Intelligent Systems (AI-2)

Computer Science cpsc422, Lecture 11

Sept, 27, 2017
Lecture Overview

• Recap of BNs Representation and Exact Inference
• Start Belief Networks Approx. Reasoning
  • Intro to Sampling
  • First Naïve Approx. Method: Forward Sampling
  • Second Method: Rejection Sampling
Realistic BNet: Liver Diagnosis

Source: Onisko et al., 1999
Bnets to assess and manage Climate Change

Journal of Environmental Management
Volume 202, Part 1, 1 November 2017, Pages 320-331

Reviewing Bayesian Networks potentials for climate change impacts assessment and management: A multi-risk perspective
One Recent Example from that review

Environmental Modelling & Software Journal
Volume 80, June 2016, Pages 132-142

A Bayesian Belief Network to assess rate of changes in coral reef ecosystems

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Carbonate Budget BBN (CARBNET)

- We propose a Bayesian Belief Network (BBN) approach, which offers a methodological framework to address uncertainty (Bennett et al., 2013, Kelly et al., 2013).
- Can aid sustainable coral reef management and prevent further decline.
- Help evaluate effects of anthropogenic and climatic disturbances on the reef framework.
- Consider impacts of implementing management interventions or decision options in order to maximize their benefit (Uusitalo et al., 2015).
- CARBNET: developed to evaluate coral reef CaCO$_3$ (carbonate) balance under changing environmental conditions and across reef bioregions.
58 nodes and 94 cause-effect links.
CARBNET Engineering

- Variables identified through literature search
- Nodes representing different levels of spatial resolution were used to capture changes that may occur at different spatial scales.
- Presence/absence of reef-building and erosive organisms or reef growth and erosion processes are captured at the smallest scale of reef depth, but also for an entire reef (‘Site’), sub-region (‘Reef type’, ‘Reef topography’) or region (‘Coral reef region’).
- The CARBNET conceptualisation was proposed to twenty experts in the field of coral reef management and ecology to identify flaws in the network structure and address structural bias before model parameterisation.
Another Example

Predicting water quality responses to a changing climate: building an integrated modelling framework

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Abstract The future management of freshwater resources for human and environmental needs requires an integrated set of tools for predicting the relationship between climate change, water quality and ecological responses. In this paper, we present the early phases of a project for building a Bayesian network (BN) based framework to link ecological and water quality responses to features of the flow regime in the Molonglo and Yass rivers in southeastern Australia. At this stage, the objective is to conceptualize the modelling components and define causal links. Expert elicitation was used to identify important drivers and interactions which influence water quality attributes and related ecological responses.

Key words Bayesian network models; water quality; prediction; climate change; integrated modelling

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A group of 14 experts and decision makers were involved in a half-day workshop to define the important variables, links and states of variables.

The next phase of the project involves using available data to construct the conditional probability tables and populate the BN structure.
Revise (in)dependencies
Conditional Independencies

Or, blocking paths for probability propagation. Three ways in which a path between X to Y can be blocked, (1 and 2 given evidence $E$)

1. $Z$
2. $Z$
3. $Z$

Note that, in 3, X and Y become dependent as soon as I get evidence on Z or on any of its descendants.
Independence (Markov Blanket)

What is the minimal set of nodes that must be observed in order to make node $X$ independent from all the non-observed nodes in the network.
Variable elimination algorithm:

**Summary**

\[ P(Z, Y_1..., Y_j, Z_1..., Z_j) \]

To compute \( P(Z| Y_1=v_1, ..., Y_j=v_j) \):

1. Construct a factor for each conditional probability.
2. Set the observed variables to their observed values.
3. Given an elimination ordering, simplify/decompose sum of products
   - For all \( Z_i \): Perform products and sum out \( Z_i \)
4. Multiply the remaining factors (all in \( Z \))
5. Normalize: divide the resulting factor \( f(Z) \) by \( \sum_Z f(Z) \).
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Approximate Inference

Basic idea:
  • Draw N samples from known prob. distributions
  • Use those samples to estimate unknown prob. distributions

Why sample?
  • Inference: getting a sample is faster than computing the right answer (e.g. with variable elimination)
We use **Sampling**

**Sampling** is a process to **obtain samples** adequate to **estimate** an **unknown probability**

**How do we get samples?**

![Diagram showing the process of sampling]

- **Samples**
- Known prob. distribution(s)

- Estimates for unknown (hard to compute) distribution(s)
Generating Samples from a Known Distribution

