Intelligent Systems (AI-2)

Computer Science cpsc422, Lecture 18

Feb, 26, 2021

Slide Sources
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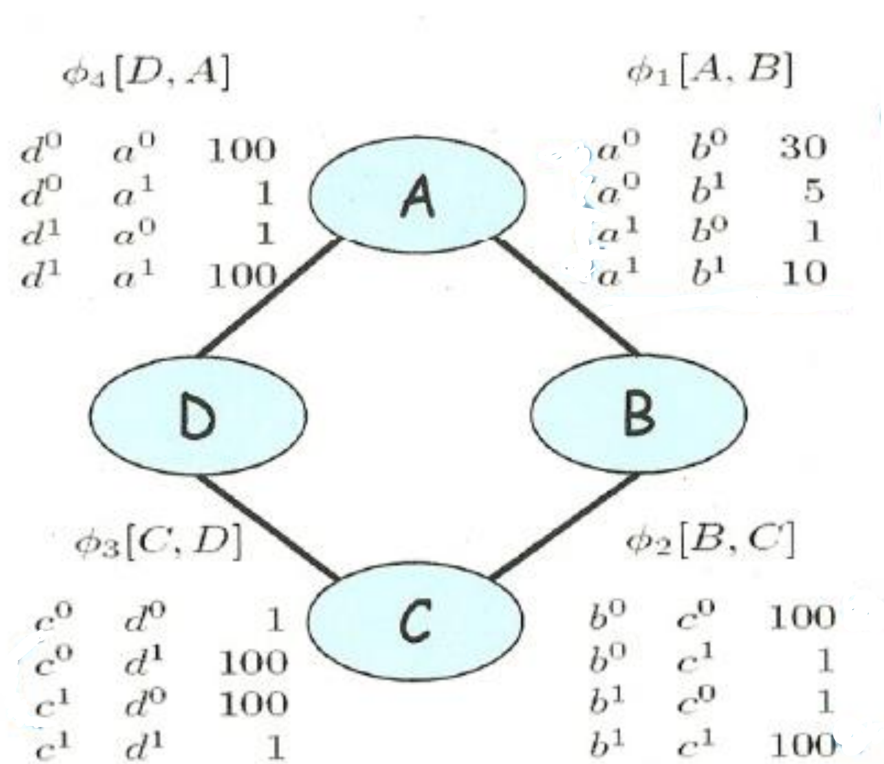
D. Koller, Stanford CS - Probabilistic Graphical Models

Lecture Overview

Probabilistic Graphical models

- **Recap Markov Networks**
- **Inference in Markov Networks (Exact and Approx.)**
- **Conditional Random Fields**

Parameterization of Markov Networks



X set of random vars: A factor is $\Phi(Val(X)) \rightarrow \mathbb{R}$

Factors define the local interactions (like CPTs in Bnets)

What about the global model? What do you do with Bnets?

How do we combine local models?

As in BNets by multiplying them!

$$\tilde{P}(A,B,C,D) = \phi_1(A,B) \times \phi_2(B,C) \times \phi_3(C,D) \times \phi_4(A,D)$$

$$P(A,B,C,D) = \frac{1}{Z}\tilde{P}(A,B,C,D)$$

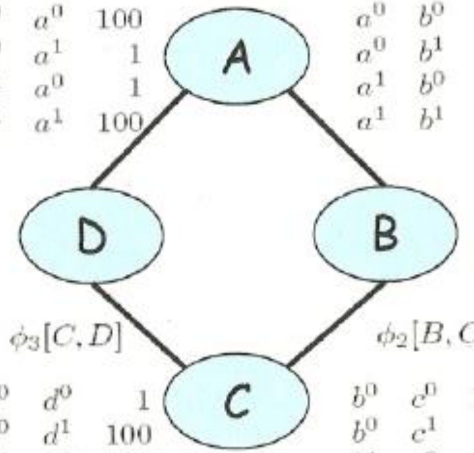
Assignment				Unnormalized	Normalized
a^0	b^0	c^0	d^0	300000	.04
a^0	b^0	c^0	d^1	300000	.04
a^0	b^0	c^1	d^0	300000	.04
a^0	b^0	c^1	d^1	30	4.1×10^{-6}
a^0	b^1	c^0	d^0	500	.
a^0	b^1	c^0	d^1	500	.
a^0	b^1	c^1	d^0	5000000	.69
a^0	b^1	c^1	d^1	500	.
a^1	b^0	c^0	d^0	100	.
a^1	b^0	c^0	d^1	1000000	.
a^1	b^0	c^1	d^0	100	.
a^1	b^0	c^1	d^1	100	.
a^1	b^1	c^0	d^0	10	.
a^1	b^1	c^0	d^1	100000	.
a^1	b^1	c^1	d^0	100000	.
a^1	b^1	c^1	d^1	100000	.

