# Intelligent Systems (AI-2)

## Computer Science cpsc422, Lecture 22

## Oct, 28, 2019

Slide credit: some from Prof. Carla P. Gomes (Cornell)
some slides adapted from Stuart Russell (Berkeley), some from Prof. Jim Martin (Univ. of Colorado)

# Lecture Overview

- **SAT : example**
- **First Order Logics**
  - Language and Semantics
  - Inference

# Satisfiability problems (SAT)

Consider a CNF sentence, e.g.,

$(\neg D \lor \neg B \lor C) \land (B \lor \neg A \lor \neg C) \land (\neg C \lor \neg B \lor E)$
$\land (E \lor \neg D \lor B) \land (B \lor E \lor \neg C)$

*Is there an interpretation in which this sentence is true (i.e., that is a model of this sentence )?*

Many combinatorial problems can be reduced to checking the satisfiability of propositional sentences ……and returning a model

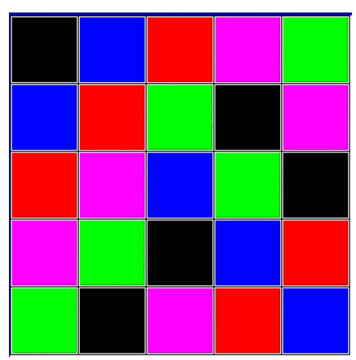
# Encoding the Latin Square Problem in Propositional Logic

In **combinatorics** and in experimental design, a **Latin square** is

- an ***n* × *n* array**
- filled with ***n* different sym**bols,
- each occurring exactly once in each row and exactly once in each column.
- Here is an example:

| | | |
|---|---|---|
| A | B | C |
| C | A | B |
| B | C | A |

Here is another one:

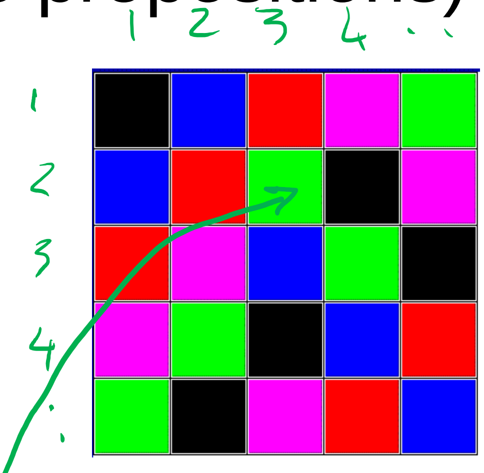


# Encoding Latin Square in Propositional Logic: Propositions

## Variables must be binary! (They must be propositions)

Each variables represents a color assigned to a cell ***i j***.

Assume colors are encoded as an integer ***k***




$$x_{ijk} \in \{0,1\}$$

Assuming colors are encoded as follows

(black, 1) (red, 2) (blue, 3) (green, 4) (purple, 5)

$x_{234} = 1$

$x_{233} = 0$ True or false, ie. 1 or 0 with respect to the interpretation represented by the picture?

How many vars/propositions overall? $n^3$

# Encoding Latin Square in Propositional Logic

## Variables must be binary! (They must be propositions)

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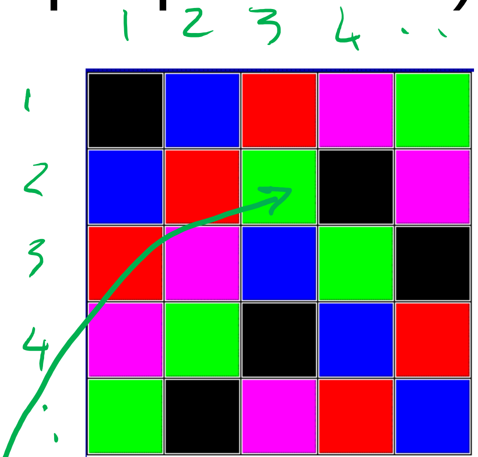


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How many vars/propositions overall? $n^3$

