Intelligent Systems (AI-2)

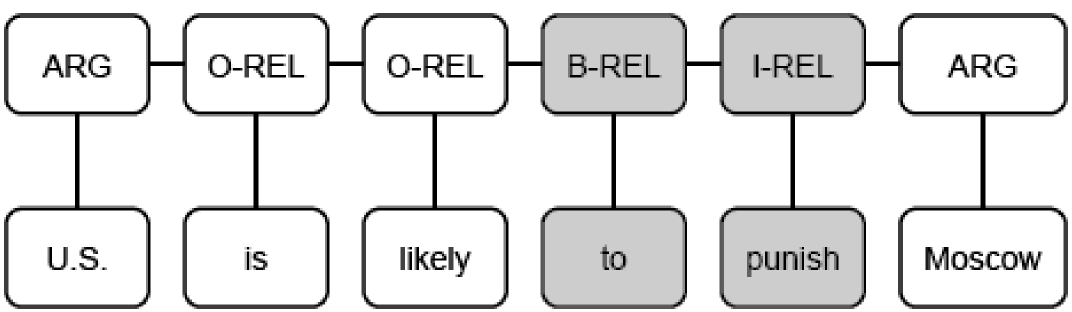
Computer Science cpsc422, Lecture 20

Oct, 21, 2019

Slide credit: some slides adapted from Stuart Russell (Berkeley), some from Padhraic Smyth (UCIrvine)

PhD thesis I reviewed 3-4 years ago…
University of Alberta
EXTRACTING INFORMATION NETWORKS FROM TEXT

We model *predicate detection* as a sequence labeling problem — …. We adopt the BIO encoding, a widely-used technique in NLP.
Our method, called Meta-CRF, is based on Conditional Random Fields (CRF) .
CRF is a graphical model that estimates a conditional probability distribution, denoted p(yjx), over label sequence y given the token sequence x.



422 big picture

StarAI (statistical relational AI)

Hybrid: Det +Sto

- *Prob CFG*
- *Prob Relational Models*
- *Markov Logics*

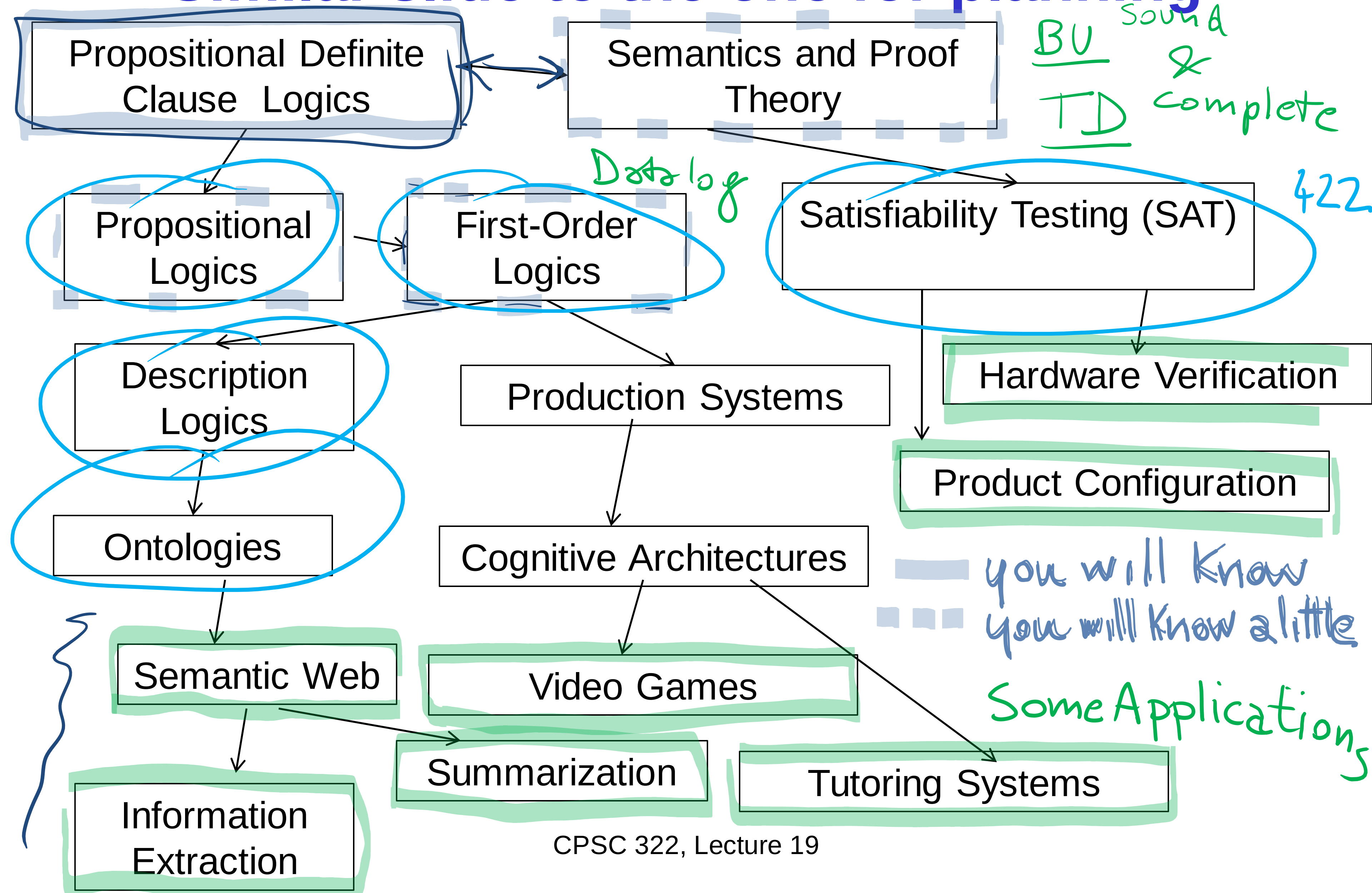
	Deterministic	Stochastic
Query	*Logics* *First Order Logics* *Ontologies* • Full Resolution • SAT	*Belief Nets* Approx. : Gibbs *Markov Chains and HMMs* Forward, Viterbi…. Approx. : Particle Filtering *Undirected Graphical Models* *Markov Networks* *Conditional Random Fields*
Planning		*Markov Decision Processes and Partially Observable MDP* • Value Iteration • Approx. Inference *Reinforcement Learning*

