Intelligent Systems (AI-2)

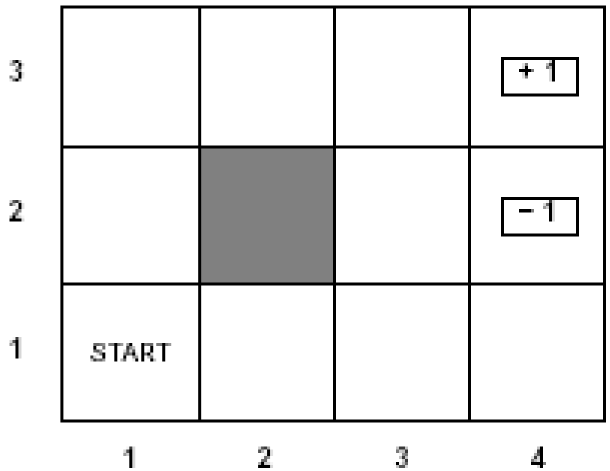
Computer Science cpsc422, Lecture 5

Sep, 18, 2017

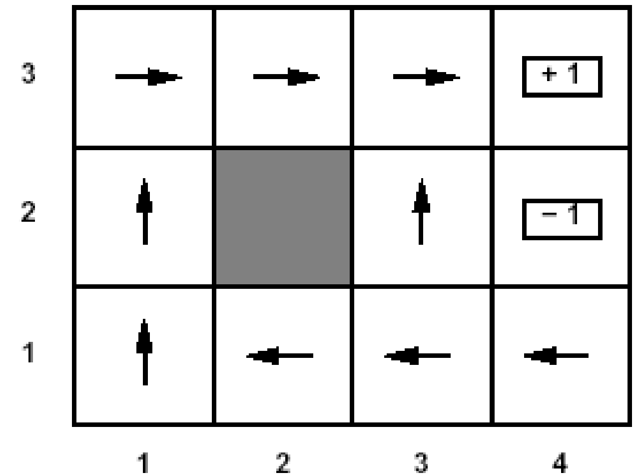
Slide credit POMDP: C. Conati and P.Viswanathan

Optimal policy

➤ Reward structure for our example



➤ This is the policy that we obtain by applying Value Iteration to our example

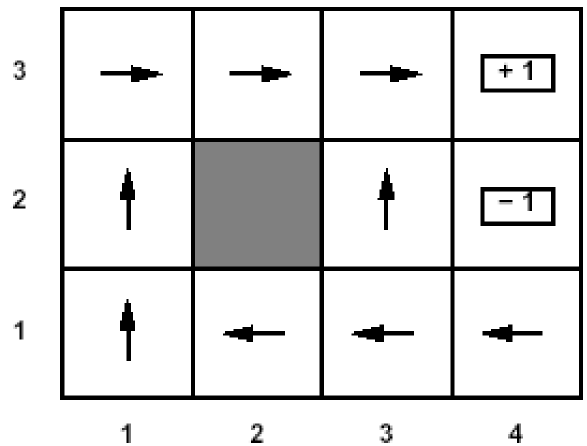


$$R(s) = \begin{cases} -0.04 & \text{(small penalty) for nonterminal states} \\ \pm 1 & \text{for terminal states} \end{cases}$$

Rewards and Optimal Policy

Optimal Policy when reward in non-terminal states is −0.04

computed by Value Iteration



Is it possible that the optimal policy changes if the reward in the non-terminal states changes?

