

Election Theory

How voters and parties behave strategically in democratic systems

Mark Crowley

Department of Computer Science
University of British Columbia

January 30, 2006

Sources

- Jeff Gill and Jason Gainous. "Why does voting get so complicated? A review of theories for analyzing democratic participation." *Statistical Science*, 17(4):383–404, 2002.
- Kevin M. Quinn and Andrew D. Martin. "An integrated computational model of multiparty electoral competition." *Statistical Science*, 17(4):405–412, 2002.

Outline

- 1 Voting Theory
 - Goals of Voting
 - Voting Systems
 - Limitations of Voting Systems
- 2 Modeling Voters
 - Spatial Modeling - Issue Space
 - Probabilistic Modeling of Voter Behaviour
 - Cost-Benefit Modeling - Participation
- 3 Modeling Policy
 - Choosing Policy as a Strategic Game
 - Multiparty Proportional Representation
 - Study of Dutch Parliamentary Elections

Outline

- 1 **Voting Theory**
 - **Goals of Voting**
 - Voting Systems
 - Limitations of Voting Systems
- 2 **Modeling Voters**
 - Spatial Modeling - Issue Space
 - Probabilistic Modeling of Voter Behaviour
 - Cost-Benefit Modeling - Participation
- 3 **Modeling Policy**
 - Choosing Policy as a Strategic Game
 - Multiparty Proportional Representation
 - Study of Dutch Parliamentary Elections

What is the goal of an election?

- Have everyone vote their 'conscience'
- Outrage the fewest people
- Make the largest number of people happy
- Have every party honestly states their true beliefs and policies
- Achieve responsible government
- Avoid a completely irresponsible government
- All of the above?

If you are a *rational* voter . . .

Utility: you have preferences over outcomes

Purposefulness: you act to increase utility

Certainty: you don't like risky decisions

Sincerity: you act honestly, vote for the party that you agree with most

Comparability: you believe that $a > b$ and $b > c \implies a > c$

Outline

- 1 **Voting Theory**
 - Goals of Voting
 - **Voting Systems**
 - Limitations of Voting Systems
- 2 **Modeling Voters**
 - Spatial Modeling - Issue Space
 - Probabilistic Modeling of Voter Behaviour
 - Cost-Benefit Modeling - Participation
- 3 **Modeling Policy**
 - Choosing Policy as a Strategic Game
 - Multiparty Proportional Representation
 - Study of Dutch Parliamentary Elections

There are many voting schemes

- 1 Unanimity Voting
- 2 Plurality Voting
- 3 Approval Voting
- 4 Cumulative Voting
- 5 Condorcet Voting
- 6 Borda Count
- 7 Hare Procedure
- 8 Coombs Procedure

There are many voting schemes

Unanimity Voting: Everyong has to agree, come to a cooperative deal to balance utilities

Majority/Plurality Voting: Runoff elections required for true majority, sometimes it can make sense to vote for your third choice

- antiplurality
- inefficiency

There are many voting schemes

Approval Voting: voters select all candidates they approve of

- $2^K - 1$ strategies for K candidates
- strong incentive to vote strategically

Cumulative Voting: multiple votes allowed on the same candidate

- better for minorities?
- lots of strategic voting, would have avoided French election problem

There are many voting schemes

Approval Voting: voters select all candidates they approve of

- $2^K - 1$ strategies for K candidates
- strong incentive to vote strategically

Cumulative Voting: multiple votes allowed on the same candidate

- better for minorities?
- lots of strategic voting, would have avoided French election problem

There are many voting schemes

Approval Voting: voters select all candidates they approve of

- $2^K - 1$ strategies for K candidates
- strong incentive to vote strategically

Cumulative Voting: multiple votes allowed on the same candidate

- better for minorities?
- lots of strategic voting, would have avoided French election problem

There are many voting schemes

Approval Voting: voters select all candidates they approve of

- $2^K - 1$ strategies for K candidates
- strong incentive to vote strategically

Cumulative Voting: multiple votes allowed on the same candidate

- better for minorities?
- lots of strategic voting, would have avoided French election problem

There are many voting schemes

Approval Voting: voters select all candidates they approve of

- $2^K - 1$ strategies for K candidates
- strong incentive to vote strategically

Cumulative Voting: multiple votes allowed on the same candidate

- better for minorities?
- lots of strategic voting, would have avoided French election problem

There are many voting schemes

Condorcet Voting: (1785) All candidates ranked and compared in pairwise elections, whoever has the most wins is elected.