Applications of AI

Representation

Reasoning Technique

Logics in AI (322): Similar slide to the one for planning



Relationships between different Logics (better with colors)

First Order Logic

$\forall X \exists Y\, p(X,Y) \Leftrightarrow \neg q(Y)$

$p(a_1, a_2)$

$\neg q(a_5)$

...

Propositional Logic

$\neg(p \vee q) \rightarrow (r \wedge s \wedge f),$

p, r

Datalog

$p(X) \leftarrow q(X) \wedge r(X,Y)$

$r(X,Y) \leftarrow s(Y)$

$s(a_1), q(a_2)$

PDCL

$p \leftarrow s \wedge f$

$r \leftarrow s \wedge q \wedge p$

r

p

Lecture Overview

- **Basics Recap: Interpretation / Model /..**
- Propositional Logics
- Satisfiability, Validity
- Resolution in Propositional logics

Basic definitions from 322 (Semantics)

Definition (interpretation)
An interpretation I assigns a truth value to each atom.

Definition (truth values of statements cont'**):** A knowledge base *KB* is true in I if and only if every clause in *KB* is true in I.

PDC Semantics: Knowledge Base (KB)

- A **knowledge base KB** is true in **I** if and only if every clause in KB is true in I.

	p	q	r	s
I_1	*true*	*true*	*false*	*false*

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Which of the three KB below is *true* in I_1 ?

A

p
r
$s \leftarrow q \wedge p$

B

p
q
$s \leftarrow q$

C

p
$q \leftarrow r \wedge s$

PDC Semantics: Knowledge Base (KB)

- A **knowledge base KB** is true in I if and only if every clause in KB is true in I.

	p	q	r	s
I_1	*true*	*true*	*false*	*false*

KB_1

p
r
$s \leftarrow q \land p$

KB_2

p
q
$s \leftarrow q$

KB_3

p
$q \leftarrow r \land s$

Which of the three KB above is True in I_1 ? ***KB_3***

Basic definitions from 322 (Semantics)

Definition (interpretation)
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Definition (truth values of statements cont'**):** A knowledge base *KB* is true in I if and only if every clause in *KB* is true in I.

Definition (model)
A model of a set of clauses (a KB) is an interpretation in which all the clauses are *true*.

Example: Models

$$KB = \begin{cases} p \leftarrow q. \\ q. \\ r \leftarrow s. \end{cases}$$

	p	q	r	s
I_1	*true*	*true*	*true*	*true*
I_2	*false*	*false*	*false*	*false*
I_3	*true*	*true*	*false*	*false*
I_4	*true*	*true*	*true*	*false*
I_5	*true*	*true*	*false*	*true*

Which interpretations are models?

