## Intelligent Systems (AI-2)

#### Computer Science cpsc422, Lecture 2

Sep, 6, 2019

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#### WAITLISTS INFO

https://www.cs.ubc.ca/students/undergrad/courses/waitlists

#### **Lecture Overview**

## Value of Information and Value of Control

Recap Markov Chain

Markov Decision Processes (MDPs)

Formal Specification and example

## 422 big picture

StarAl (statistical relational Al)
Hybrid: Det +Sto
Prob CFG Parsing
Prob Relational Models
Markov Logics

**Deterministic** Stochastic

Logics
First Order Logics
Ontologies

- Full Resolution
- SAT

Query

**Planning** 

**Belief Nets** 

**Approx. : Gibbs** 

Markov Chains and HMMs

Forward, Viterbi....

**Approx.**: Particle Filtering

Undirected Graphical Models
Markov Networks
Conditional Random Fields

Markov Decision Processes and

Partially Observable MDP

- Value Iteration
- Approx. Inference

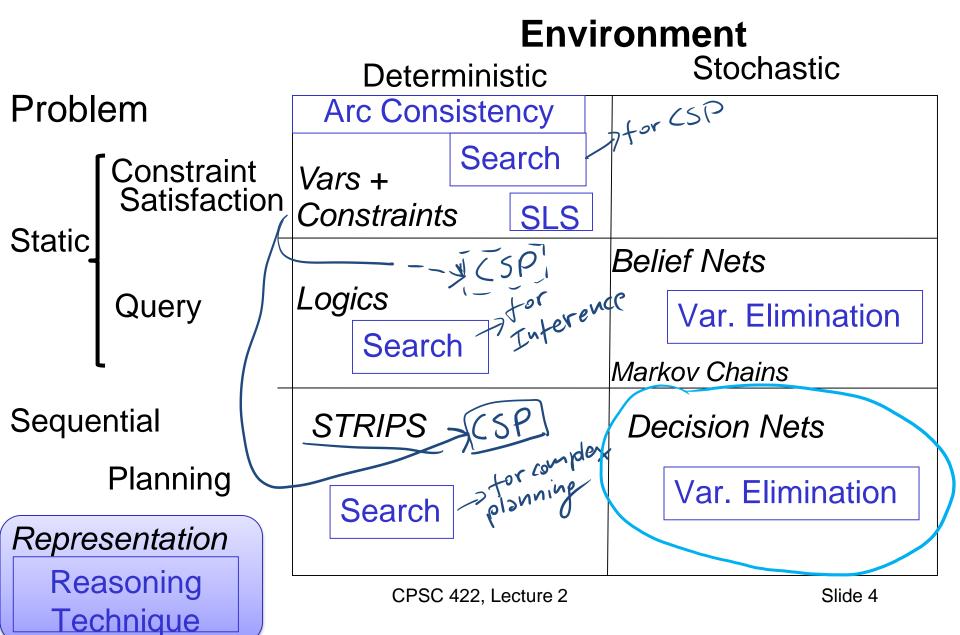
Reinforcement Learning

Applications of Al

Representation

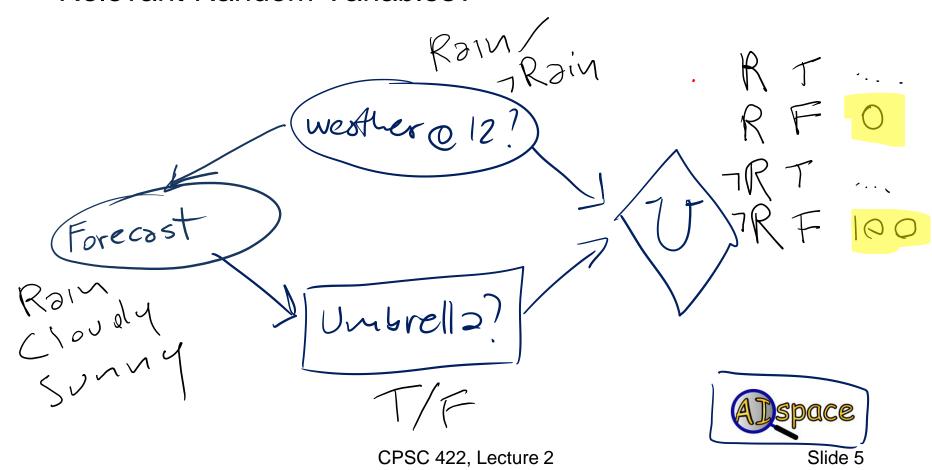
Reasoning Technique

## **Cpsc 322 Big Picture**



#### Simple Decision Net

- Early in the morning. Shall I take my umbrella today? (I'll have to go for a long walk at noon)
- Relevant Random Variables?