# Encoding Latin Square in Propositional Logic: Clauses

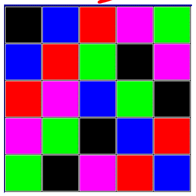
- Some color must be assigned to each cell (clause of length n);

$$\forall_{ij} (x_{ij1} \vee x_{ij2} \ldots x_{ijn}) \qquad \forall_{ik} (x_{i1k} \vee x_{i2k} \cdots x_{ink})$$

A. B.

j

i



k = color

- No color is repeated in the same row (sets of negative binary clauses);

$$\forall_{ik} (\neg x_{i1k} \vee \neg x_{i2k}) \wedge (\neg x_{i1k} \vee \neg x_{i3k}) \ldots (\neg x_{i1k} \vee \neg x_{ink}) \cdots (\neg x_{i(n-1)k} \vee \neg x_{ink})$$

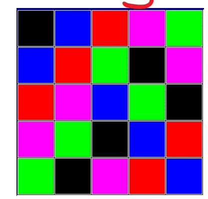
How many clauses?

# Encoding Latin Square in Propositional Logic: Clauses

- Some color must be assigned to each cell (clause of length n);

$$\forall_{ij}(x_{ij1} \vee x_{ij2} \ldots x_{ijn})$$

$i$ ↓ $j$ →



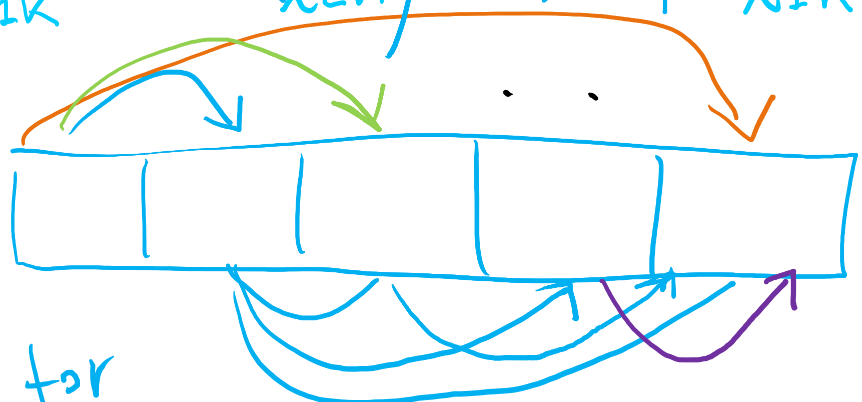
$k$ = color

- No color repeated in the same row (sets of negative binary clauses);

$$\forall_{ik}(\neg x_{i1k} \vee \neg x_{i2k}) \wedge (\neg x_{i1k} \vee \neg x_{i3k}) \ldots (\neg x_{i1k} \vee \neg x_{ink}) \cdots (\neg x_{i(n-1)k} \vee \neg x_{ink})$$

$n * n = n^2$

$\neg(x_{i1k} \wedge x_{i2k}) \Rightarrow \neg x_{i1k} \vee \neg x_{i2k}$



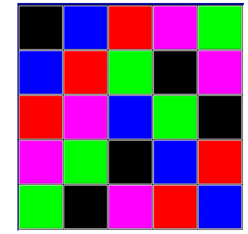

same for

$\frac{n*(n-1)}{2}$

How many clauses?

$n^2 * \frac{n*(n-1)}{2}$ $O(n^4)$

# Encoding Latin Square Problems in Propositional Logic: FULL MODEL



$n^3$

Variables: $x_{ijk}$ $cell\ i,j\ has\ color\ k;\ \ i,j,k = 1,2,\ldots,n.$ $\quad x_{ijk} \in \{0,1\}$

Each variables represents a color assigned to a cell.

