## Introduction to

## **Artificial Intelligence (AI)**

Computer Science cpsc502, Lecture 6

Sep, 27, 2011

CPSC 502, Lecture 6

### Today Sept 27

- Finish R&R systems in deterministic environments
- Logics
  - Propositional, Definite Clause Logic: Syntax Semantics + Two different proof Procedures
  - Reasoning with individuals and relations
  - Full Propositional Logics and First-Order Logics

### **R&Rsys we'll cover in this course**



### What you already know for sure about logic...

#### From programming: Some logical operators



Logic is the language of Mathematics. To define formal structures (e.g., sets, graphs) and to proof statements about those

We are going to look at Logic as a **Representation and Reasoning System** that can be used to **formalize a domain (e.g., an electrical system, an organization)** and to **reason about it** CPSC 502, Lecture 6 Slide 4

### Why Logics?

 "Natural" to express knowledge about the world (more natural than a "flat" set of variables & constraints) "Every 502 student will pass the final"

Final (f1) Course (C1) Name-of(C1, 502)  $Course-of(t_1, C_1)$ 

Vz Student(z) ~ Registred(z, c,)  $\Rightarrow$  pass  $(f_1, Z)$ 

- It is easy to incrementally add knowledge
- It is easy to check and debug knowledge
- Provide language for asking complex queries Well understood formal properties

### **Ambitious Plan for today**

Datalog First Order Logic  $p(X) \leftarrow q(X) \wedge r(X,Y)$  $\forall X \exists Y p(X, X) \Leftrightarrow \neg q(Y)$  $r(X,Y) \leftarrow S(Y)$  $P(\partial_1,\partial_2)$  $S(\partial_1), Q(\partial_2)$  $-q(\partial_5)$ PDCL Propositional Logic pt snf  $7(p \vee q) \longrightarrow (r \wedge s \wedge f)_{f}$ rESAGAP P, r Slide 7 CPSC 502, Lecture 6

### **Propositional Logic**

- A very simple form of Logics: Propositional
- The primitive elements are propositions: Boolean variables that can be {*true, false*}

   *P*<sub>1</sub>
   *P*<sub>2</sub>
- The goal is to illustrate the basic ideas: syntax, semantics, proof procedure
- This is a starting point for more complex logics (e.g., first-order logic)

 Boolean nature can be exploited for efficiency. So more complex logics are often mapped in propositional form for inference

### **Propositional logic: Complete Language**

The **proposition** symbols  $p_1, p_2 \dots$  etc are sentences

- If S is a sentence, ¬S is a sentence (negation)
- If  $S_1$  and  $S_2$  are sentences,  $S_1 \wedge S_2$  is a sentence (conjunction)
- If  $S_1$  and  $S_2$  are sentences,  $S_1 \lor S_2$  is a sentence (disjunction)
- If  $S_1$  and  $S_2$  are sentences,  $S_1 \Rightarrow S_2$  is a sentence (implication)
- If  $S_1$  and  $S_2$  are sentences,  $S_1 \Leftrightarrow S_2$  is a sentence (biconditional)

Sample Formula  $((P_1 \vee P_2) \wedge P_3) \iff ((P_2 \Rightarrow \neg P_4) \vee P_5)$ 

### **Propositional Definite Clauses Logic**

- Propositional Definite Clauses: our first logical representation and reasoning system.
   (very simple!)
- Only two kinds of statements:
  - that a proposition is true
  - that a proposition is true if one or more other propositions are true  $P_1 \leftarrow P_3 \land P_4$
- Why still useful?
  - Adequate in many domains (with some adjustments)
  - Reasoning steps easy to follow by humans
  - Inference linear in size of your set of statements
  - Similar formalisms used in cognitive architectures

### **Propositional Definite Clauses: Syntax**

### Definition (atom)

An atom is a symbol starting with a lower case letter

Definition (body) $p_2 \land \ldots \land p_n$ A body is an atom or is of the form  $b_1 \land b_2$  where  $b_1$ and  $b_2$  are bodies.Definition (definite clause)

A definite clause is an atom or is a rule of the form  $(h \leftarrow b)$  where *h* is an atom and *b* is a body. (Read this as ``*h* if *b*.")