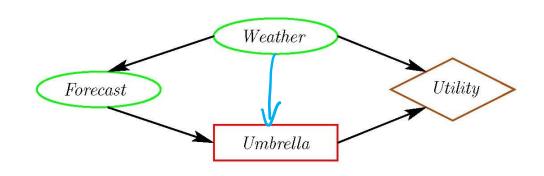


#### Polices for Umbrella Problem

 A policy specifies what an agent should do under each circumstance (for each decision, consider the parents of the decision node)

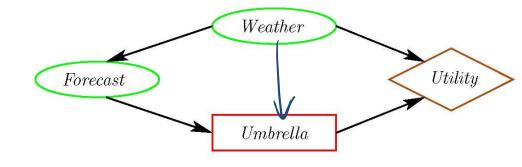
In the *Umbrella* case:

## Value of Information



- Early in the morning. I listen to the weather forecast, shall I take my umbrella today? (I'll have to go for a long walk at noon)
- What would help the agent make a better Umbrella decision?

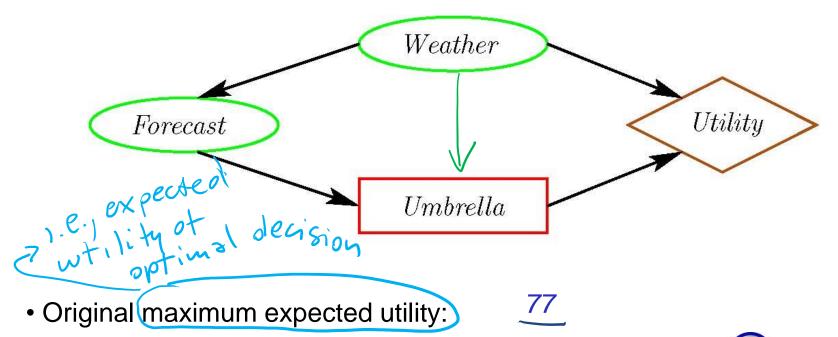
# Value of Information



- The value of information of a <u>random variable</u> X for decision D is: EU (Knowing X) EU (unt Knowing)
   the utility of the network with an arc from X to D minus the utility of the network without the arc.
- Intuitively:
  - The value of information is always
  - It is positive only if the agent changes its policy

## Value of Information (cont.)

 The value of information provides a bound on how much you should be prepared to pay for a sensor. How much is a perfect weather forecast worth?



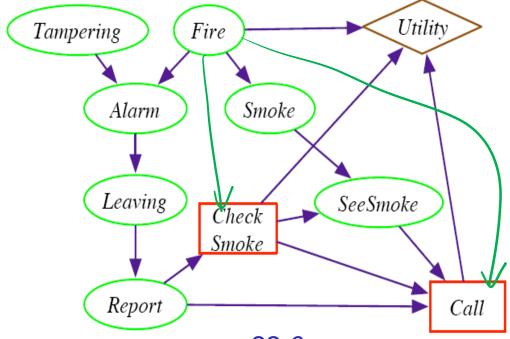
Maximum expected utility when we know Weather: 91

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• Better forecast is worth at most: /4

#### Value of Information

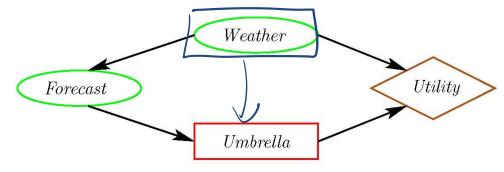
• The value of information provides a bound on how much you should be prepared to pay for a sensor. How much is a **perfect** fire sensor worth?