Basic definitions from 322 (Semantics)

Definition (interpretation)
An interpretation I assigns a truth value to each atom.

Definition (truth values of statements cont'**):** A knowledge base *KB* is true in I if and only if every clause in *KB* is true in I.

Definition (model)
A model of a set of clauses (a KB) is an interpretation in which all the clauses are *true*.

Definition (logical consequence)
If *KB* is a set of clauses and *G* is a conjunction of atoms, *G* is a logical consequence of *KB*, written $KB \models G$, if *G* is *true* in every model of *KB*.

Example: Logical Consequences

	p	q	r	s
I_1	true	true	true	true
I_2	true	true	true	false
I_3	true	true	false	false
I_4	true	true	false	true
I_5	false	true	true	true
I_6	false	true	true	false
I_7	false	true	false	false
I_8	false	true	false	true
...	....	...	...	...

Models

$$KB = \begin{cases} p \leftarrow q. \\ q. \\ r \leftarrow s. \end{cases}$$

$2^4 = 16$ interpretations in total, only 3 are models

remaining 8 cannot be models because q is false

F

Which of the following is true?

- $KB \models q$, $KB \models p$, $KB \not\models s$, $KB \not\models r$

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Is it true that if

M(KB) is the set of all models of KB

$M(\alpha)$ is the set of all models of α

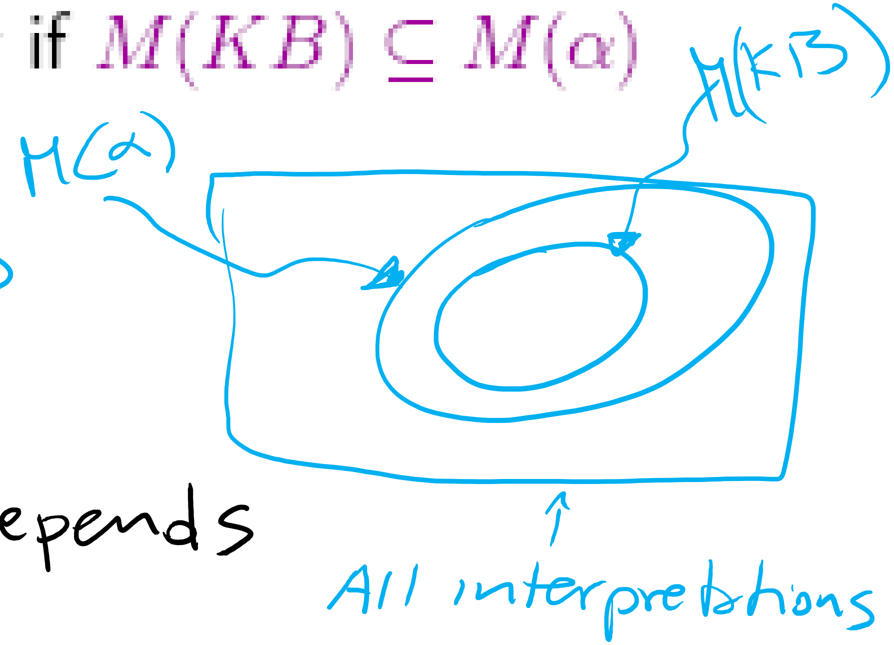
Then $KB \models \alpha$ if and only if $M(KB) \subseteq M(\alpha)$

α true in all the models of KB

A. yes

B. no

C. It depends



Basic definitions from 322 (Proof Theory)

Definition (soundness)

A proof procedure is sound if $KB \vdash G$ implies $KB \models G$.

Definition (completeness)

A proof procedure is complete if $KB \models G$ implies $KB \vdash G$.