A. Yes !

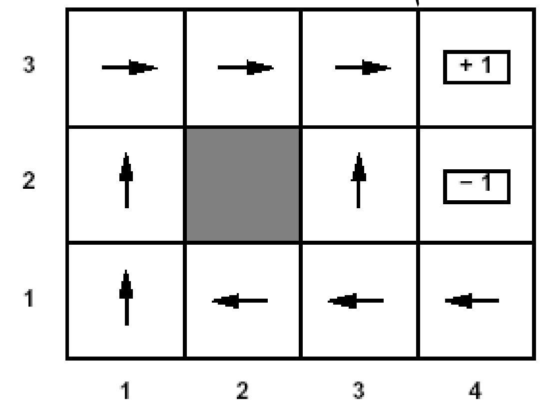
B. No

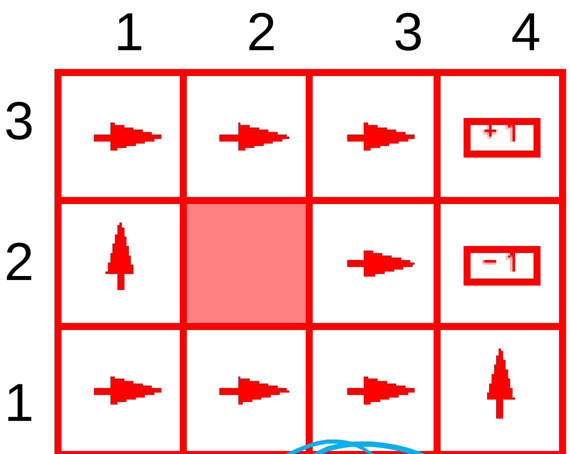
Rewards and Optimal Policy

If r = −2, what would be a reasonable policy?

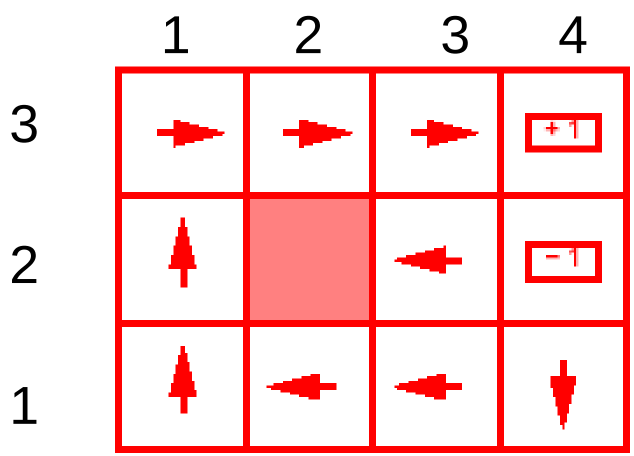
Optimal Policy

$r = -.04$





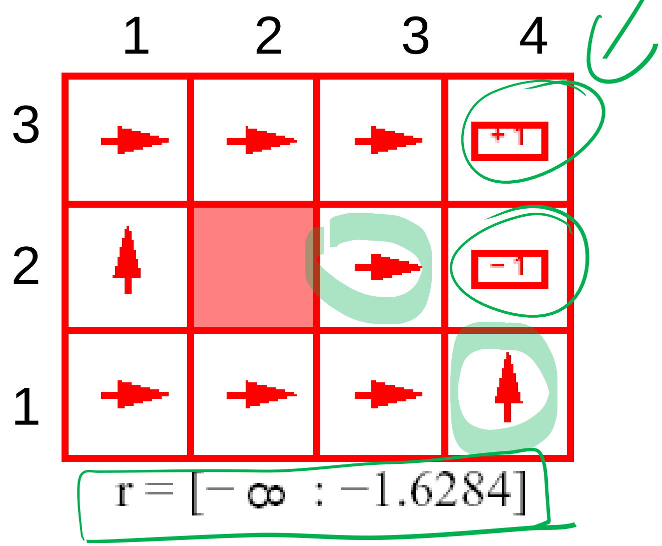
A.

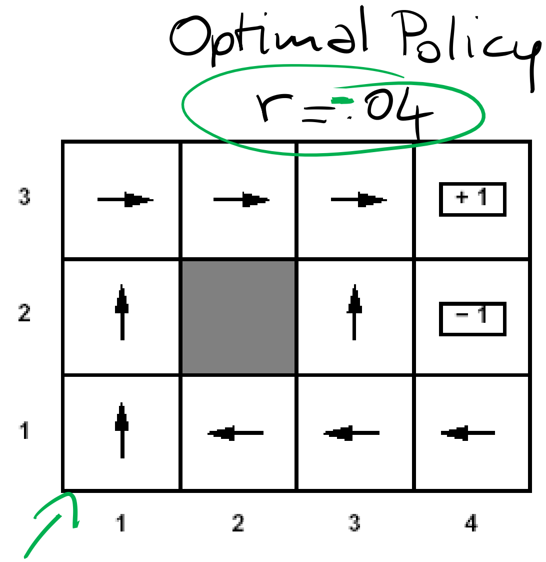


B.