- Original maximum expected utility: -22.6
- Maximum expected utility when we know Fire:
- Perfect fire sensor is worth: 20.6



## Value of Control



 What would help the agent to make an even better Umbrella decision? To maximize its utility.

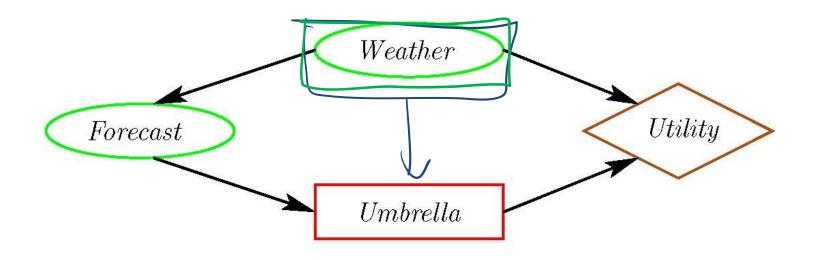
|          | Weather | Umbrella | Value |
|----------|---------|----------|-------|
|          | Rain    | true     | 70    |
|          | Rain    | false    | 0     |
|          | noRain  | true     | 20    |
| <b>→</b> | noRain  | false    | 100   |

• The value of control of a variable X is:

the utility of the network when you make X a decision variable **minus** the utility of the network when X is a random variable.

#### Value of Control

What if we could control the weather?

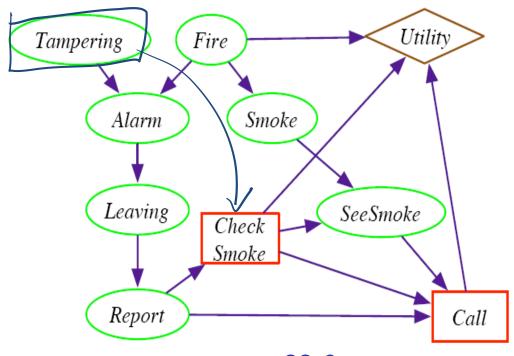


- Original maximum expected utility: 77
- Maximum expected utility when we control the weather: 100
- Value of control of the weather: 23



#### **Value of Control**

What if we control Tampering?



- Original maximum expected utility: -22.6
- Maximum expected utility when we control the Tampering: -20.7
- Value of control of Tampering:
- Let's take a look at the optimal policy
- Conclusion: do not tamper with fire alarms!



#### **Lecture Overview**

#### Value of Information and Value of Control

#### Recap Markov Chain

Markov Decision Processes (MDPs)

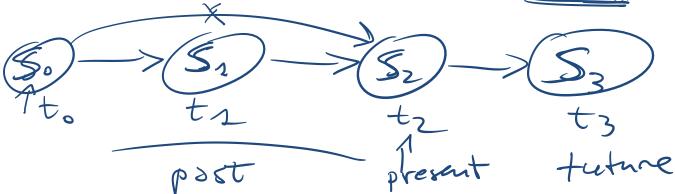
Formal Specification and example

## **Lecture Overview (from my 322)**

- Recap
- Temporal Probabilistic Models
- Start Markov Models
  - Markov Chain
  - Markov Chains in Natural Language Processing

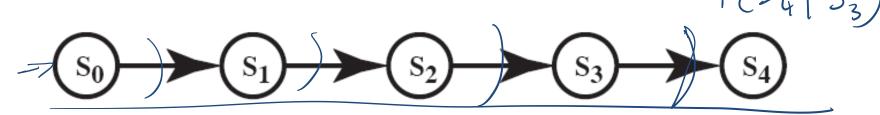
## **Simplest Possible DBN**

• One random variable for each time slice: let's assume  $S_t$  represents the state at time t. with domain  $\{v_1 \dots v_n\}$ 



- Each random variable depends only on the previous one
- Thus  $P(S_{t+1}|S_0,...,S_t) = P(S_{t+1}|S_t)$
- Intuitively S<sub>t</sub> conveys all of the information about the history that can affect the future states.
- \_ "The future is independent of the past given the present."