$\phi_4[D,A]$

d^0	a^0	100
d^0	a^1	1
d^1	a^0	1
d^1	a^1	100

$\phi_1[A,B]$

a^0	b^0	30
a^0	b^1	5
a^1	b^0	1
a^1	b^1	10



$\phi_3[C,D]$

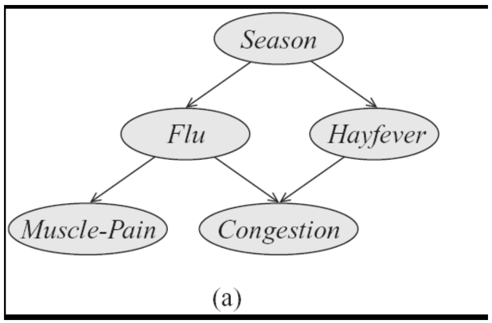
c^0	d^0	1
c^0	d^1	100
c^1	d^0	100
c^1	d^1	1

$\phi_2[B,C]$

b^0	c^0	100
b^0	c^1	1
b^1	c^0	1
b^1	c^1	100

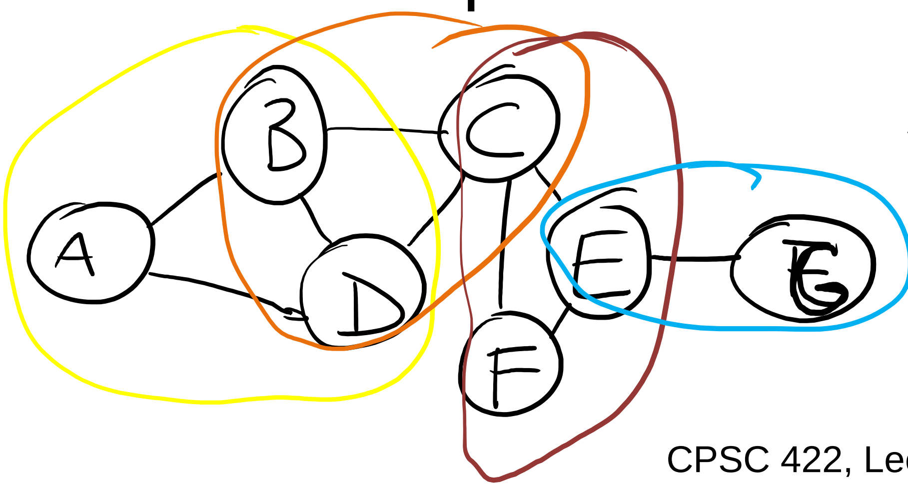
Step Back…. From structure to factors/potentials

In a Bnet the joint is factorized….



(a)