### Propositional Definite Clauses Semantics: Interpretation

Semantics allows you to relate the symbols in the logic to the domain you're trying to model. An **atom** can be  $\dots$   $\top$ 

**Definition (interpretation)** An interpretation *I* assigns a truth value to each atom.

If your domain can be represented by four atoms (propositions):

qpsrTTFF 24

So an interpretation is just a <u>possible</u> world

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### PDC Semantics: Body

We can use the **interpretation** to determine the truth value of **clauses** and **knowledge bases**:

**Definition (**truth values of statements): A body  $b_1 \wedge b_2$  is true in *I* if and only if  $b_1$  is true in *I* and  $b_2$  is true in *I*.

	p	q	r	S	PAY	PARAS
I <sub>1</sub>	true	true	true	true	T	
$I_2$	false	false	false	false	F	+
l <sub>3</sub>	true	true	false	false	F	
$I_4$	true	true	true	false		ī
$I_5$	true	true	false	true	F	F
	I		0.0		0	

### **PDC Semantics: definite clause**

**Definition (truth values of statements cont'):** A rule  $h \leftarrow b$  is false in *I* if and only if <u>b</u> is true in *I* and <u>h</u> is false in *I*.



In other words: *"if b is true I am claiming that h must be true, otherwise I am not making any claim"* CPSC 502, Lecture 6 Slide 17

### PDC Semantics: Knowledge Base

**Definition (**truth values of statements cont'): A knowledge base *KB* is true in *I* if and only if every clause in *KB* is true in *I*.



### **Models and Satisfiability**

Definition (model) A model of a set of clauses (a KB) is an interpretation in which all the clauses are *true*.

**Definition (satisfiability)** A set of clauses (a KB) is satisfiable if it has at least one model

Example: Models											
				$\int p \leftarrow q.$							
$KB = \begin{cases} q. \end{cases}$											
	р	q	r	$s$ $r \leftarrow s$ .							
71	true	true	true	true M	Which interpretations are						
I <sub>2</sub>	false	false	false	false $ imes$	models?						
l <sub>3</sub>	true	true	false	false M							
$I_4$	true	true	true	false M							
$I_5$	true	true	false	true 🗙							

### **Logical Consequence**

**Definition (logical consequence)** If *KB* is a set of clauses and *G* is a conjunction of atoms, *G* is a logical consequence of *KB*, written  $KB \models G$ , if *G* is *true* in every model of *KB*.

- we also say that *G* logically follows from *KB*, or that *KB* entails *G*.
- In other words,  $KB \models G$  if there is no interpretation in which KB is *true* and G is *false*.

**Example: Logical Consequences** S р q r  $KB = \begin{cases} p \leftarrow q. \ \checkmark \\ \underline{q.} \\ r \leftarrow s. \ \checkmark \end{cases}$  $\mathbf{I}_1$ true true true true Smodels false true  $I_2$ true true false **1**3 false true true false true <u>irríe</u> true true true 2<sup>4</sup> = 16 interpretations in total, only 3 are models false true false true false true false false false true false false true true remaining 8 connot F o convidels be models beconse 9 is tolse 1 Which of the following is true? KB = q, KB = p, KB = s, KB = r 1) CPSC 502, Lecture 6 Slide 22

# One simple way to prove that G logically follows from a KB

- Collect all the models of the KB
- Verify that G is true in all those models

Caset of atoms P1, P2, .... Any problem with this approach? check of the interpretations 2" interpretations

 The goal of proof theory is to find proof procedures that allow us to prove that a logical formula follows form a KB avoiding the above is logically entailed by

### **Soundness and Completeness**

- If I tell you I have a proof procedure for PDCL
- What do I need to show you in order for you to trust my procedure?
  - KB ⊢ G means G can be derived by my proof procedure from KB.
  - Recall  $KB \models G$  means G is true in all models of KB.

**Definition (soundness)** 

A proof procedure is sound if  $KB \vdash G$  implies  $KB \models G$ .