Borda Count: (1781) For K candidates voters rank them and the highest get $K - 1$ points, the lowest get none, candidate with the most points wins.

- how you order your irrelevant alternatives can alter the winner

Both these systems force equal distances between preferences, no way to express intensity of feeling about a candidate.

There are many voting schemes

Condorcet Voting: (1785) All candidates ranked and compared in pairwise elections, whoever has the most wins is elected.

Borda Count: (1781) For K candidates voters rank them and the highest get $K - 1$ points, the lowest get none, candidate with the most points wins.

- how you order your irrelevant alternatives can alter the winner

Both these systems force equal distances between preferences, no way to express intensity of feeling about a candidate.

There are many voting schemes

Hare Procedure: also known as Single Transferable Vote.

- all candidates ranked
- if no one receives $> 50\%$ of first place votes drop the lowest and use second place votes
- can have multiple candidates per riding

Coombs Procedure: another proportional method similar to Hare

- if someone needs to be dropped its the candidate with the *most* last place votes

Outline

- 1 **Voting Theory**
 - Goals of Voting
 - Voting Systems
 - **Limitations of Voting Systems**
- 2 **Modeling Voters**
 - Spatial Modeling - Issue Space
 - Probabilistic Modeling of Voter Behaviour
 - Cost-Benefit Modeling - Participation
- 3 **Modeling Policy**
 - Choosing Policy as a Strategic Game
 - Multiparty Proportional Representation
 - Study of Dutch Parliamentary Elections

Condorcet's Paradox

Condorcet Voting may have *no winner*.

Position of Preference	A	B	C
Most	Decrease	Increase	Status quo
Next	Status quo	Decrease	Increase
Last	Increase	Status quo	Decrease

Arrow's Impossibility Theorem

With reasonable assumptions about voter preferences Arrow (1951) showed that having all four of the following is impossible:

Unrestricted domain: Voters are free to rank candidates in any order.

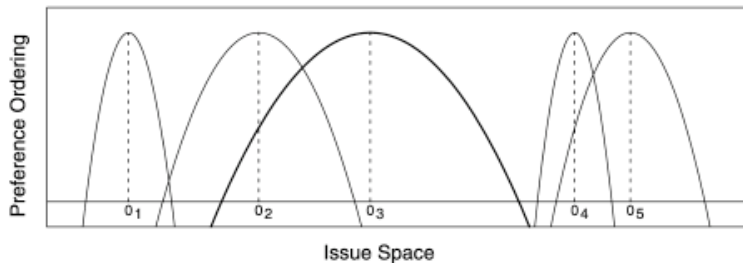
IIA: Deciding which of x or y will win should only involve preference on x and y .

Pareto: If everyone prefers x to y then x must do better than y .

Nondictatorship: No one voter can determine the ranking between two candidates with just their vote regardless of the votes of others.

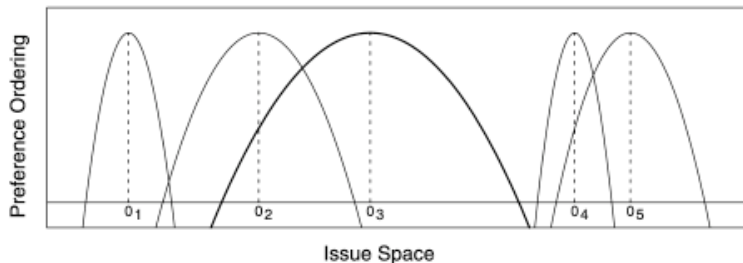
Median Voter Theorem

This theorem by Black (1958) drops the unrestricted domain requirement. Each voter has a unimodal peak along a spectrum on one issue.



Median Voter Theorem

Median voter is guaranteed to be in the majority.
 Parties will tend to move policy towards the centre.



Outline

- 1 Voting Theory
 - Goals of Voting
 - Voting Systems
 - Limitations of Voting Systems
- 2 Modeling Voters
 - Spatial Modeling - Issue Space
 - Probabilistic Modeling of Voter Behaviour
 - Cost-Benefit Modeling - Participation
- 3 Modeling Policy
 - Choosing Policy as a Strategic Game
 - Multiparty Proportional Representation
 - Study of Dutch Parliamentary Elections

Preferences are modelled in Issue Space

Instead of one dimension, multiple dimensions are used.