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PDCL

$$p \leftarrow s \wedge f$$

$$r \leftarrow s \wedge q \wedge p$$

r

p

Propositional logic: Syntax

Atomic sentences = single proposition symbols

- E.g., P, Q, R
- Special cases: True = always true, False = always false

Complex sentences:

- If S is a sentence, $\neg$S is a sentence (negation)
- If S_1 and S_2 are sentences, $S_1 \wedge S_2$ is a sentence (conjunction)
- If S_1 and S_2 are sentences, $S_1 \vee S_2$ is a sentence (disjunction)
- If S_1 and S_2 are sentences, $S_1 \Rightarrow S_2$ is a sentence (implication)
- If S_1 and S_2 are sentences, $S_1 \Leftrightarrow S_2$ is a sentence (biconditional)

Propositional logic: Semantics

Each interpretation specifies true or false for each proposition symbol

E.g.

p	q	r
false	*true*	*false*

Rules for evaluating truth with respect to an interpretation I :

$\neg S$	is true iff	S is false	
$S_1 \wedge S_2$	is true iff	S_1 is true and	S_2 is true
$S_1 \vee S_2$	is true iff	S_1 is true or	S_2 is true
$S_1 \Rightarrow S_2$	is true iff	S_1 is false or	S_2 is true
i.e.,	is false iff	S_1 is true and	S_2 is false
$S_1 \Leftrightarrow S_2$	is true iff	$S_1 \Rightarrow S_2$ is true and $S_2 \Rightarrow S_1$ is true	

Simple recursive process evaluates an arbitrary sentence, e.g.,

$(\neg p \wedge (q \vee r)) \Leftrightarrow \neg p$ = $(\neg F \wedge (T \vee F)) \Leftrightarrow \neg F$

$(T \wedge T) \Leftrightarrow T$

$T \Leftrightarrow T$ → $(T \Rightarrow T) \wedge (T \Rightarrow T)$

$T \wedge T$

T

Logical equivalence

Two sentences are logically equivalent iff true in same interpretations

$\alpha \equiv \beta$ if and only if $\alpha \models \beta$ and $\beta \models \alpha$

they have the same models

$$(\alpha \wedge \beta) \equiv (\beta \wedge \alpha) \quad \text{commutativity of } \wedge$$
$$(\alpha \vee \beta) \equiv (\beta \vee \alpha) \quad \text{commutativity of } \vee$$
$$((\alpha \wedge \beta) \wedge \gamma) \equiv (\alpha \wedge (\beta \wedge \gamma)) \quad \text{associativity of } \wedge$$
$$((\alpha \vee \beta) \vee \gamma) \equiv (\alpha \vee (\beta \vee \gamma)) \quad \text{associativity of } \vee$$
$$\neg(\neg\alpha) \equiv \alpha \quad \text{double-negation elimination}$$
$$(\alpha \Rightarrow \beta) \equiv (\neg\beta \Rightarrow \neg\alpha) \quad \text{contraposition}$$
$$(\alpha \Rightarrow \beta) \equiv (\neg\alpha \vee \beta) \quad \text{implication elimination}$$
$$(\alpha \Leftrightarrow \beta) \equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) \quad \text{biconditional elimination}$$
$$\neg(\alpha \wedge \beta) \equiv (\neg\alpha \vee \neg\beta) \quad \text{De Morgan}$$
$$\neg(\alpha \vee \beta) \equiv (\neg\alpha \wedge \neg\beta) \quad \text{De Morgan}$$
$$(\alpha \wedge (\beta \vee \gamma)) \equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) \quad \text{distributivity of } \wedge \text{ over } \vee$$
$$(\alpha \vee (\beta \wedge \gamma)) \equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) \quad \text{distributivity of } \vee \text{ over } \wedge$$

$(\alpha \wedge \beta)$	$\equiv (\beta \wedge \alpha)$	commutativity of $\wedge$
$(\alpha \vee \beta)$	$\equiv (\beta \vee \alpha)$	commutativity of $\vee$
$((\alpha \wedge \beta) \wedge \gamma)$	$\equiv (\alpha \wedge (\beta \wedge \gamma))$	associativity of $\wedge$
$((\alpha \vee \beta) \vee \gamma)$	$\equiv (\alpha \vee (\beta \vee \gamma))$	associativity of $\vee$
$\neg(\neg \alpha)$	$\equiv \alpha$	double-negation elimination
$(\alpha \Rightarrow \beta)$	$\equiv (\neg \beta \Rightarrow \neg \alpha)$	contraposition
$(\alpha \Rightarrow \beta)$	$\equiv (\neg \alpha \vee \beta)$	implication elimination
$(\alpha \Leftrightarrow \beta)$	$\equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha))$	biconditional elimination
$\neg(\alpha \wedge \beta)$	$\equiv (\neg \alpha \vee \neg \beta)$	De Morgan
$\neg(\alpha \vee \beta)$	$\equiv (\neg \alpha \wedge \neg \beta)$	De Morgan
$(\alpha \wedge (\beta \vee \gamma))$	$\equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma))$	distributivity of $\wedge$ over $\vee$
$(\alpha \vee (\beta \wedge \gamma))$	$\equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma))$	distributivity of $\vee$ over $\wedge$

Can be used to rewrite formulas….