In a Markov Network you have one factor for each maximal clique



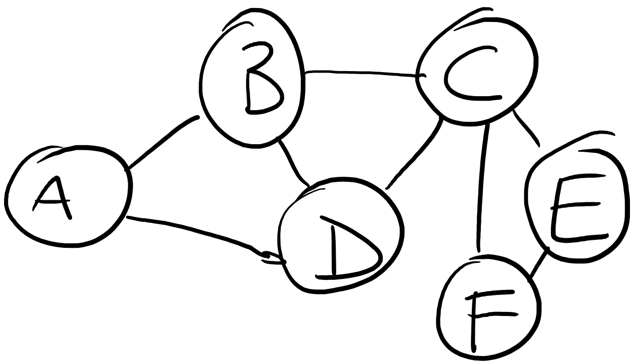
$\Phi_1(A\,B\,D)$

$\Phi_2(B\,D\,C)$ $\Phi_4(E\,G)$

$\Phi_3(C\,E\,F)$

General definitions

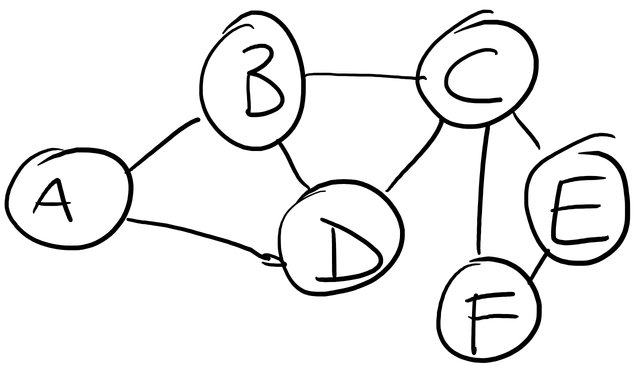
Two nodes in a Markov network are **independent** if and only if every path between them is cut off by evidence



eg for A C

So the **markov blanket** of a node is…?

eg for C



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Variable elimination algorithm for Bnets

Given a network for $P(Z, Y_1, \ldots, Y_j, Z_1, \ldots, Z_i)$ **:**

To compute $P(Z \mid Y_1=v_1, \ldots, Y_j=v_j)$ **:**

1. Construct a factor for each conditional probability.
2. Set the observed variables to their observed values.
3. Given an elimination ordering, simplify/decompose sum of products
4. Perform products and sum out Z_i
5. Multiply the remaining factors Z
6. Normalize: divide the resulting factor $f(Z)$ by $\sum_Z f(Z)$.

Variable elimination algorithm for Markov Networks…..

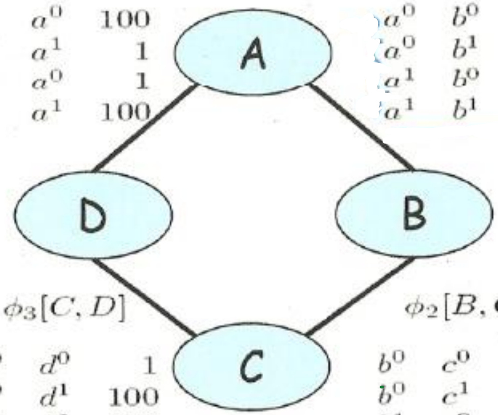
Variable Elimination on MN: Example

$\phi_4[D, A]$

d^0	a^0	100
d^0	a^1	1
d^1	a^0	1
d^1	a^1	100

$\phi_1[A, B]$

a^0	b^0	30
a^0	b^1	5
a^1	b^0	1
a^1	b^1	10



$\phi_3[C, D]$

c^0	d^0	1
c^0	d^1	100
c^1	d^0	100
c^1	d^1	1

$\phi_2[B, C]$

b^0	c^0	100
b^0	c^1	1
b^1	c^0	1
b^1	c^1	100

Example compute

$P(D \mid b^0)$ — Z — $B = Y_1$

Set observed vars

Elimination ordering: A C

$\alpha \sum \quad \sum$

Now it is just a matter of multiplying factors and summing out vars

Normalize at the end!

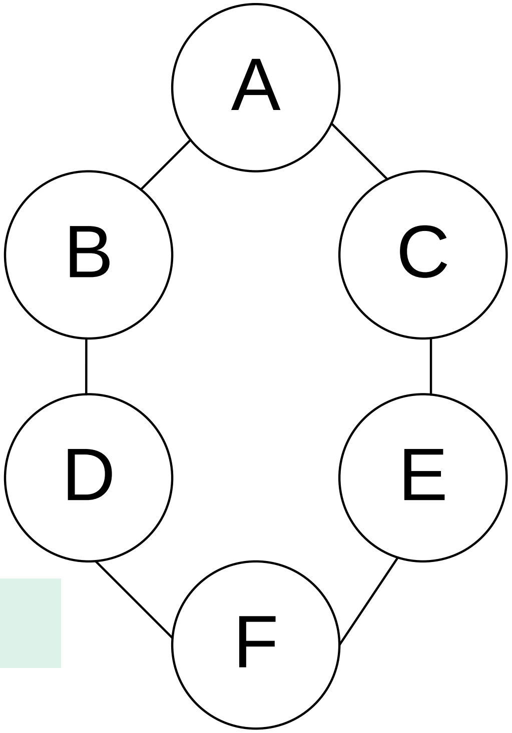
Gibbs sampling for Markov Networks

Example: P(D | C=0)

Note: never change evidence!