Rewards and Optimal Policy

Optimal Policy when $r < -1.6284$

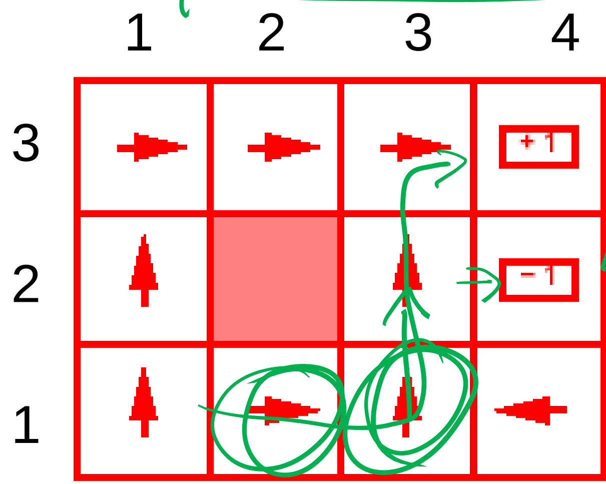




Why is the agent heading straight into (2,4) from its surrounding states?

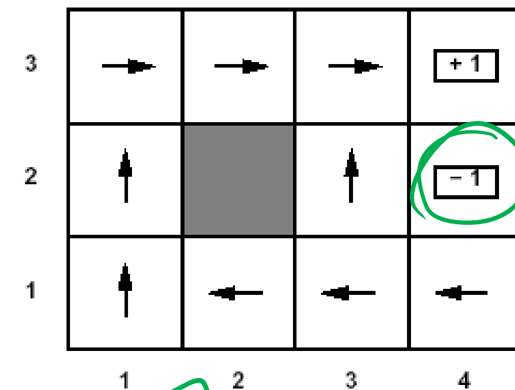
Rewards and Optimal Policy

Optimal Policy when $-0.427 < r < -0.085$



$r = [-0.4278 : -0.0850]$

Optimal Policy
$r = -.04$

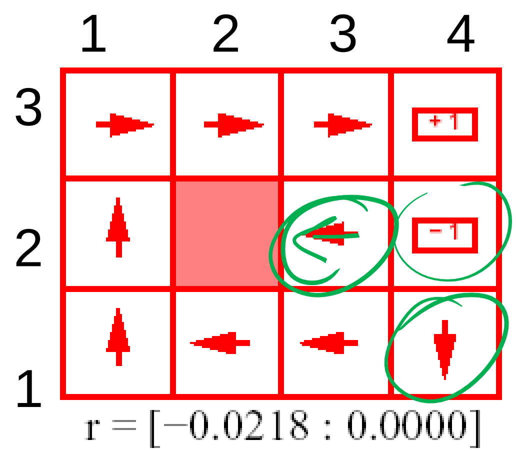


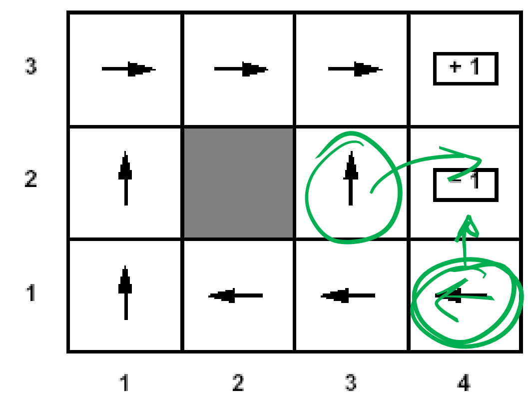
The cost of taking a step is high enough to make the agent take the shortcut to (3,4) from (1,3)