Clauses: $O(n^4)$

- Some color must be assigned to each cell (clause of length n);

$$\forall_{ij}(x_{ij1} \vee x_{ij2} \ldots x_{ijn})$$

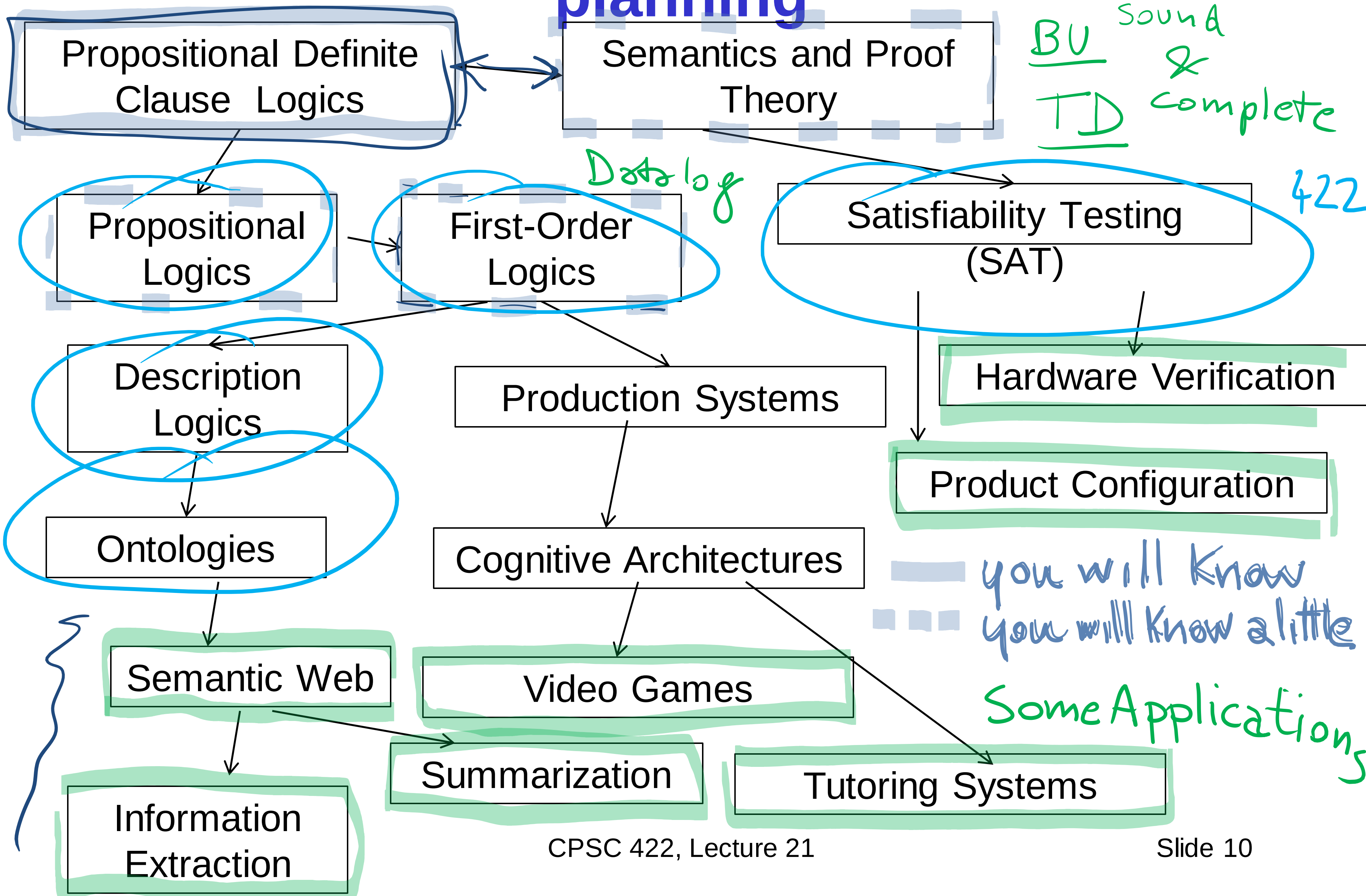
- No color repeated in the same row (sets of negative binary clauses);

$$\forall_{ik}(\neg x_{i1k} \vee \neg x_{i2k}) \wedge (\neg x_{i1k} \vee \neg x_{i3k}) \ldots (\neg x_{i1k} \vee \neg x_{ink}) \cdots (\neg x_{i(n-1)k} \vee \neg x_{ink})$$