Resample non-evidence variables in a pre-defined order or a random order

Suppose we begin with A



What do we need to sample?

a. P(A | B=0**)**

b. P(A | B=0, C=0**)**

c. P(B=0, C=0**| A)**

A	B	C	D	E	F
1	0	0	1	1	0

Initial assigmnet

Gibbs sampling MN: what to sample

For Bnets $P(x_i'|mb(X_i)) = \alpha P(x_i'|parents(X_i))\Pi_{Z_j \in Children(X_i)} P(z_j|parents(Z_j))$

For Markov Networks just the product of the factors involving X (normalized)

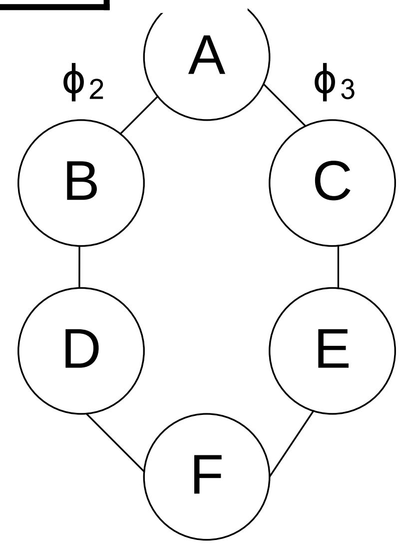
	A=1	A=0
C=1	1	2
C=0	3	4

Resample probability distribution of P(A|BC)

	A=1	A=0
B=1	1	5
B=0	4.3	0.2

B=0 ; C=0

A	B	C	D	E	F
1	0	0	1	1	0
?	0	0	1	1	0



$$\Phi_2 \times \Phi_3 =$$

A=1	A=0
12.9	0.8

A=1	A=0
0.95	0.05

Example: Gibbs sampling

Resample probability distribution of B given A D

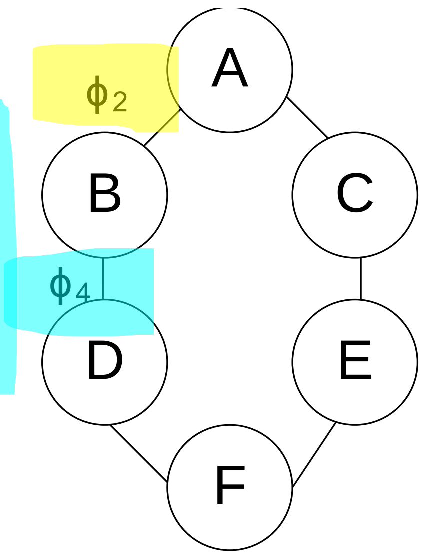
	A=1	A=0
B=1	1	5
B=0	4.3	0.2

A	B	C	D	E	F
1	0	0	1	1	0
1	0	0	1	1	0
1	?	0	1	1	0

$\phi_2 \times \phi_4 =$

B=1	B=0
1	??

	D=1	D=0
B=1	1	2
B=0	2	1



B=1	B=0
0.11	0.89

A. 10

B. 0.4

C. 8.6

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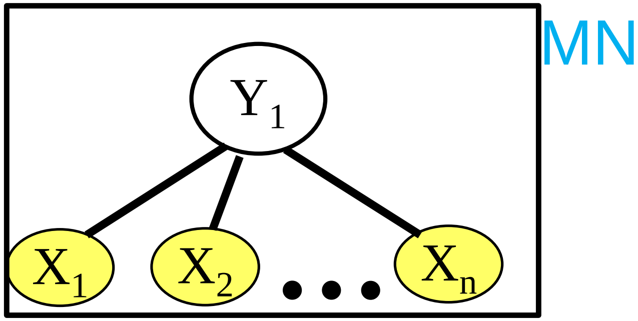
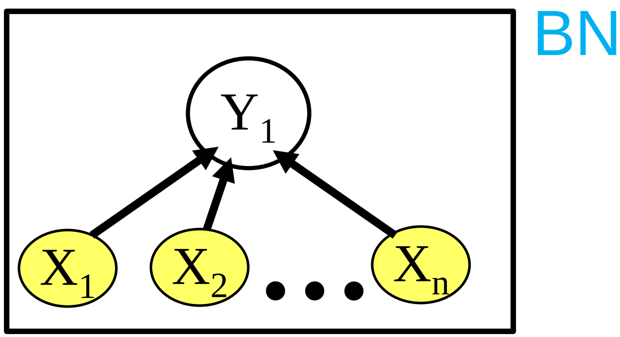
We want to model $P(Y_1 | X_1 .. X_n)$

