Game
Theory
m- Online

# Bidding in Second-Price Auctions 

Game Theory Course:<br>Jackson, Leyton-Brown \& Shoham

## Second-Price

## Theorem <br> Truth-telling is a dominant strategy in a second-price auction.

- In fact, we know this already (do you see why?)
- However, we'll look at a simpler, direct proof.


## Second-Price proof

## Theorem

Truth-telling is a dominant strategy in a second-price auction.

## Proof.

Assume that the other bidders bid in some arbitrary way. We must show that i's best response is always to bid truthfully. We'll break the proof into two cases:
I. Bidding honestly, $i$ would win the auction
2. Bidding honestly, $i$ would lose the auction

## Second-Price proof (2)




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- If $i$ bids lower, he will either still win and still pay the same amount...or lose and get utility of zero.


## Second-Price proof (3)



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## Second-Price proof (3)



- Bidding honestly, $i$ is not the winner
- If $i$ bids lower, he will still lose and still pay nothing
- If $i$ bids higher, he will either still lose and still pay nothing...or win and pay more than his valuation.


## English and Japanese auctions

- A much more complicated strategy space
- extensive form game
- bidders are able to condition their bids on information revealed by others
- in the case of English auctions, the ability to place jump bids
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## Theorem <br> Under the independent private values model (IPV), it is a dominant strategy for bidders to bid up to (and not beyond) their valuations in both Japanese and English auctions.