- No color repeated in the same column (sets of negative binary clauses);

$$\forall_{jk}(\neg x_{1jk} \vee \neg x_{2jk}) \wedge (\neg x_{1jk} \vee \neg x_{3jk}) \ldots (\neg x_{1jk} \vee \neg x_{njk}) \cdots (\neg x_{(n-1)jk} \vee \neg x_{njk})$$

# Logics in AI: Similar slide to the one for planning



# Relationships between different Logics (better with colors)

First Order Logic

$\forall X \exists Y p(X,Y) \Leftrightarrow \neg q(Y)$

$p(a_1, a_2)$

$\neg q(a_5)$

...

Datalog

$p(X) \leftarrow q(X) \wedge r(X,Y)$

$r(X,Y) \leftarrow s(Y)$

$s(a_1), q(a_2)$

Propositional Logic

$\neg(p \vee q) \rightarrow (r \wedge s \wedge f),$

$p, r$

PDCL

$p \leftarrow s \wedge f$

$r \leftarrow s \wedge q \wedge p$

$r$

$p$

# Lecture Overview

- Finish SAT (example)
- **First Order Logics**
  - Language and Semantics
  - Inference

# Representation and Reasoning in Complex domains (from 322)

- In complex domains expressing knowledge with **propositions** can be quite limiting

p
q
r
s

$up\_s_2$
$up\_s_3$
$ok\_cb_1$
$ok\_cb_2$
$live\_w_1$
$connected\_w_1\_w_2$

- It is often natural to consider **individuals** and their **properties**

$up(s_2)$
$up(s_3)$
$ok(cb_1)$
$ok(cb_2)$
$live(w_1)$
$connected(w_1, w_2)$

There is no notion that the system can reason about

$\boxed{\begin{array}{l} up\_s_2 \\ up\_s_3 \end{array}}$

up are about the same property

$live\_w_1$
$connected\_w_1\_w_2$

$w_1$ are about the same individual

# (from 322) What do we gain….

By breaking propositions into relations applied to individuals?

- Express **knowledge** that **holds for set of individuals** (by introducing variables )

$$live(W) \leftarrow connected\_to(W,W1) \wedge live(W1) \wedge wire(W) \wedge wire(W1).$$

- We can **ask generic queries** (i.e., containing vars variables )

? *connected_to(W, $w_1$)*

# "Full" First Order Logics (FOL)

LIKE DATALOG: Whereas propositional logic assumes the world contains facts, FOL (like natural language) assumes the world contains

- Objects: people, houses, numbers, colors, baseball games, wars, …
- Relations: red, round, prime, brother of, bigger than, part of, comes between, …
- Functions: father of, best friend, one more than, plus, …

FURTHERMORE WE HAVE

- More Logical Operators:….
- Equality: coreference (two terms refer to the same object)
- Quantifiers
  - ✓ Statements about unknown objects
  - ✓ Statements about classes of objects

# Syntax of FOL

| | |
|---|---|
| **Constants** | KingJohn, 2, ,... |
| **Predicates** | Brother, >,... |
| **Functions** | Sqrt, LeftLegOf,... |
| **Variables** | x, y, a, b,... |
| **Connectives** | $\neg$, $\Rightarrow$, $\wedge$, $\vee$, $\Leftrightarrow$ |
| **Equality** | = |
| **Quantifiers** | $\forall$, $\exists$ |

# Atomic sentences

**Term** is a *function* (*$term_1$,...,$term_n$*) or *constant* or *variable*

**Atomic sentence** is *predicate* (*$term_1$,...,$term_n$*) or *$term_1$ = $term_2$*

E.g.,

predicate(constant, constant)

- *Brother(KingJohn, RichardTheLionheart)*

predicate(function(function(constant)), (function(function(constant)))

- *> (Length( LeftLegOf(Richard)), Length(LeftLegOf(KingJohn)))*

# Complex sentences

Complex sentences are made from atomic sentences using connectives

$$\neg S, \quad S_1 \wedge S_2, \quad S_1 \vee S_2, \quad S_1 \Rightarrow S_2, \quad S_1 \Leftrightarrow S_2,$$

E.g.