- Where does each voter fit in the space?
- Where does each party fit?
- No longer a guaranteed majority median, though there can sometimes be a related concept called the *core*.
- Some work (Hotelling, 1929; Davis & Hinich, 1967) focusses on distance from competitors in policy space as a negative factor.

Preferences are modelled in Issue Space

Instead of one dimension, multiple dimensions are used.

- Where does each voter fit in the space?
- Where does each party fit?
- No longer a guaranteed majority median, though there can sometimes be a related concept called the *core*.
- Some work (Hotelling, 1929; Davis & Hinich, 1967) focusses on distance from competitors in policy space as a negative factor.

Preferences are modelled in Issue Space

Instead of one dimension, multiple dimensions are used.

- Where does each voter fit in the space?
- Where does each party fit?
- No longer a guaranteed majority median, though there can sometimes be a related concept called the *core*.
- Some work (Hotelling, 1929; Davis & Hinich, 1967) focusses on distance from competitors in policy space as a negative factor.

Preferences are modelled in Issue Space

Instead of one dimension, multiple dimensions are used.

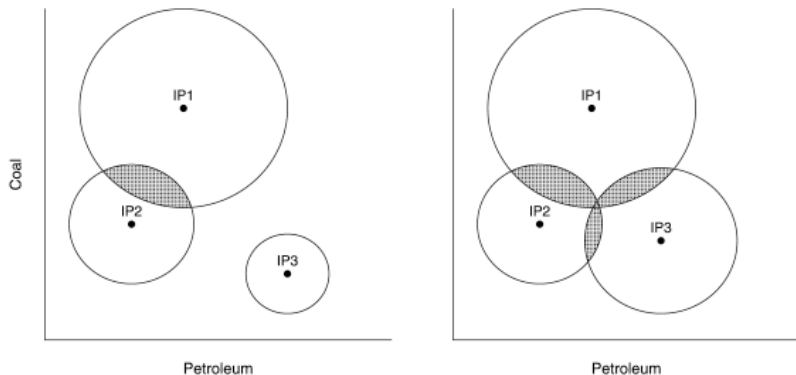
- Where does each voter fit in the space?
- Where does each party fit?
- No longer a guaranteed majority median, though there can sometimes be a related concept called the *core*.
- Some work (Hotelling, 1929; Davis & Hinich, 1967) focusses on distance from competitors in policy space as a negative factor.

Preferences are modelled in Issue Space

Instead of one dimension, multiple dimensions are used.

- Where does each voter fit in the space?
- Where does each party fit?
- No longer a guaranteed majority median, though there can sometimes be a related concept called the *core*.
- Some work (Hotelling, 1929; Davis & Hinich, 1967) focusses on distance from competitors in policy space as a negative factor.

Preferences are modelled in Issue Space



Outline

- 1 Voting Theory
 - Goals of Voting
 - Voting Systems
 - Limitations of Voting Systems
- 2 Modeling Voters
 - Spatial Modeling - Issue Space
 - **Probabilistic Modeling of Voter Behaviour**
 - Cost-Benefit Modeling - Participation
- 3 Modeling Policy
 - Choosing Policy as a Strategic Game
 - Multiparty Proportional Representation
 - Study of Dutch Parliamentary Elections

Unpredictable Voters

Probabilistic voting models (Hinich, 1977; Ordeshook, 1986) have no impact on the voting process but are used to predict the outcome or understand voter behaviour

- A probabilistic voter does not have discrete, deterministic utilities.
- Sometimes they will vote for alternatives with lower expected utility.
- Candidates believing this model have more incentive to be vague about policy.

Unpredictable Voters

- Burden (1997) shows how deterministic and probabilistic models can lead to different predictions using the same data.
- They use this to model the probability that a voter will abstain from voting because of alienation or indifference.
- Leads to more certainty in strategy for creating a policy (Coughlin and Nitzan, 1981).

Outline

- 1 Voting Theory
 - Goals of Voting
 - Voting Systems
 - Limitations of Voting Systems
- 2 Modeling Voters
 - Spatial Modeling - Issue Space
 - Probabilistic Modeling of Voter Behaviour
 - **Cost-Benefit Modeling - Participation**
- 3 Modeling Policy
 - Choosing Policy as a Strategic Game
 - Multiparty Proportional Representation
 - Study of Dutch Parliamentary Elections

Why Vote?