# Simplest Possible DBN (cont') $P(s_4 \mid S_3)$



• How many CPTs do we need to specify?  $4 P(S_1|S_0) P(S_2|S_1) etc.$ 



A. 1

**C.2** 

**D.** 3

**B.** 4

- Stationary process assumption: the mechanism that regulates how state variables change overtime is stationary, that is it can be described by a single transition model
- · P(St|St-1) is the same for all t

### **Stationary Markov Chain (SMC)**



A stationary Markov Chain: for all t >0

- $P(S_{t+1}|S_0,...,S_t) = P(S_{t+1}|S_t)$  and Markov assumption
- $P(S_{t+1}|S_t)$  is the same 5 + 30

So we only need to specify?



**A.** 
$$P(S_{t+1}|S_t)$$
 and  $P(S_0)$ 

B. 
$$P(S_0)$$

$$\mathbf{C} \cdot P(S_{t+1}|S_t)$$

$$\mathbf{D.} P(S_t | S_{t+1})$$

## **Stationary Markov Chain (SMC)**



A stationary Markov Chain: for all t >0

- $P(S_{t+1}|S_0,...,S_t) = P(S_{t+1}|S_t)$  and Markov assumption
- $P(S_{t+1}|S_t)$  is the same 5+84 on  $S_t$

We only need to specify  $P(S_t)$  and  $P(S_{t+1}|S_t)$ 

- Simple Model, easy to specify
- Often the natural model <</li>
- The network can extend indefinitely
- Variations of SMC are at the core of most Natural sed in the Language Processing (NLP) applications! also used by Googless)

  Page Rank also (used by web pages)

**Stationary Markov-Chain: Example** 

Domain of variable S<sub>i</sub> is {t, q, p, a, h, e}

Probability of initial state  $P(S_0)$ 

Stochastic Transition Matrix  $P(S_{t+1}|S_t)$ 

Which of these two is a possible STM?

 $S_{t+1}$ 

| t | .6 |
|---|----|
| q | .4 |
| р | 0  |
| а | P  |
| h | O  |
| е |    |

$$S_{t}$$
 + 1

|   | t q p |    | р  | a h |    | е |
|---|-------|----|----|-----|----|---|
| t | 0     | .3 | 0  | .3  | .4 | 0 |
| q | .4    | 0  | .6 | 0   | 0  | 0 |
| р | 0     | 0  | 1  | 0   | 0  | 0 |
| а | 0     | 0  | .4 | .6  | 0  | 0 |
| h | 0     | 0  | 0  | 0   | 0  | 1 |
| е | 1     | 0  | 0  | 0   | 0  | 0 |

|   | ,  | 4 | ٢ | J | - | O |
|---|----|---|---|---|---|---|
| t | 1  | 0 | 0 | 0 | 0 | 0 |
| q | 0  | 1 | 0 | 0 | 0 | 0 |
| р | .3 | 0 | 1 | 0 | 0 | 0 |
| а | 0  | 0 | 0 | 1 | 0 | 0 |
| h | 0  | 0 | 0 | 0 | 0 | 1 |
|   | 1  |   |   |   |   |   |

5>1

A.Left one only

C. Both

clicker.

**B.** Right one only

D. None

## **Stationary Markov-Chain: Example**

Domain of variable S<sub>i</sub> is {t, q, p, a, h, e}

We only need to specify...

$$P(S_0)$$

Probability of initial state

| t | .6 |  |  |  |  |
|---|----|--|--|--|--|
| q | .4 |  |  |  |  |
| р | 0  |  |  |  |  |
| а | 9  |  |  |  |  |
| h | 0  |  |  |  |  |
| е | 0  |  |  |  |  |

1.

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**Stochastic Transition Matrix** 

$$P(S_{t+1}|S_t)$$

St) St+2)
6 values 6 values

|                   |          |    |    |    |    |    | V   |  |
|-------------------|----------|----|----|----|----|----|-----|--|
|                   |          | t  | σ  | р  | а  | h  | е   |  |
|                   | t        | 0  | .3 | 0  | .3 | .4 | 0   |  |
| 7                 | q        | .4 | 0  | .6 | 0  | 0  | 0 6 | P(St+1 St=9)                                 |
|                   | р        | 0  | 0  | 1  | 0  | 0  | 0   | $P(S_{t+1} S_{t}=9)$<br>$P(S_{t+1} S_{t}=9)$ |
| $S_t \rightarrow$ | а        | 0  | 0  | .4 | .6 | 0  | 0   |  |
| 7                 | h        | 0  | 0  | 0  | 0  | 0  | 1   |  |
|                   | <b>e</b> | 1  | 0  | 0  | 0  | 0  | 0   |  |