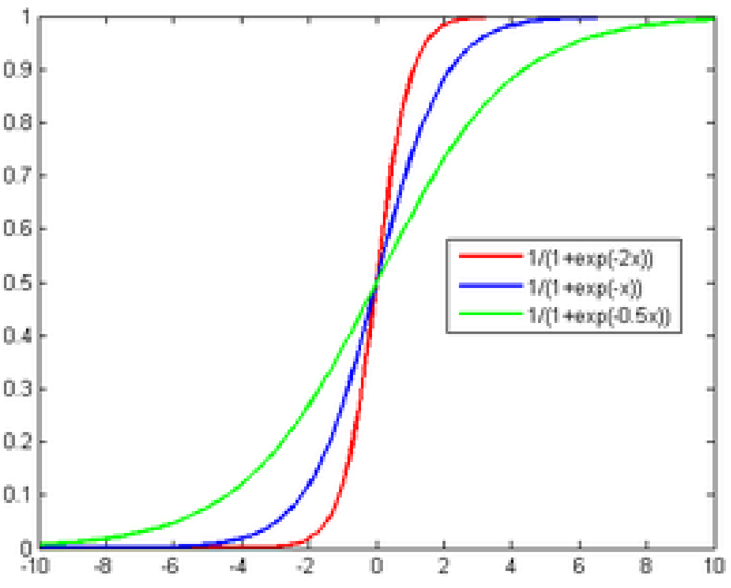
… where all the X_i are always observed





- Which model is simpler, MN or BN?

- Naturally aggregates the influence of different parents



Conditional Random Fields (CRFs)

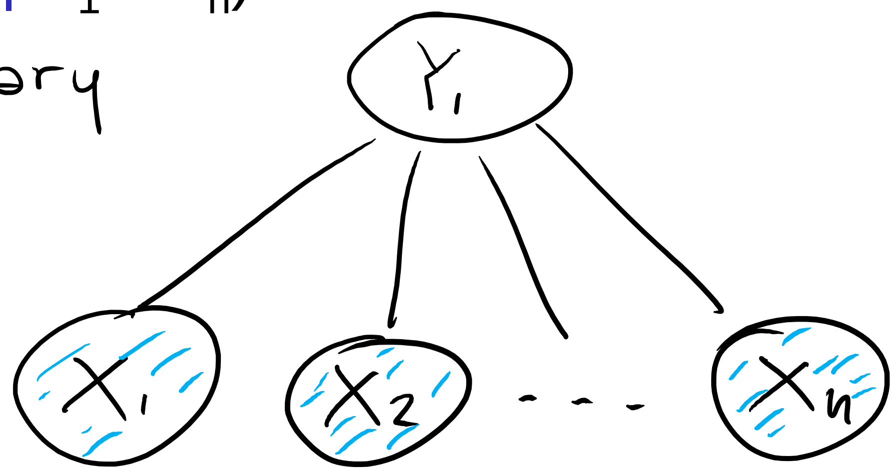
- Model $P(Y_1 .. Y_k | X_1 .. X_n)$
- Special case of Markov Networks where all the X_i are always observed