Anthony Downs (1957) calculated the costs and benefits of voting

EU_W : Expected Utility of my candidate winning

P_V : Probability of my vote making a difference in the outcome

C_V : Cost of going out to vote

$$EU_W \times P_V < C_V$$

The paradox of not voting.

Why Vote?

Anthony Downs (1957) calculated the costs and benefits of voting

EU_W : Expected Utility of my candidate winning

P_V : Probability of my vote making a difference in the outcome

C_V : Cost of going out to vote

$$EU_W \times P_V < C_V$$

The paradox of not voting.

Why Vote?

Anthony Downs (1957) calculated the costs and benefits of voting

EU_W : Expected Utility of my candidate winning

P_V : Probability of my vote making a difference in the outcome

C_V : Cost of going out to vote

$$EU_W \times P_V < C_V$$

The paradox of not voting.

Why Vote?

Anthony Downs (1957) calculated the costs and benefits of voting

EU_W : Expected Utility of my candidate winning

P_V : Probability of my vote making a difference in the outcome

C_V : Cost of going out to vote

$$EU_W \times P_V < C_V$$

The paradox of not voting.

Why Vote?

Anthony Downs (1957) calculated the costs and benefits of voting

EU_W : Expected Utility of my candidate winning

P_V : Probability of my vote making a difference in the outcome

C_V : Cost of going out to vote

$$EU_W \times P_V < C_V$$

The paradox of not voting.

Outline

- 1 Voting Theory
 - Goals of Voting
 - Voting Systems
 - Limitations of Voting Systems
- 2 Modeling Voters
 - Spatial Modeling - Issue Space
 - Probabilistic Modeling of Voter Behaviour
 - Cost-Benefit Modeling - Participation
- 3 Modeling Policy
 - **Choosing Policy as a Strategic Game**
 - Multiparty Proportional Representation
 - Study of Dutch Parliamentary Elections

Strategic Voting leads to Strategic Policies

- Voters consider strategic voting in most systems
- So policy needs to be created *relative* to other parties in order to win
- Most research assumes that parties determine policies to maximize their vote count, which often makes sense in plurality systems
- Quinn & Martin (2002) postulate that this is not always so, especially in proportional systems. They may often choose policies to maximize their chance of the final cabinet implementing part of it.

Plurality vs. Proportional

- Most of the literature is focussed on plurality rule systems with two parties
- Parties still display "Downsian" convergence of policy in such systems, or moving towards the centre. As long as:
 - parties want as many seats as possible
 - parties do not have high confidence in what the electorate will decide
- Nash equilibria for policies only exist at 'core points' which rarely exist
- Multiparty proportional usually assumed to be the same, not studied much
- Many of the world's democracies use some form of proportional representation (PR)

Outline

- 1 Voting Theory
 - Goals of Voting
 - Voting Systems
 - Limitations of Voting Systems
- 2 Modeling Voters
 - Spatial Modeling - Issue Space
 - Probabilistic Modeling of Voter Behaviour
 - Cost-Benefit Modeling - Participation
- 3 Modeling Policy
 - Choosing Policy as a Strategic Game
 - **Multiparty Proportional Representation**
 - Study of Dutch Parliamentary Elections

Multiparty PR is different

- One party winning a majority of seats is rare
- Policy is determined by the coalition that is formed from the largest parties
- "The power to determine policy is not monotonically increasing in vote shares or seat shares"

To model the outcome or how parties should pick their policies we need to model cabinet formation.

Broken Assumption: Are parties motivated by maximizing their seats, or the resulting government policy?

Multiparty PR is different

- One party winning a majority of seats is rare
- Policy is determined by the coalition that is formed from the largest parties
- "The power to determine policy is not monotonically increasing in vote shares or seat shares"

To model the outcome or how parties should pick their policies we need to model cabinet formation.

Broken Assumption: Are parties motivated by maximizing their seats, or the resulting government policy?

Multiparty PR is different

- One party winning a majority of seats is rare
- Policy is determined by the coalition that is formed from the largest parties
- "The power to determine policy is not monotonically increasing in vote shares or seat shares"

To model the outcome or how parties should pick their policies we need to model cabinet formation.

Broken Assumption: Are parties motivated by maximizing their seats, or the resulting government policy?

Multiparty PR is different

- One party winning a majority of seats is rare
- Policy is determined by the coalition that is formed from the largest parties
- "The power to determine policy is not monotonically increasing in vote shares or seat shares"

To model the outcome or how parties should pick their policies we need to model cabinet formation.