Rewards and Optimal Policy

Optimal Policy when $-0.0218 < r < 0$

Optimal Policy
r = -.04



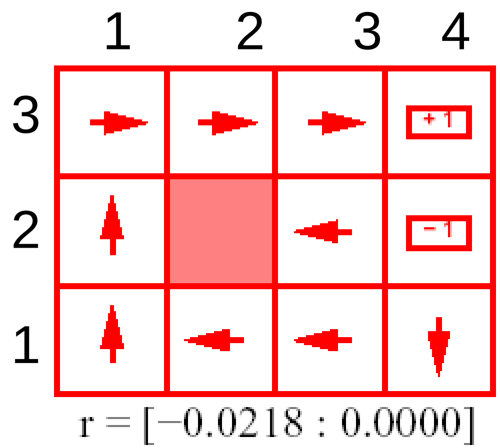


Why is the agent heading straight into the obstacle from (2,3)?And into the wall in (1,4)?

see next slide .....

Rewards and Optimal Policy

Optimal Policy when $-0.0218 < r < 0$



Stay longer in the grid is not penalized as much as before. The agent is willing to take longer routes to avoid (2,4)

- This is true even when it means banging against the obstacle a few times when moving from (2,3)

Rewards and Optimal Policy

Optimal Policy when $r > 0$?

Which means the agent is rewarded for every step it takes

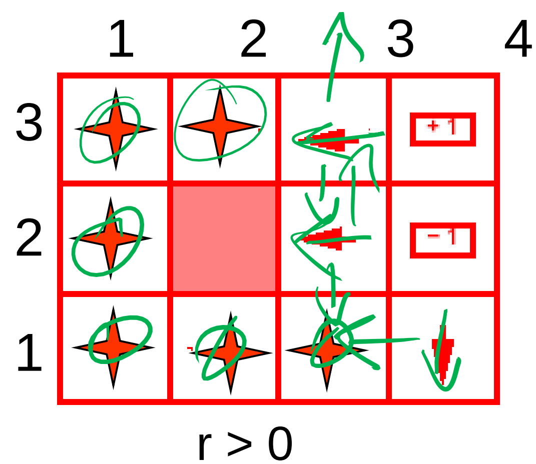
Rewards and Optimal Policy

Optimal Policy when $r > 0$

Which means the agent is rewarded for every step it takes

it avoids terminal states completely

state where every action belong to an optimal policy



MDPs scalability (not required)

- Modern optimal algorithms draw from a vast repertoire of techniques, like **graph algorithms**, **heuristic search**, **compact value function representations**, and **simulation-based approaches**. E.g.,
  - Only compute V for states “reachable” from S_0
  - Do not compute V for really bad states (based on heuristics)

- An enormous number of approximation algorithms have been suggested that exploit several intuitions, such as **inadmissible heuristics, interleaving planning and execution**, special processing for dead-end states, **domain determinization** ideas, **hybridizing multiple algorithms**, and **hierarchical problem decompositions**.

Markov Models

Markov Chains

Noisy observations

Hidden Markov Model

Noisy Actions Rewards

Partially Observable Markov Decision Processes (POMDPs)

Markov Decision Processes (MDPs)

Lecture Overview

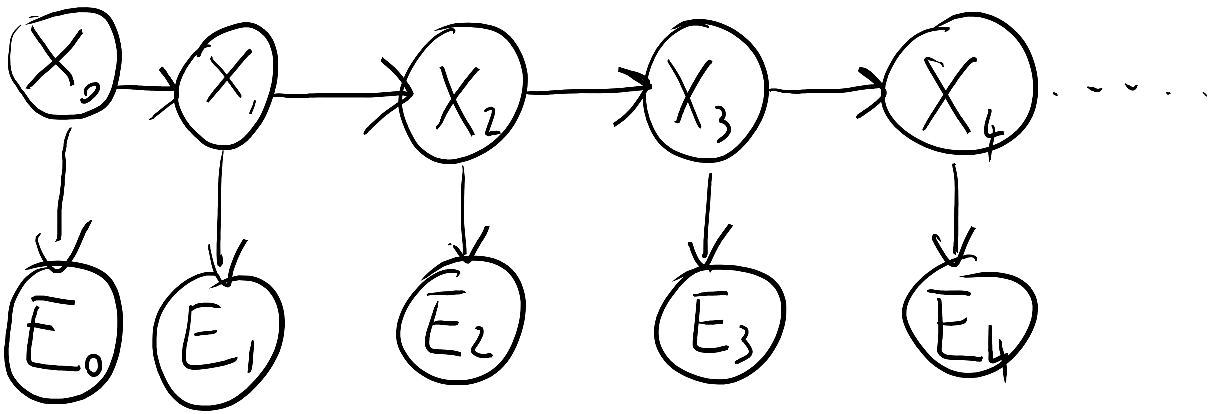
<u>**Filtering for HMM** (more when we will do temporal models)</u>