**Definition (completeness)** 

A proof procedure is complete if  $KB \models G$  implies  $KB \vdash G$ .

### **Bottom-up Ground Proof Procedure**

One rule of derivation, a generalized form of *modus ponens*:  $f = (h \leftarrow b_1 \land \dots \land b_m)$  is a clause in the knowledge base, and each  $b_i$  has been derived, then h can be derived.

You are forward chaining on this clause. (This rule also covers the case when m=0.)

### **Bottom-up proof procedure**

 $(KB \vdash G)$  if  $G \subseteq C$  at the end of this procedure:



### repeat

select clause " $h \leftarrow b_1 \land \dots \land b_m$ " in *KB* such that  $b_i \in C$  for all *i*, and  $h \notin C$ ;  $C := C \cup \{h\}$ until no more clauses can be selected.



### **Top-down Ground Proof Procedure**

**Key Idea:** search backward from a query *G* to determine if it can be derived from *KB*.



### **Top-down Proof Procedure: Basic elements**

**Notation**: An answer clause is of the form:

 $yes \leftarrow a_1 \land a_2 \land \dots \land a_m$ 

Express query as an answer clause (e.g., query  $a_1 \land a_2 \land \dots \land a_m$ )  $yes \leftarrow \ge 1 \land \ldots \land \ge 1$ 

Rule of inference (called <u>SLD Resolution</u>) Given an answer clause of the form:



### **Rule of inference: Examples**

**Rule of inference** (called SLD Resolution) Given an answer clause of the form:

KB  $yes \leftarrow a_1 \land a_2 \land \dots \land a_m$ and the clause:

 $a_i \leftarrow b_1 \land b_2 \land \dots \land b_p$ You can generate the answer clause  $yes \leftarrow a_1 \land \dots \land a_{i-1} \land b_1 \land b_2 \land \dots \land b_p \land a_{i+1} \land \dots \land a_m$ KB clause  $yes \leftarrow b \land c.$   $b \leftarrow k \land f. \Rightarrow Yes \in K \land f \land C$ KB e => yes f

 $Ves \leftarrow e \land f.$ 

### (successful) Derivations

An answer is an answer clause with m = 0. That is, it is the answer clause yes ←.

- A (successful) derivation of query "?q<sub>1</sub> Λ ... Λ q<sub>k</sub>" from KB is a sequence of answer clauses γ<sub>0</sub>, γ<sub>1</sub>,..., γ<sub>n</sub> such that
  - $\gamma_0$  is the answer clause  $yes \leftarrow q_1 \land \dots \land q_k$
  - $\gamma_i$  is obtained by resolving  $\gamma_{i-1}$  with a clause in *KB*, and
  - $\gamma_n$  is an answer. yes  $\leftarrow$ .
- An unsuccessful derivation.....

• Successful Derivation: When by applying the inference rule you obtain the answer clause yes ←.

$$\begin{array}{cccc} a \leftarrow e \wedge f. & a \leftarrow b \wedge c. & b \leftarrow k \wedge f. & \swarrow B \\ \hline c \leftarrow e. & d \leftarrow k. & e. \\ f \leftarrow j \wedge e. & f \leftarrow c. & j \leftarrow c. \end{array}$$

$$yes \leftarrow a.$$

$$u \leq e \land f$$

$$u \leq f$$

$$u \leq f$$

$$u \leq f$$

$$u \leq c$$

$$u \leq e$$

$$u \leq e$$

$$yes \leftarrow a.$$
  
 $\eta \leftarrow b \land c$   
 $\eta \leftarrow k \land + \land c$   
 $\eta$ 

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### Search Graph



### Systematic Search in different R&R systems

Constraint Satisfaction (Problems): V

- State: assignments of values to a subset of the variables
- Successor function: assign values to a "free" variable
- Goal test: set of constraints
- Solution: possible world that satisfies the constraints
- Heuristic function: none (all solutions at the same distance from start)

Planning (forward) : V

- State possible world
- Successor function states resulting from valid actions
- Goal test assignment to subset of vars
- Solution sequence of actions
- Heuristic function empty-delete-list (solve simplified problem)