*Sibling(KingJohn, Richard)* $\Rightarrow$ *Sibling(Richard, KingJohn)*

$\forall$x *P(x)* is true in an interpretation ***I*** iff *P* is true with *x* being each possible object in ***I***

$\exists$x P(x) is true in an interpretation I iff P is true with x being some possible object in I

# Truth in first-order logic

Like in Prop. Logic a sentences is true with respect to an interpretation

$\neg A \wedge (B \Rightarrow C)$,

$2^3$

| A | B | C | |
|---|---|---|---|
| T | F | F | × |
| F | T | T | ✓ |

In FOL interpretations are **much more complex** but still same idea possible configuration of the world

symbols: CONSTANTS → objects; Predicates → relations; Functions → functions

2 objects △ □

2 CONSTANT SYMBOLS $\{c_1, c_2\}$ → $c_1$ → △, $c_2$ → □

1 unary Predicate $P$ → $\{\triangle\}$

1 binary Predicate $Q$ → $\{\{\triangle, \triangle\}\}$

IS $\forall x\, P(x)$ TRUE?

A. yes
B. no

iclicker

# Truth in first-order logic

Like in Prop. Logic a sentences is true with respect to an interpretation

$\neg A \wedge B \Rightarrow C,$ $\quad 2^3$

| A | B | C | |
|---|---|---|---|
| T | F | F | ✗ |
| T | T | T | ✓ |

In FOL interpretations are much more complex but still same idea: possible configuration of the world

symbols $\left\{ \begin{array}{l} \text{CONSTANTS} \rightarrow \text{objects} \\ \text{Predicates} \rightarrow \text{relations} \\ \text{Functions} \rightarrow \text{functions} \end{array} \right.$

2 objects △ □

2 CONSTANT SYMBOLS $\{c_1, c_2\}$ → $c_1$ → △, $c_2$ → □

1 unary Predicate P → {△}

1 binary Predicate Q → {{△, △}}

is $\forall x\, P(x)$ TRUE? NO

but if P → {△, □} Yes!