For a random variable $X$ with
- values $\{x_1, \ldots, x_k\}$
- Probability distribution $P(X) = \{P(x_1), \ldots, P(x_k)\}$

Partition the interval $[0, 1]$ into $k$ intervals $p_i$, one for each $x_i$, with length $P(x_i)$

To generate one sample
- Randomly generate a value $y$ in $[0, 1]$ (i.e. generate a value from a uniform distribution over $[0, 1]$).
- Select the value of the sample based on the interval $p_i$ that includes $y$

From probability theory:

$$P(y \subset p_i) = \text{Length}(p_i) = P(x_i)$$
From Samples to Probabilities

Count total number of samples $m$
Count the number $n_i$ of samples $x_i$
Generate the frequency of sample $x_i$ as $n_i / m$
This frequency is your estimated probability of $x_i$
Sampling for Bayesian Networks (BN)

➢ Suppose we have the following BN with two binary variables

It corresponds to the joint probability distribution

- \( P(A,B) = P(B|A)P(A) \)

➢ To sample from \( P(A,B) \) i.e., unknown distribution

- we first sample from \( P(A) \). Suppose we get \( A = 0 \).
- In this case, we then sample from ...
- If we had sampled \( A = 1 \), then in the second step we would have sampled from
Prior (Forward) Sampling

\[
P(C)
\]

\[
\begin{array}{c|c}
+c & 0.5 \\
-c & 0.5 \\
\end{array}
\]

\[
P(S | C)
\]

\[
\begin{array}{c|c|c}
+c & +s & 0.1 \\
- & s & 0.9 \\
- & c & 0.5 \\
- & s & 0.5 \\
\end{array}
\]

\[
P(W | S, R)
\]

\[
\begin{array}{c|c|c|c}
+ & s & + & r & +w & 0.99 \\
 & - & w & 0.01 \\
- & r & + & w & 0.90 \\
 & - & w & 0.10 \\
- & s & + & r & +w & 0.90 \\
 & - & w & 0.10 \\
- & r & + & w & 0.01 \\
 & - & w & 0.99 \\
\end{array}
\]

\[
P(R | C)
\]

\[
\begin{array}{c|c|c}
+c & +r & 0.8 \\
- & r & 0.2 \\
- & c & 0.2 \\
- & r & 0.8 \\
\end{array}
\]

Samples:

+\(C\), -\(S\), +\(R\), +\(W\)
-\(C\), +\(S\), -\(R\), +\(W\)
...

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We’ll get a bunch of samples from the BN:

+ c, -s, +r, +w
+c, +s, +r, +w
-c, +s, +r, -w
+c, -s, +r, +w
-c, -s, -r, +w

If we want to know P(W)

• We have counts <+w:4, -w:1>
• Normalize to get P(W) =<+w:, -w:>
• This will get closer to the true distribution with more samples
Example

Can estimate anything else from the samples, besides $P(W)$, $P(R)$, etc:

- $c$, $-s$, $r$, $w$
- $c$, $s$, $r$, $w$
- $c$, $s$, $r$, $w$
- $c$, $s$, $r$, $w$

- What about $P(C|+w)$? $P(C|+r,+w)$? $P(C|-r,-w)$?

\[
\begin{bmatrix}
  0 & 1 \\
  0 & 0
\end{bmatrix}
\quad \begin{bmatrix}
  +c & -c \\
  +c & -c
\end{bmatrix}
\quad \begin{bmatrix}
  1 & 0 \\
  0 & 0
\end{bmatrix}
\]

A. $\begin{bmatrix}
  0 & 1 \\
  0 & 0
\end{bmatrix}$  B. $\begin{bmatrix}
  +c & -c \\
  +c & -c
\end{bmatrix}$  C. $\begin{bmatrix}
  1 & 0 \\
  0 & 0
\end{bmatrix}$

D. None of the above

Can use/generate fewer samples when we want to estimate a probability conditioned on evidence?
Rejection Sampling

Let’s say we want $P(S| +w)$

- Ignore (reject) samples which don’t have $W=+w$
- This is called rejection sampling
- It is also consistent for conditional probabilities (i.e., correct in the limit)

See any problem as the number of evidence vars increases?
Or if...

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Learning Goals for today’s class

➢ You can:

• Motivate the need for approx inference in Bnets
• Describe and compare Sampling from a single random variable
• Describe and Apply Forward Sampling in BN
• Describe and Apply Rejection Sampling
TODO for Mon

- Read textbook 6.4.2
- Assignment-2 will be out on the weekend: Start working on it
- Next research paper will be this coming Wed