  - Simple case $P(Y_1 | X_1 \ldots X_n)$

all vars are binary

$Y_1 = \{0, 1\}$

$\forall i \; X_i = \{0, 1\}$



Some notation: exp and indicator function

exp and indicator function

$\exp(z)$

e^{z}

$\mathbb{1}(\text{Predicate}(x)) \rightarrow 1$ or 0

Exaples

$X = \{1\ 2\ 3\ 4\ 5\}$ ← set

$\sum_{x_i \in X} x_i =$

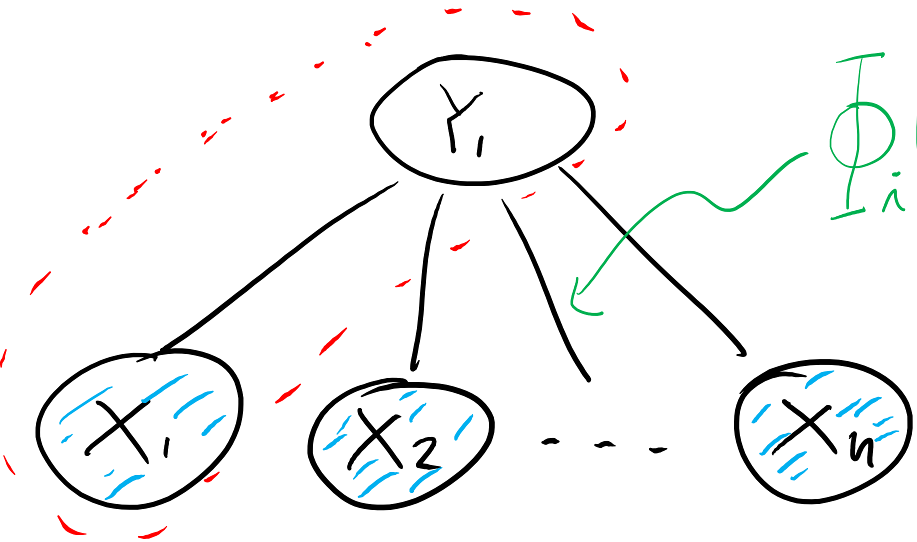
$\sum \mathbb{1}\,\text{Even}(x_i)$

$\sum \mathbb{1}\,\text{Prime}(x_i)$

$=$

$=$

What are the Parameters?



$$\Phi_i(X_i, Y_1) = \exp\{w_i * \mathbb{1}\{X_i = 1, Y_1 = 1\}\}$$

one such factor for each clique

also $\Phi_0(Y_1) = \exp\{w_0 \mathbb{1}\{Y = 1\}\}$

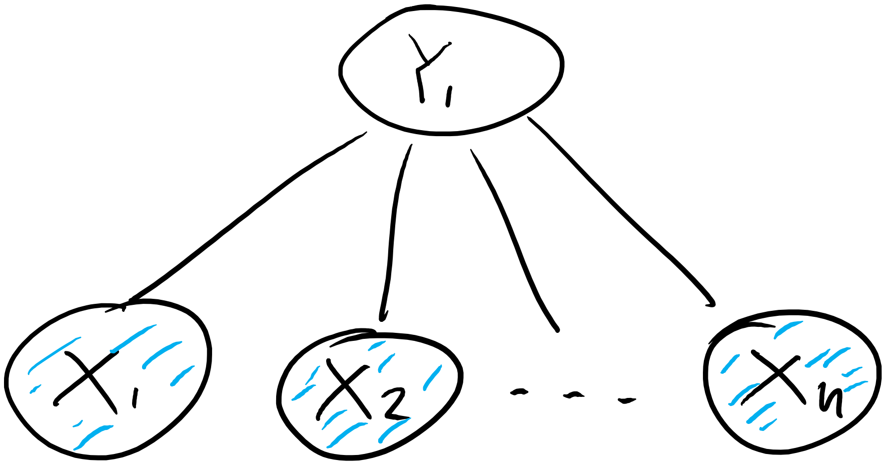
Example $w_2 = 1.5$ $\Phi_2(X_2, Y_1)$

X_2	Y_1	Φ_2
1	1	$e^{1.5}$
0	1	1
1	0	1
0	0	1

Example $w_0 = .4$

Y_1	Φ_0
0	1
1	$e^{.4}$

Let's derive the probabilities we need



To compute

$P(\mathbf{Y_1} | X_1 \ldots X_n) = P(\mathbf{Y_1}, X_1 \ldots X_n) / P(X_1 \ldots X_n)$

We compute

$P(\mathbf{Y_1} = 1 | X_1 \ldots X_n) = P(\mathbf{Y_1} = 1, X_1 \ldots X_n) / P(X_1 \ldots X_n)$