Broken Assumption: Are parties motivated by maximizing their seats, or the resulting government policy?

Multiparty PR is different

- One party winning a majority of seats is rare
- Policy is determined by the coalition that is formed from the largest parties
- "The power to determine policy is not monotonically increasing in vote shares or seat shares"

To model the outcome or how parties should pick their policies we need to model cabinet formation.

Broken Assumption: Are parties motivated by maximizing their seats, or the resulting government policy?

Outline

- 1 Voting Theory
 - Goals of Voting
 - Voting Systems
 - Limitations of Voting Systems
- 2 Modeling Voters
 - Spatial Modeling - Issue Space
 - Probabilistic Modeling of Voter Behaviour
 - Cost-Benefit Modeling - Participation
- 3 Modeling Policy
 - Choosing Policy as a Strategic Game
 - Multiparty Proportional Representation
 - Study of Dutch Parliamentary Elections

The Dutch Electoral System

Quinn & Martin (2002) is a study of the Dutch electoral system for the 1989 parliamentary elections.

- It is *fully* proportional, any party with more than .67% of the vote gets a seat
- One voting district for the whole country
- After seats are allocated the largest party tries to form a coalition
- Any of its candidate partners can veto the alliance
- Then the next largest party tries

The Dutch Electoral System

Quinn & Martin (2002) is a study of the Dutch electoral system for the 1989 parliamentary elections.

- It is *fully* proportional, any party with more than .67% of the vote gets a seat
- One voting district for the whole country
- After seats are allocated the largest party tries to form a coalition
- Any of its candidate partners can veto the alliance
- Then the next largest party tries

The Dutch Electoral System

Quinn & Martin (2002) is a study of the Dutch electoral system for the 1989 parliamentary elections.

- It is *fully* proportional, any party with more than .67% of the vote gets a seat
- One voting district for the whole country
- After seats are allocated the largest party tries to form a coalition
- Any of its candidate partners can veto the alliance
- Then the next largest party tries

The Dutch Electoral System

Quinn & Martin (2002) is a study of the Dutch electoral system for the 1989 parliamentary elections.

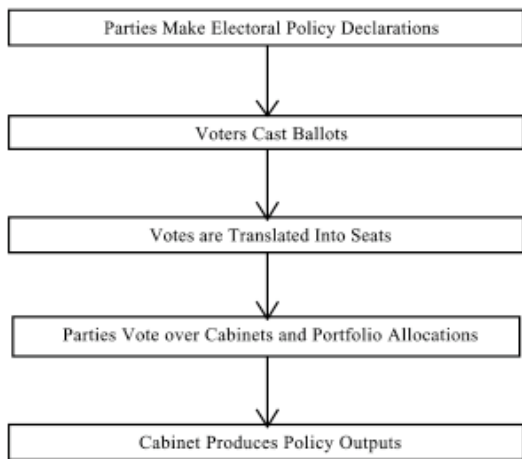
- It is *fully* proportional, any party with more than .67% of the vote gets a seat
- One voting district for the whole country
- After seats are allocated the largest party tries to form a coalition
- Any of its candidate partners can veto the alliance
- Then the next largest party tries

The Dutch Electoral System

Quinn & Martin (2002) is a study of the Dutch electoral system for the 1989 parliamentary elections.

- It is *fully* proportional, any party with more than .67% of the vote gets a seat
- One voting district for the whole country
- After seats are allocated the largest party tries to form a coalition
- Any of its candidate partners can veto the alliance
- Then the next largest party tries

The Dutch democracy has many stages



A Study of Policy Space

- The authors also took a national survey of about 1800 people to determine:
 - their location in issue space across five issues (abortion, nuclear power, state anti-poverty policy, euthanasia, deployment of nuclear weapons)
 - their view of each of the four major party's locations in issue space
 - reduced issue space to two dimensions Social (religion) and Economic

A Study of Policy Space

- The authors also took a national survey of about 1800 people to determine:
 - their location in issue space across five issues (abortion, nuclear power, state anti-poverty policy, euthanasia, deployment of nuclear weapons)
 - their view of each of the four major party's locations in issue space
 - reduced issue space to two dimensions Social (religion) and Economic