$(p \Rightarrow \neg(q \wedge r))$

$\neg p \vee \neg(q \wedge r)$

$\neg p \vee \neg q \vee \neg r$

$(\alpha \land \beta)$	$\equiv$	$(\beta \land \alpha)$	commutativity of $\land$
$(\alpha \lor \beta)$	$\equiv$	$(\beta \lor \alpha)$	commutativity of $\lor$
$((\alpha \land \beta) \land \gamma)$	$\equiv$	$(\alpha \land (\beta \land \gamma))$	associativity of $\land$
$((\alpha \lor \beta) \lor \gamma)$	$\equiv$	$(\alpha \lor (\beta \lor \gamma))$	associativity of $\lor$
$\neg(\neg\alpha)$	$\equiv$	α	double-negation elimination
$(\alpha \Rightarrow \beta)$	$\equiv$	$(\neg\beta \Rightarrow \neg\alpha)$	contraposition
$(\alpha \Rightarrow \beta)$	$\equiv$	$(\neg\alpha \lor \beta)$	implication elimination
$(\alpha \Leftrightarrow \beta)$	$\equiv$	$((\alpha \Rightarrow \beta) \land (\beta \Rightarrow \alpha))$	biconditional elimination
$\neg(\alpha \land \beta)$	$\equiv$	$(\neg\alpha \lor \neg\beta)$	De Morgan
$\neg(\alpha \lor \beta)$	$\equiv$	$(\neg\alpha \land \neg\beta)$	De Morgan
$(\alpha \land (\beta \lor \gamma))$	$\equiv$	$((\alpha \land \beta) \lor (\alpha \land \gamma))$	distributivity of $\land$ over $\lor$
$(\alpha \lor (\beta \land \gamma))$	$\equiv$	$((\alpha \lor \beta) \land (\alpha \lor \gamma))$	distributivity of $\lor$ over $\land$

$(p \Rightarrow \neg(q \land r$

$\neg p \lor \neg(q \land r$

Can be used to rewrite formulas….

$(p \Rightarrow \neg(q \land r))$

$\neg(q \land r) \lor \neg p$

$(q \land r) \Rightarrow \neg p$

$\neg q \lor \neg r \lor \neg p$

Validity and satisfiability

A sentence is valid if it is true in **all** interpretations

e.g., $True$, $A \lor \neg A$, $A \Rightarrow A$, $(A \land (A \Rightarrow B)) \Rightarrow B$

Validity is connected to inference via the Deduction Theorem:

$KB \models \alpha$ if and only if $(KB \Rightarrow \alpha)$ is valid

A sentence is satisfiable if it is true in **some** interpretation

e.g., $A \lor B$, C

A sentence is unsatisfiable if it is true in **no** interpretations

e.g., $A \land \neg A$

Satisfiability is connected to inference via the following:

$KB \models \alpha$ if and only if $(KB \land \neg\alpha)$ is unsatisfiable

i.e., prove α by *reductio ad absurdum*

Validity and Satisfiability

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$\langle \alpha$ is valid iff $\neg\alpha$ unsatisfiable $\rangle$

$\langle \alpha$ is satisfiable iff $\neg\alpha$ is valid $\rangle$

The statements above are:

A: All false

B: Some true Some false

C: All true

Validity and Satisfiability

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(α is valid iff ¬α unsatisfiable) T

true in all models — cannot be true in any model

(α is satisfiable iff ¬α is valid) F

true in some models — true in all models

The statements above are:

A: All false

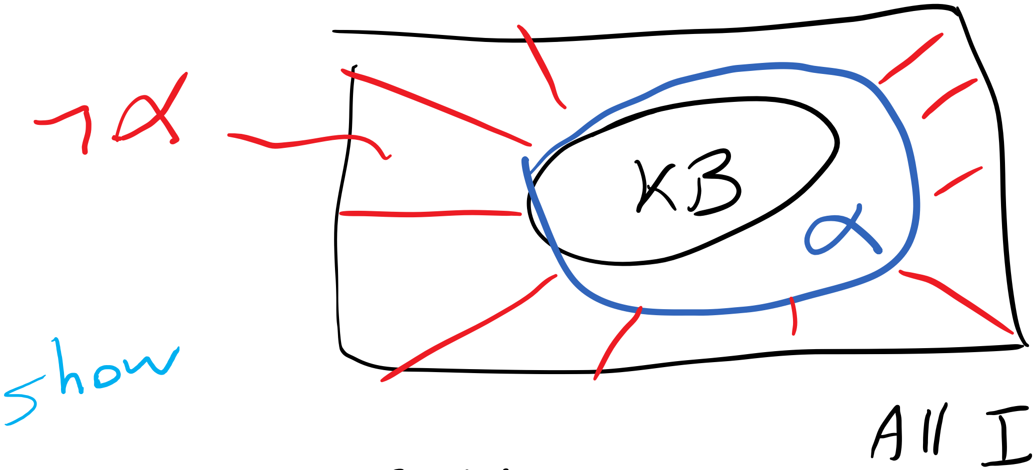
B: Some true Some false

C: All True

Lecture Overview

- Basics Recap: Interpretation / Model /
- Propositional Logics
- Satisfiability, Validity
- **Resolution in Propositional logics**

Proof by resolution



Key ideas

proof

$KB \models \alpha$

show

equivalent to : $KB \wedge \neg\alpha$ unsatifiable

(there is no I in which both are true)

- Simple Representation for Conjunctive Normal Form
- Simple Rule of Derivation

Resolution

Conjunctive Normal Form (CNF)

Rewrite $KB \wedge \neg\alpha$ into **conjunction of disjunctions**

literals

$$\underbrace{(A \vee \neg B)}_{\text{Clause}} \wedge \underbrace{(B \vee \neg C \vee \neg D)}_{\text{Clause}}$$

- Any KB can be converted into CNF !

Example: Conversion to CNF

$A \Leftrightarrow (B \vee C)$

1. Eliminate $\Leftrightarrow$, replacing $\alpha \Leftrightarrow \beta$ with $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$.
   $(A \Rightarrow (B \vee C)) \wedge ((B \vee C) \Rightarrow A)$

2. Eliminate $\Rightarrow$, replacing $\alpha \Rightarrow \beta$ with $\neg\alpha \vee \beta$.
   $(\neg A \vee B \vee C) \wedge (\neg(B \vee C) \vee A)$

3. Using de Morgan's rule replace $\neg(\alpha \vee \beta)$ with $(\neg\alpha \wedge \neg \beta)$:
   $(\neg A \vee B \vee C) \wedge ((\neg B \wedge \neg C) \vee A)$

4. Apply distributive law ($\vee$ over $\wedge$) and flatten:
   $(\neg A \vee B \vee C) \wedge (\neg B \vee A) \wedge (\neg C \vee A)$

Example: Conversion to CNF

$A \Leftrightarrow (B \lor C)$

5. KB is the conjunction of all of its sentences (all are true), so write each clause (disjunct) as a sentence in KB:

…
$(\neg A \lor B \lor C)$
$(\neg B \lor A)$
$(\neg C \lor A)$
…

Resolution Deduction step

Resolution: inference rule for CNF: sound and complete! *

$$(A \vee B \vee C)$$
$$(\neg A)$$
$$\text{-------------}$$
$$\therefore (B \vee C)$$

"If A or B or C is true, but not A, then B or C must be true."

$$(A \vee B \vee C)$$
$$(\neg A \vee D \vee E)$$
$$\text{-------------}$$
$$\therefore (B \vee C \vee D \vee E)$$

"If A is false then B or C must be true,
or if A is true then D or E must be true,

hence since A is either true or false,
B or C or D or E must be true."

$$(A \vee B)$$
$$(\neg A \vee B)$$
$$\text{---------}$$
$$\therefore (B \vee B) \equiv B$$

Simplification

Learning Goals for today's class

You can:

- Describe relationships between different logics
- Apply the definitions of Interpretation, model, logical entailment, soundness and completeness
- Define and apply satisfiability and validity
- Convert any formula to CNF
- Justify and apply the resolution step

Next Class Wed

- Finish Resolution

- Another proof method for Prop. Logic
  Model checking - Searching through truth assignments. Walksat.

- Start First Order Logics

Midterm, Fri, Oct 25,
we will start at 4pm sharp

How to prepare…

- Go to **Office Hours** (I am offering one more Thur 9-10)
- **Learning Goals** (look at the end of the slides for each lecture – complete list has been posted)
- Revise all the **clicker questions** and **practice exercises**
- Practice material has been posted
- Check questions and answers on Piazza