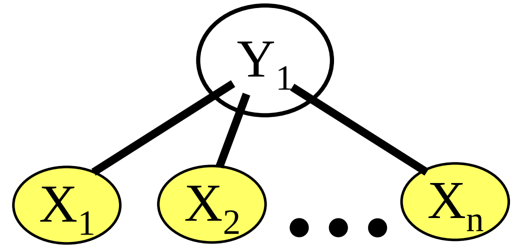
$P(Y_1 = 1, X_1 \ldots X_n) + P(Y_1 = 0, X_1 \ldots X_n)$

a a b

Let's derive the probabilities we need

$$\phi_i(X_i, Y_1) = \exp\{w_i * \mathbb{1}\{X_i = 1, Y_1 = 1\}\}$$

$$\phi_0(Y_1) = \exp\{w_0 * \mathbb{1}\{Y_1 = 1\}\}$$



$$\tilde{P}(Y_1 = 1, x_1, x_2, \ldots, x_n) = \Phi_0(Y_1) * \prod_{i=1}^{n} \Phi_i(x_i, Y_1)$$

A. $e^{\sum_1^n w_i}$

B. $e^{w_0 + \sum_1^n w_i * x_i}$

D $e^{w_0 + \sum_1^n w_i}$

C. $e^{w_0 + \sum_1^n x_i}$

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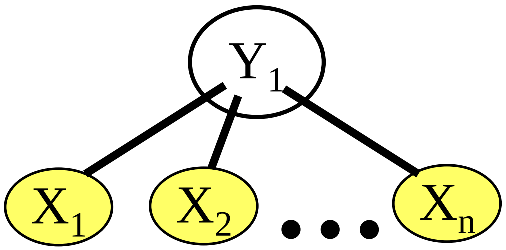
Let's derive the probabilities we need

$$\phi_i(X_i, Y_1) = \exp\{w_i \mathbb{1}\{X_i = 1, Y_1 = 1\}\}$$



$$\phi_0(Y_1) = \exp\{w_0 \mathbb{1}\{Y_1 = 1\}\}$$

$$\tilde{P}(Y_1 = 0, x_1, x_2, \ldots, x_n) = \Phi_0(Y_1) * \prod_{i=1}^{n} \Phi_i(x_i, Y_1)$$

A. 1

B. e^{w_0}

C. 0

D. $e^{\sum_{1}^{n} w_i}$

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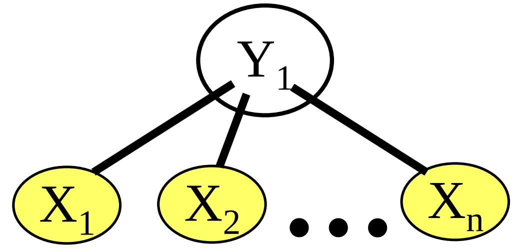
Let's derive the probabilities we need

(a) $\tilde{P}(Y_1 = 1, x_1, \ldots, x_n) = \exp(w_0 + \sum_{i=1}^{n} w_i x_i)$

(b) $\tilde{P}(Y_1 = 0, x_1, \ldots, x_n)$ ...

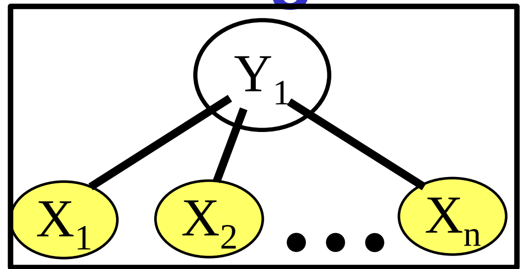


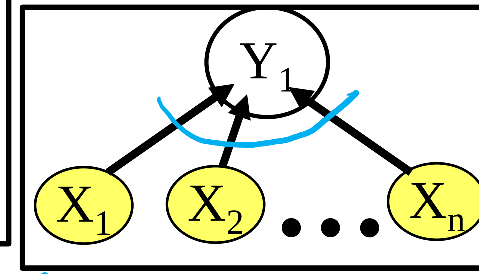
$$P(Y_1 = 1 \mid x_1, \ldots, x_n) = \frac{\tilde{P}(Y_1 = 1, x_1, \ldots, x_n)}{P(x_1, \ldots, x_n)}$$ ← sum of (a) and (b)