#### Logical Inference (top Down)

- State answer clause 4es
- Successor function states resulting from substituting one atom with all the clauses of which it is the head

start state:

query as an

answer dauso

- Goal test empty answer clause yes
- Solution start state
- Heuristic function number of atoms in given state

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## Representation and Reasoning in Complex domains

- In complex domains expressing knowledge with **propositions** can be quite limiting  $up s_2$  $up s_3$  $ok cb_1$  $ok cb_2$
- It is often natural to consider individuals and their properties

 $up(s_2)$  $up(\bar{s_3})$  $\dot{ok}(\dot{cb_1})$  $ok(cb_2)$ live\_w<sub>1</sub>  $live(\bar{w_1})$ connected  $W_1 W_2$ connected( $w_1, w_2$ ) There is no notion that the system  $up_{s_{2}}$ ore about the up are about live\_w, wh 2 the same property connected\_w\_w2 up\_s<sub>2</sub> up\_s<sub>3</sub> some CPSC 502. Lecture 6 Slide 38

## What do we gain....

By breaking propositions into relations applied to individuals?

Express knowledge that holds for set of individuals
 (by introducing variables)

 $live(W) <- connected_to(W,W1) \land live(W1) \land wire(W) \land wire(W1).$ 

We can ask generic queries (i.e., containing
 Vars )
 Variabless

? connected\_to(W,  $w_1$ )

### Datalog: a relational rule language

It expands the syntax of PDCL....

A variable is a symbol starting with an upper case letter

A constant is a symbol starting with lower-case letter or a sequence of digits. alan

W1

A term is either a variable or a constant.

A predicate symbol is a symbol starting with lower-case letter. port-of live N



A definite clause is either an atom (a fact) or of the form:  $\begin{array}{c}
 h \leftarrow b_1 \wedge \dots \wedge b_m \\
 where h and the b_i are atoms (Read this as ``h if b.")$   $\begin{array}{c}
 Im (X,Y) \leftarrow Im (X,Z) \wedge prA_of(Z,Y)
\end{array}$ 

A knowledge base is a set of definite clauses

### Datalog: Top Down Proof

Extension of TD for PDCL.

How do you deal with variables?

Example: in(alan, r123).  $part_of(r123, cs_building)$ .  $in(X,Y) <- part_of(Z,Y) & in(X,Z)$ . Query: in(alan, cs\_building).  $ges <- in(alan, cs_building)$ .  $ges <- in(alan, cs_building)$ .

### **Datalog: queries with variables**

n(alan, r123). part\_of(r123,cs\_building). one answer in(X,Y) <- in(X,Z). & part\_of(Z,Y)</pre> -123  $Yes(x_1) \leftarrow In(\partial \partial n, Z) \& port - of(Z \times 1)$ in(alan, X1). Query Yes(X1) <- in(alan, X1 yes (cs\_building)

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## **Full Propositional Logics**

### **Literal:** an atom or a negation of an atom $P \neg q$ **Clause:** is a disjunction of literals $p \lor \neg r \lor q$ Conjunctive Normal Form (CNF): a conjunction of clauses INFERENCE: KBEXX formula (P) (qv7r) (qvp)

- Convert all formulas in KB and 
   In CNF
- Apply Resolution Procedure (at each step combine two clauses containing complementary literals into a new pvg rvig -> pvr one)
- Termination

DEFs.

- KBXX • No new clause can be added
- Two clause resolve into an empty clause  $KB \rightarrow \propto$

## Propositional Logics: Satisfiability (SAT problem)

- Does a set of formulas have a model? Is there an interpretation in which all the formulas are true?
- (Stochastic) Local Search Algorithms can be used for this task!
- Evaluation Function: number of unsatisfied clauses
- WalkSat: One of the simplest and most effective algorithms:
- Start from a randomly generated interpretation
- Pick an unsatisfied clause
- Pick an proposition to flip (randomly 1 or 2)
  - 1. To minimize # of unsatisfied clauses
  - 2. Randomly

## Full First-Order Logics (FOLs)

We have constant symbols, predicate symbols and function symbols

So interpretations are much more complex (but the same basic idea – one possible configuration of the world) constant symbols => individuals, entities predicate symbols => relations function symbols => functions

### **INFERENCE:**

- Semidecidable: algorithms exists that says yes for every entailed formulas, but no algorithm exists that also says no for every non-entailed sentence
- Resolution Procedure can be generalized to FOL

## TODO for next Tue

Thurs class is canceled – I'll be visiting SAP labs in France for a workshop on Business Intelligence This week Focus on
Complete the assignment!
Study carefully handout on Probability (critical for the rest of the lectures!)