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#### **Markov-Chain: Inference**

Probability of a sequence of states S<sub>0</sub> ... S<sub>T</sub>

$$P(S_{0},...,S_{T}) = P(S_{0}) P(S_{1}|S_{0}) P(S_{2}|S_{1})$$

$$P(S_{t+1}|S_{t})$$

$$P(S_{t+1}|S_{t})$$

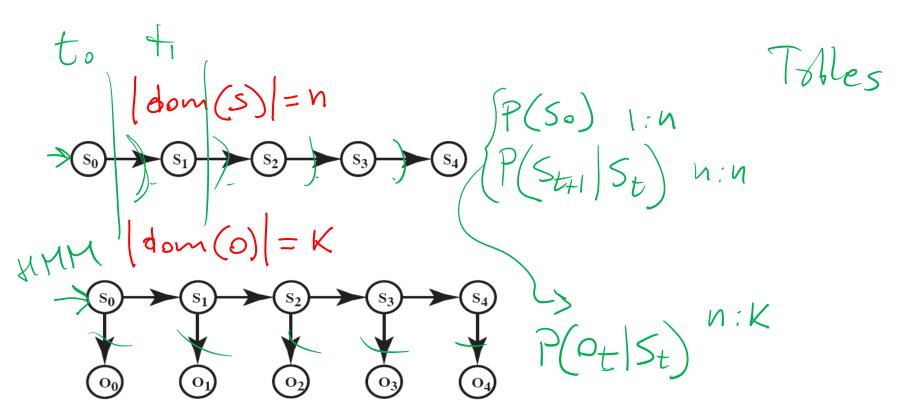
$$P(S_{0}) = P(S_{0}) P(S_{0}) P(S_{t+1}|S_{t})$$

$$P(S_{0}) = P(S_{0}) P(S_{0}) P(S_{0}|S_{0}) P(S_{0}|S_{0}|S_{0}) P(S_{0}|S_{0}|S_{0}|S_{0}) P(S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S_{0}|S$$

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## Recap: Markov Models



#### **Lecture Overview**

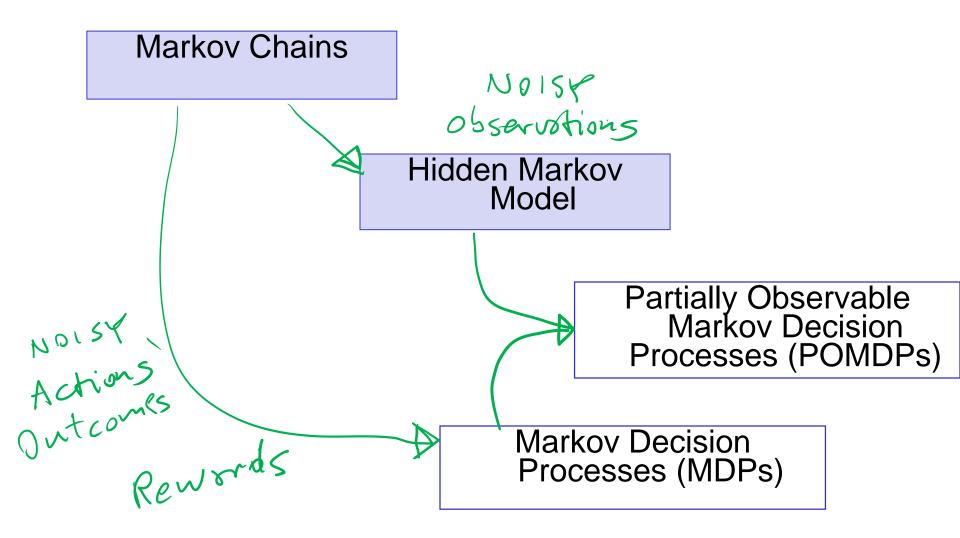
#### Value of Information and Value of Control

### Recap Markov Chain

### **Markov Decision Processes (MDPs)**

Formal Specification and example

#### **Markov Models**



# Combining ideas for Stochastic planning

What is a key limitation of decision networks?