$$= \frac{\exp(w_0 + \sum w_i x_i)}{\ldots + \exp(w_0 + \sum w_i x_i)}$$

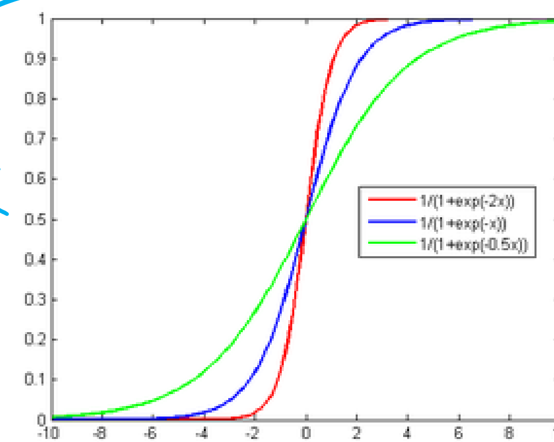
$$\frac{e^z}{1 + e^z}$$ or

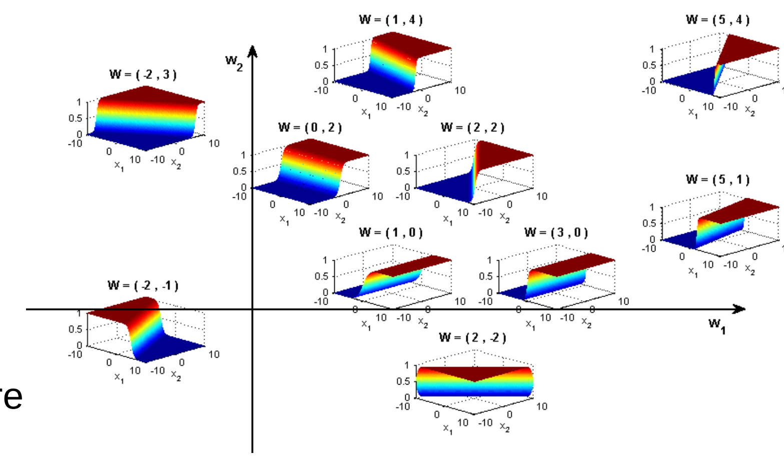
Sigmoid Function used in Logistic Regression





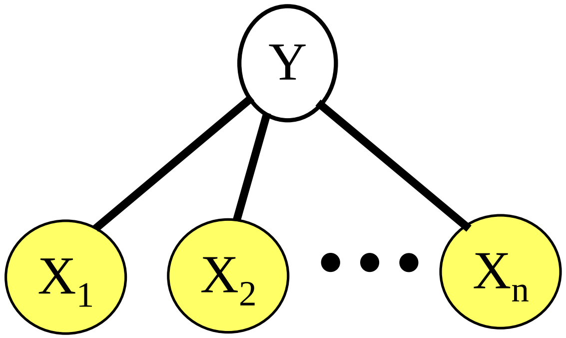
- Great practical interest
- Number of param w_i is linear instead of exponential in the number of parents
- Natural model for many real-world applications
- Naturally aggregates the influence of different parents





Logistic Regression as a Markov Net (CRF)

Logistic regression is a simple Markov Net (a CRF) *aka* **naïve markov model**



- But only models the **conditional distribution**, P(*Y* | ***X***) and not the full joint P(*X*,*Y*)

Learning Goals for today's class

You can:

- Perform Exact and Approx. Inference in Markov Networks
- Describe a natural parameterization for a Naïve Markov model (which is a simple CRF)
- Derive how P(Y|X) can be computed for a Naïve Markov model
- Explain the discriminative vs. generative distinction and its implications

Assignment 2 – due on Mon

Next class Mon Linear-chain CRFs

To Do Revise generative temporal models (HMM)

Midterm, Mon, March 8,

How to prepare….

- Go to **Office Hours**
- **Learning Goals** (look at the end of the slides for each lecture – complete list will be posted)
- Revise all the **clicker questions, practice exercises, assignments**
- M**ore practice material** will be posted
- Check questions and answers on Piazza