A Study of Policy Space

- The authors also took a national survey of about 1800 people to determine:
 - their location in issue space across five issues (abortion, nuclear power, state anti-poverty policy, euthanasia, deployment of nuclear weapons)
 - their view of each of the four major party's locations in issue space
 - reduced issue space to two dimensions Social (religion) and Economic

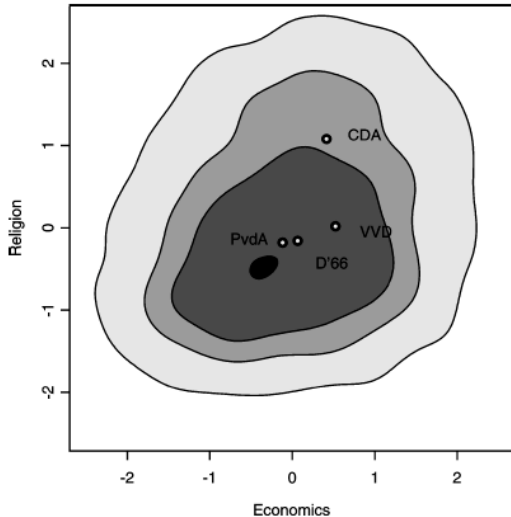
A Study of Policy Space

- The authors also took a national survey of about 1800 people to determine:
 - their location in issue space across five issues (abortion, nuclear power, state anti-poverty policy, euthanasia, deployment of nuclear weapons)
 - their view of each of the four major party's locations in issue space
 - reduced issue space to two dimensions Social (religion) and Economic

A Study of Policy Space

- The authors also took a national survey of about 1800 people to determine:
 - their location in issue space across five issues (abortion, nuclear power, state anti-poverty policy, euthanasia, deployment of nuclear weapons)
 - their view of each of the four major party's locations in issue space
 - reduced issue space to two dimensions Social (religion) and Economic

Parties searching for voters



Bang for your policy change

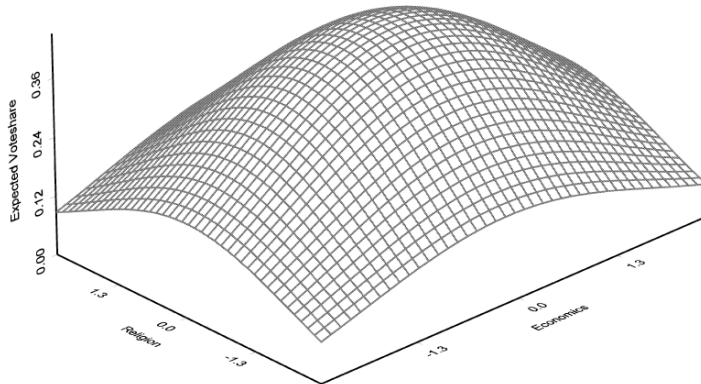


FIG. 3. *Expected CDA vote share at various electoral declarations conditional on empirical locations of the other Dutch parties, 1989.*

Bang for your policy change

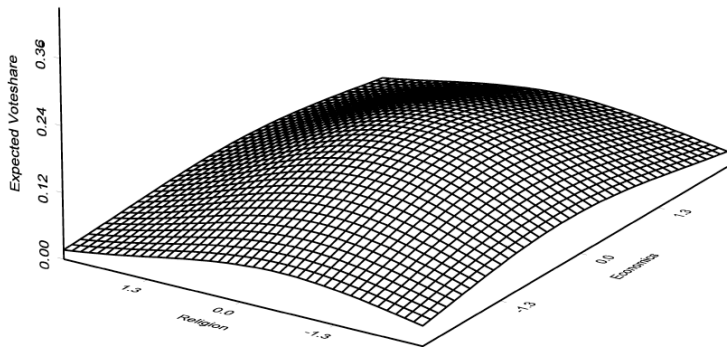


FIG. 5. *Expected VVD vote share at various electoral declarations conditional on empirical locations of the other Dutch parties, 1989.*

