p,df)}$
5. if $t > t_{(p,df)}$, can **reject null hypothesis at *p***

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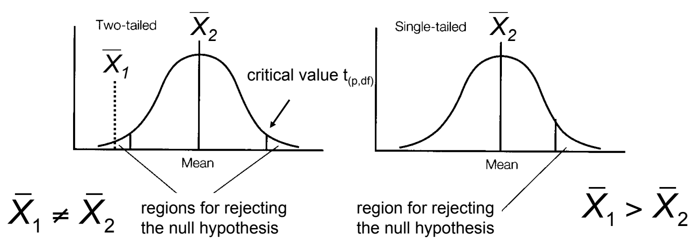
what does this look like graphically?



null hypothesis rejection area:

- two-tailed: divided equally between left/right
- single-tailed: all on one side

region(s) for rejecting the null hypothesis:

the area of the normal distribution that equals the chance you might be wrong

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you now know

How to answer the following:

what is the experimental method?
what is an experimental hypothesis?
how do I plan an experiment?
why are statistics used?
within- & between-subject comparisons: how do they differ?
how do I compute a t-test?
what are the different types of t-tests?

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additional slides: material I assume you know

types of variables
samples & populations
normal distribution
variance and standard deviation

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types of variables
(independent or dependent)

discrete: can take on **finite** number of levels
- e.g. a 3-color display can only render in red, green or blue;
- a design may be version A, or version B

continuous: can take any value (usually within bounds)
- e.g. a response time that may be any positive number (to resolution of measuring technology)

normal: one particular **distribution** of a continuous variable

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populations and samples

statistical sample =
approximation of total possible set of, e.g.
- **people** who will ever use the system
- **tasks** these users will ever perform
- **state** users might be in when performing tasks

} ← the population

"**sample**" a representative fraction
- draw **randomly** from population
- if large enough and representative enough, the **sample mean** should lie somewhere near the **population mean**

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confidence levels

"the **sample mean** should lie somewhere near the **population mean**"

how close?
how sure are we?

a confidence interval provides an **estimate of the probability** that the statistical measure is valid:

"We are **95%** certain that selection from menus of five items is faster than that from menus of seven items"

how does this work?
important aspect of experiment design

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establishing confidence levels: normal distributions

fundamental premise of statistics:

- predict behavior of a **population** based on a **small sample**

validity of this practice depends on the **distribution**

- of the population and of the sample

many populations are **normally distributed**:

- many statistical methods for **continuous dependent variables** are based on the assumption of normality

if **your sample is normally distributed**,
your **population is likely to be**,

- and these statistical methods are valid,
- and everything is a lot easier.

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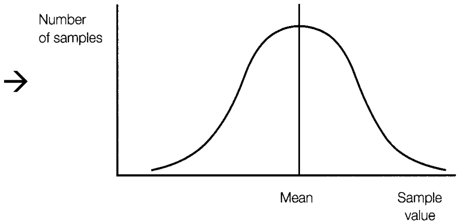
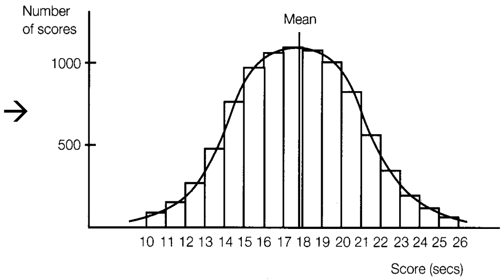
what's a normal distribution?

population →

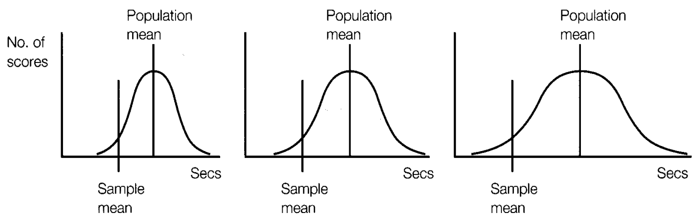


sample →



variance and standard deviation

all normal distributions are not the same:



population variance is a measure of the distribution's "spread"

all normal population distributions still have the same shape

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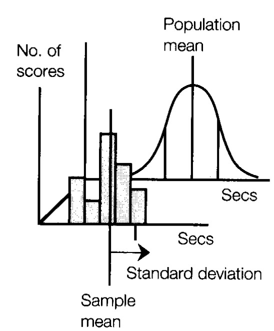
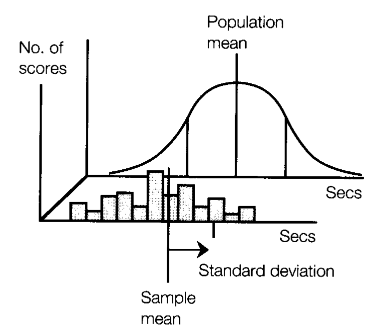
how do you get the population's variance?

estimate the population's (true) **variance**
from the (measured) sample's **standard deviation:**





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what's the big deal?

if you know you're dealing with samples from a normal distribution,

and you have a good estimate of its variance
(i.e. your sample's std dev)

then, you know the **probability** that a given sample came from that population (vs. a different one).