Also Do exercises 5.A and 12.A,B,C http://www.aispace.org/exercises.shtml

### **Expressiveness of the language**

Figure 10.1 A PDDL description of an air cargo transportation planning problem.

## Planning with Structured Rep



## Situation Calculus

- Convenient to have more expressive lang.
   "Move all the cargo from SFO to JFK"
- Use existing mechanisms for logical proof
- Strong foundation for studying planning
- Still, less used in practice than other techniques

## Situation Calculus

- Possibility Axioms (for each action)
  - SomeFormula(s)  $\Rightarrow$  Poss(a, s)
  - Alive(Agent, s) ∧ Have(Agent, Arrow, s) ⇒ Poss(Shoot, s)
- Successor-state Axiom (for each fluent)
  - Poss(a, s) ⇒(fluent is true ⇔ a made it true
     ∨ it was true and a left it alone)
  - Poss(a, s) ⇒ (Holding(Agent, g, Result(s, a)) ⇔
     a = Grab(g) ∨
     (Holding(Agent, g, s) ∧ a ≠ Release(g)))

## CS 221: Artificial Intelligence

(Some) Slide credit: Peter Norvig and Sebastian Thrun Dan Klein, Stuart Russell, Andrew Moore

# Soundness & completeness of proof procedures

• A proof procedure X is sound ...



 We proved this in general even for domains represented by thousands of propositions and corresponding KB with millions of definite clauses !

Can you think of a proof procedure for PDCL CA = fall clauses with empty bodies } KR *a* ← *e* ∧ *g*.  $KBH_AG G \leq C_A \leq C_{BU}$  $b \leftarrow f \land g$ . 3: CB= Sall atoms of KB{  $C \leftarrow e$ . KBBG  $G \in G$ That is sound but not complete? t ← C ∧ e. е. KBFAGI=>KBFG E socdet g L=> G C C A C C BU => KB + BU G => KB = G That is complete but not sound? soundness of BU KBEG =>KBEG KBEG => KBEBUG => GECBU => GECB => KBEBG CPSC 502, Lecture 6 Slide 56

### Completeness of Bottom Up (proof summary)



- Suppose  $KB \models G$ .
- Then G is true in all the models
- · Thus G is true in the minimal model
- Thus G C
- Thus G is proved by ... BU Soundness

• THUS KB BU G I.e. KB F G RELATION KB F G RELATION LESS BU GO RELATION COMPLETENESS Completeness BET 50 & COMPLETENESS COMPLETENESS

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### Sampling a discrete probability distribution e.g. Sim. Amesling. Select n' with probability P generate randou [9,1]) 17<.3 accept n' e.g. Beam Search : Select K individuals. Probability of selection proportional to their value N3 first sample SAME HERE P1= .1 -> N1 ->N2 $P_{2=}$ CPSC 502, Lecture 6 Slide 58

# Now, do you know how to implement a planner for....

- Emergency Evacuation?
- Robotics?
- Space Exploration?
- Manufacturing Analysis?
- Games (e.g., Bridge)?
- Generating Natural language <</li>
  - Product Recommendations ....







### Logics in AI: Similar slide to the one for planning



### Logics as a R&R system



reason about it

if the agent Knows ON-SW1 and live\_w3 it should be able to infer Son-ly

### Propositional (Definite Clauses) Logic: Syntax

We start from a restricted form of Prop. Logic:

Only two kinds of statements

- that a proposition is true
- that a proposition is true if one or more other propositions are true

 $(P \downarrow V P z) \Rightarrow (P \downarrow V 7 P z)$ 