Parties Seeking to Maximize Votes

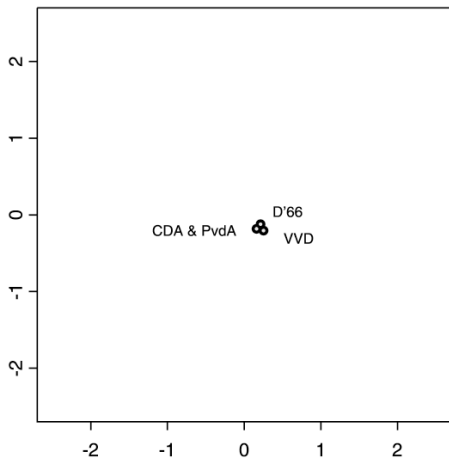


FIG. 7. *Equilibrium electoral declarations with vote-maximizing parties, 1989.*

Parties Seeking to Enact their Ideal Policy

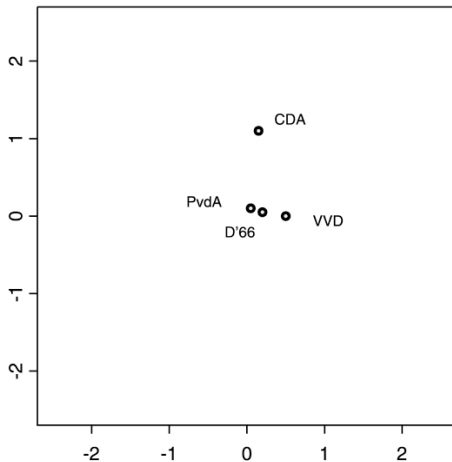
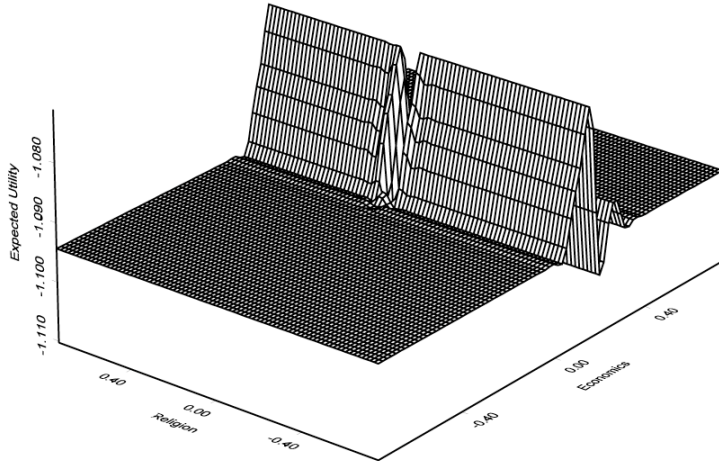


FIG. 8. *Equilibrium electoral declarations with policy-seeking parties, 1989.*

You Can't Always Get What You Want



Policy Stability Explained?

The authors conjecture that this effect accounts for stability observed by Lipset and Rokkan (1967).

- Party policies tend to remain very stable over time
- Deviating is not in their interest as it would not lead to more seats unless they passed a competitor on some issue dimension
- Canadian Reform/Alliance/Conservative electoral difficulty possibly related to this?

Limitations of the Dutch Study

- Oversimplified model of true issue space, two dimension, each with a ministry.
- Cannot deal with changes in preferences due to war, economic shock, etc.
- They assumed only the top four parties mattered, strong IIA.

Bush vs. Gore : Florida 2002

TABLE 6
Hypothetical approval votes

Approval for	Buchanan	Bush	Gore	Nader
Semi-independents	0	582,504	582,504	0
Supporters	2,281	2,281	0	0
Nader supporters	0	0	16,415	16,415
Remaining Bush	262,151	2,621,511	0	0
Remaining Gore	0	0	2,621,028	262,103
Total	264,432	3,206,296	3,219,947	278,518

Bush vs. Gore : Florida 2002

TABLE 7
Hypothetical cumulative votes

Counts for	Buchanan	Bush	Gore	Nader
Buchanan voters	$2,281 \times 3$	$2,281 \times 2$	$2,281 \times 1$	0
Bush voters	$2,912,790 \times 2$	$2,912,790 \times 3$	$2,912,790 \times 1$	0
Gore voters	0	$2,912,253 \times 1$	$2,912,253 \times 3$	$2,912,253 \times 2$
Nader voters	0	$16,415 \times 1$	$16,415 \times 2$	$16,415 \times 3$
Total	5,832,423	11,671,600	11,684,660	5,873,751

Summary

- No electoral system is perfect and the system you use to count votes can alter the outcome.
- Voting is complicated and strategic voting is probably never going to disappear.
- Choosing a party policy before an election is a complex multi-agent game where the goal may be to maximize seats, maximize votes or attain a certain coalition cabinet to further some ideal policy.

Discussion?