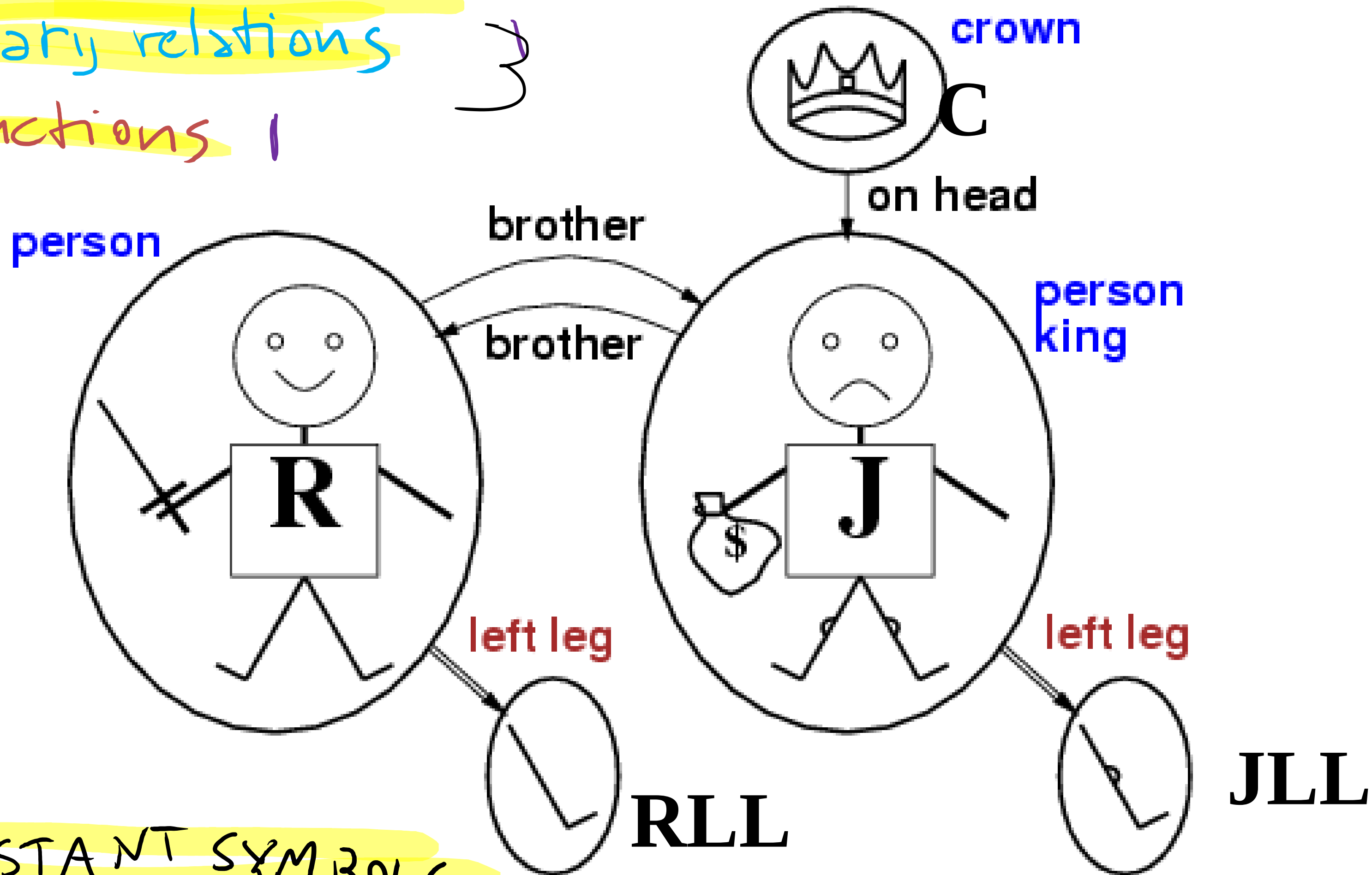
# Interpretations for FOL: Example

5 objects

binary relations 2

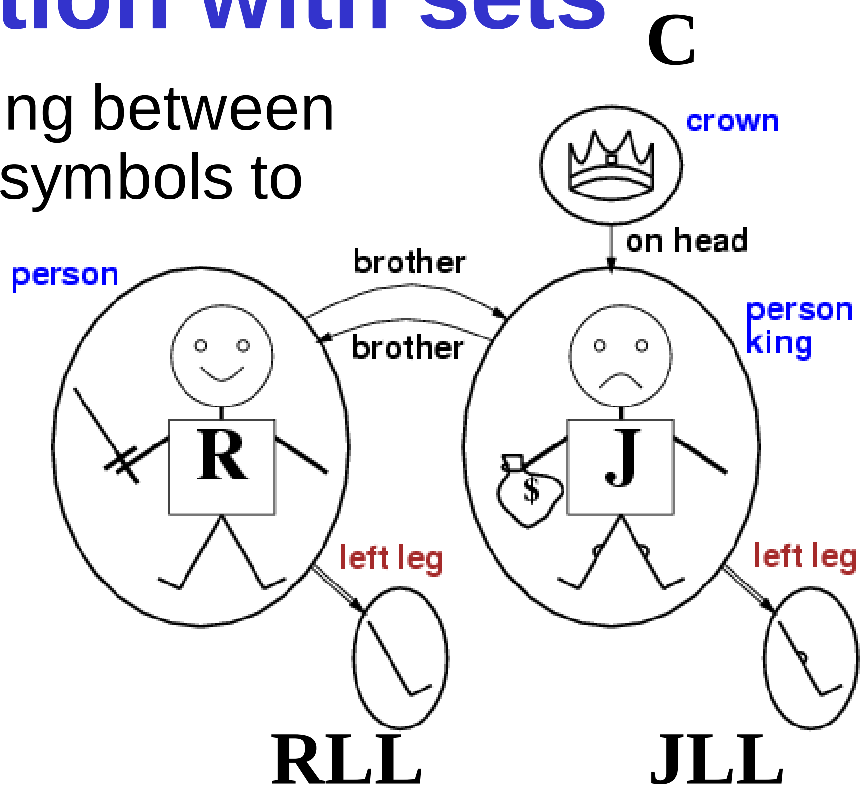
unary relations 3

functions 1




CONSTANT SYMBOLS 5

# Same interpretation with sets




Since we have a one to one mapping between symbols and object we can use symbols to refer to objects

- {R, J, RLL, JLL, C}

## Property Predicates

- Person = {R, J}
- Crown = {C}
- King = {J}

## Relational Predicates

- Brother = { <R,J>, <J,R>}
- OnHead = {<C,J>}

## Functions

- LeftLeg = {<R, RLL>, <J, JLL>}

# How many Interpretations with….

assuming unique names

5 Objects and 5 symbols  5!  J  R  C  RLL  JLL
- {R, J, RLL, JLL, C}