Represent (and optimize) only a fixed number of decisions

What is an advantage of Markov models?

The network can extend indefinitely

Goal: represent (and optimize) an indefinite sequence of decisions

#### **Decision Processes**

Often an agent needs to go beyond a fixed set of decisions – Examples?

Would like to have an ongoing decision process

Infinite horizon problems: process does not stop

Robot surviving on planet, Monitoring Nuc. Plant, ....

Indefinite horizon problem: the agent does not know when the process may stop

resolving location

Finite horizon: the process must end at a give time N

In N steps

## How can we deal with indefinite/infinite **Decision processes?**

We make the same two assumptions we made for....

The action outcome depends only on the current state

Let  $S_t$  be the state at time t ...  $P(S_{t+1} | S_t, A_t, S_{t-1}, A_{t-1})$ 

The process is stationary...

The process is stationary...

the same for M t

We also need a more flexible specification for the utility. How?

• Defined based on a reward/punishment *R(s)* that the agent receives in each state s eg.  $\leq r_0 r_2 - \cdots r_n$ 

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## **MDP:** formal specification

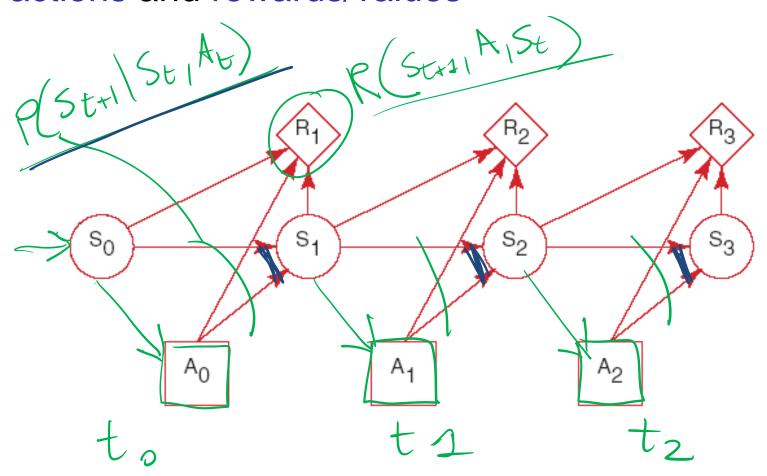
#### For an MDP you specify:

- set S of states and set A of actions
- the process' dynamics (or *transition model*)  $P(S_{t+1}|S_t, A_t)$
- The reward function
  - R(s) is used when the reward depends only on the state **s** and not on how the agent got there
  - More complex R(s, a, s') describing the reward that the agent receives when it performs action a in state s and ends up in state s'

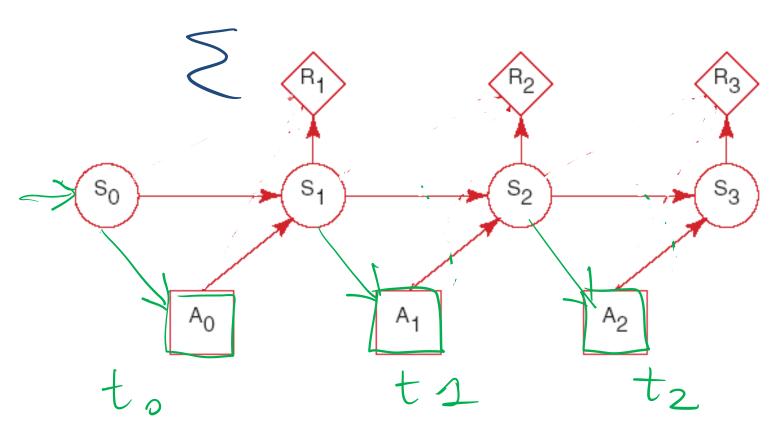
• Absorbing/stopping/terminal state 
$$S_{ab}$$
 for M action  $P(S_{ab}|a,S_{ab})=1$   $R(S_{ab},a,S_{ab})=0$ 