3 Property Predicates (Unary Relations)
- Person  R  J  RLL  JLL  C  $2^5$
- Crown  0/1  0/1  0/1  0/1  0/1
- King

2 Relational Predicates

A. $2^5$  B. $2^{25}$  C. $25^2$

- Brother  25 possibilities; each one can be 0/1 so $2^{25}$
- OnHead

1 Function
- LeftLeg  $5^5$

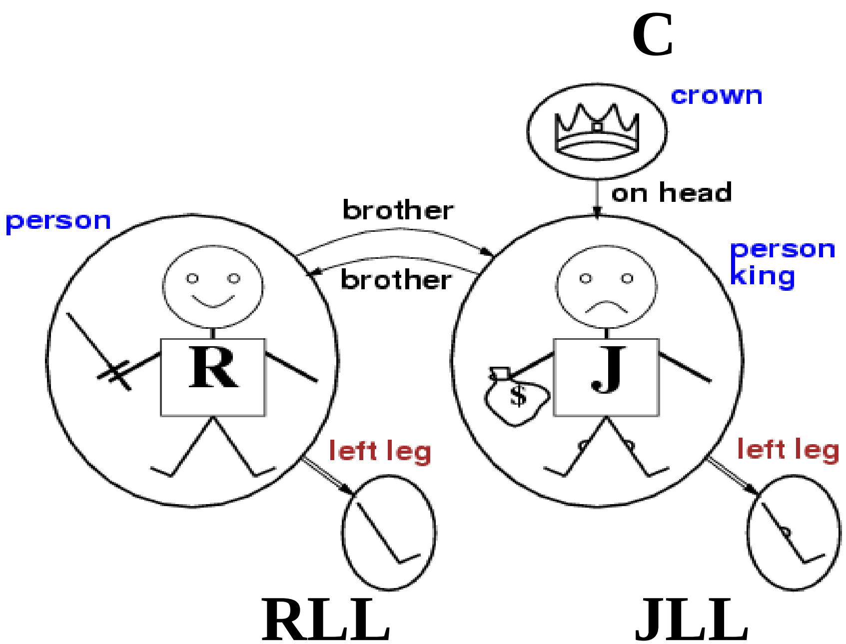
TOTAL  $5! * (2^5)^3 * (2^{25})^2 * 5^5$

# To summarize: Truth in first-order logic

- Sentences are true with respect to an **interpretation**
- World contains objects (**domain elements**)
- Interpretation specifies referents for

| | | |
|---|---|---|
| **constant symbols** | → | **objects** |
| **predicate symbols** | → | **relations** |
| **function symbols** | → | **functional relations** |

- An atomic sentence ***predicate(term$_1$,...,term$_n$)*** is true iff the **objects** referred to by ***term$_1$,...,term$_n$*** are in the **relation** referred to by ***predicate***



# Quantifiers

Allows us to express

- **Properties of collections of objects** instead of enumerating objects by name
- **Properties of an unspecified object**

Universal: “for all” $\forall$

Existential: “there exists” $\exists$

# Universal quantification

$\forall$<*variables*> <*sentence*>

Everyone at UBC is smart:

$\forall x\ At(x, UBC) \Rightarrow Smart(x)$

$\forall x$ *P* is true in an interpretation ***I*** iff *P* is true with *x* being each possible object in ***I***

Equivalent to the conjunction of instantiations of *P*

$$
\begin{aligned}
&At(KingJohn, UBC) \Rightarrow Smart(KingJohn) \\
\wedge\ &At(Richard, UBC) \Rightarrow Smart(Richard) \\
\wedge\ &At(Ralphie, UBC) \Rightarrow Smart(Ralphie) \\
\wedge\ &\ldots
\end{aligned}
$$

# Existential quantification

$\exists$<*variables*> <*sentence*>

Someone at UBC is smart:

$\exists x$ At(x, UBC) $\wedge$ Smart(x)

$\exists x$ *P* is true in an interpretation ***I*** iff *P* is true with *x* being some possible object in ***I***

Equivalent to the disjunction of instantiations of *P*

At(KingJohn, UBC) $\wedge$ Smart(KingJohn)

$\vee$ At(Richard, UBC) $\wedge$ Smart(Richard)

$\vee$ At(Ralphie, UBC) $\wedge$ Smart(Ralphie)

$\vee$ ...

# Properties of quantifiers

$\exists x\ \forall y$ is not the same as $\forall y\ \exists x$

$\exists x\ \forall y$ Loves(x,y)

- "There is a person who loves everyone in the world"

$\forall y\ \exists x$ Loves(x,y)

- "Everyone in the world is loved by at least one person"

Quantifier duality: each can be expressed using the other

$\forall x$ Likes(x,IceCream) $\neg\exists x\ \neg$Likes(x,IceCream)

$\exists x$ Likes(x,Broccoli) $\neg\forall x\ \neg$Likes(x,Broccoli)

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- Finish SAT (example)
- **First Order Logics**
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# FOL: Inference

**Resolution Procedure** can be generalized to FOL

- **Every formula** can be rewritten in **logically equivalent CNF**
    - Additional rewriting rules for quantifiers
- **Similar Resolution step,** but variables need to be unified (like in DATALOG)

$In(x, y) \lor \neg Charged(x)$ $\quad \theta = \{z/x, v/y\}$

$\neg In(z, v) \lor Connected(z)$

$\boxed{\neg Charged(x) \lor Connected(x)}$

# NLP Practical Goal for FOL: the ultimate Web question-answering system?

Map NL queries into FOPC so that answers can be effectively computed

*What African countries are not on the Mediterranean Sea?*

$$\exists c\ Country(c) \wedge \neg Borders(c, Med.Sea) \wedge In(c, Africa)$$

- *Was 2007 the first El Nino  year after 2001?*

$$ElNino(2007) \wedge \neg \exists y\ Year(y) \wedge After(y,2001) \wedge Before(y,2007) \wedge ElNino(y)$$

NL query $\xrightarrow{\text{parsing}}$ synt. tree $\xrightarrow{\lambda\text{ calculus}}$ "FOPC" query $\xrightarrow{\text{applied}}$ KB

KB $\xleftarrow{\text{into}}$ IExtraction

Web → NER

Relation identification & Extraction

# Learning Goals for today's class

**You can:**

- Explain differences between Proposition Logic and First Order Logic
- Compute number of interpretations for FOL
- Explain the meaning of quantifiers
- Describe application of FOL to NLP: Web question answering

# Next class Wed

- Ontologies (e.g., Wordnet, Probase), Description Logics…
- Midterm will be returned Fri or next Mon

## Assignment-3 will be out tonight