## **MDP** graphical specification

Basically a MDP is a Markov Chain augmented with actions and rewards/values

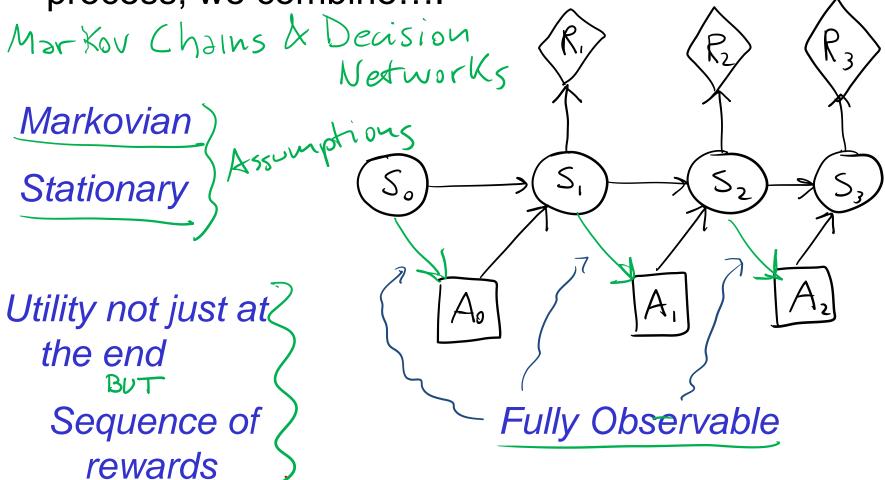


# When Rewards only depend on the state

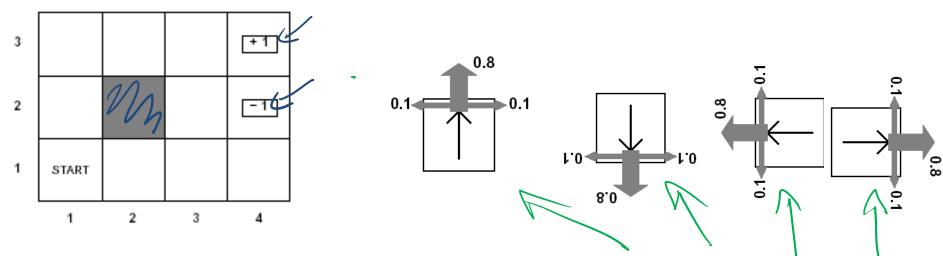


## **Summary Decision Processes: MDPs**

To manage an ongoing (indefinite... infinite) decision process, we combine....



## **Example MDP: Scenario and Actions**



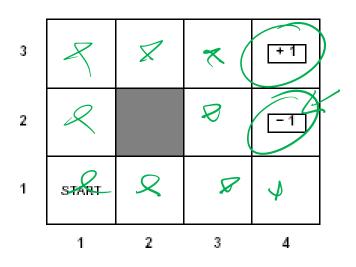
Agent moves in the above grid via actions *Up, Down, Left, Right* Each action has:

- 0.8 probability to reach its intended effect
- 0.1 probability to move at right angles of the intended direction
- If the agents bumps into a wall, it stays there

How many states? // // /2

There are two terminal states (3,4) and (2,4)

## **Example MDP: Rewards**



$$R(s) = \begin{cases} -0.04 & \text{(small penalty) for nonterminal states} \\ \pm 1 & \text{for terminal states} \end{cases}$$

## Learning Goals for today's class

#### You can:

- Define and compute Value of Information and Value of Control in a decision network
- Effectively represent indefinite/infinite decision processes with a Markov Decision Process (MDP)
- Compute the probability distribution on states given a sequence of actions in an MDP
- Define a policy for an MDP

#### **TODO for Mon**

- Read textbook 9.4
- Read textbook 9.5
  - 9.5.1 Value of a Policy
  - 9.5.2 Value of an Optimal Policy
  - 9.5.3 Value Iteration

#### CPSC 322 Review "Exam"

https://forms.gle/SpQwrXfonTZrVf4P7

#### Based on CPSC 322 material

- Logic
- Uncertainty
- Decision Theory

#### Review material (e.g., 322 slides from 2017):

https://www.cs.ubc.ca/~carenini/TEACHING/CPSC322-17S/index.html