<u>Partially Observable Markov Decision Processes</u>

- Formal Specification and example
  - Belief State
  - Belief State Update

Hidden Markov Model

- A Hidden Markov Model (HMM) starts with a Markov chain, and adds a noisy observation/evidence about the state at each time step:



- |domain(X)| = k
- |domain(E)| = h

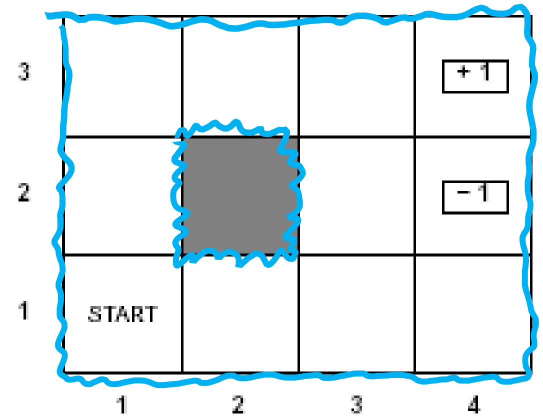
- $P(X_0)$ specifies initial conditions   K

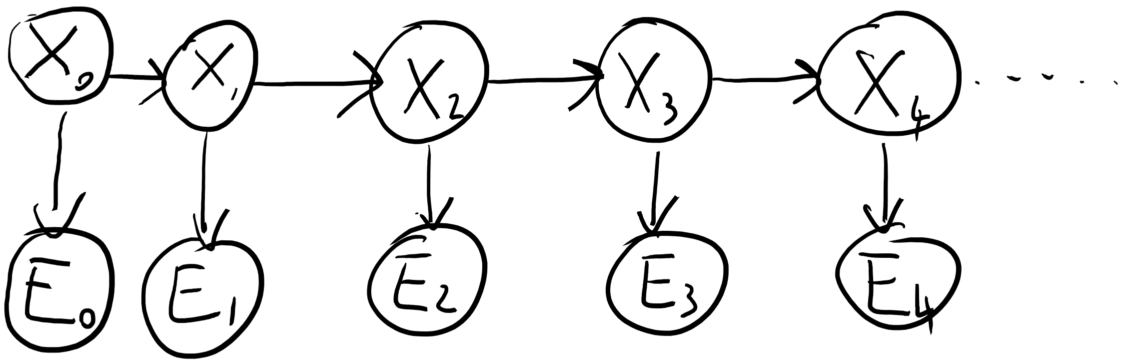
- X $P(X_{t+1}|X_t)$ specifies the dynamics   K×K

- O $P(E_t|S_t)$ specifies the sensor model   K×h {K prob. dist. over O}

Hidden Markov Model (our example with no actions)

- *E = # of walls {1w, 2w}*





- |domain(X)| = *11*
- |domain(E)| = 2

noisy sensor

- $P(X_0)$ specifies initial conditions

1,1 2,1 3,1 ...
1 0 0

perfect sensor

X_{t+1}

- $P(X_{t+1}|X_t)$ specifies the dynamics

11×11

X_t

	1w, 2w		1w	2w
1,1	0	1	.1	.9
1,2	0	1	.1	.9
...	...			
3,3	1	0	.9	.1

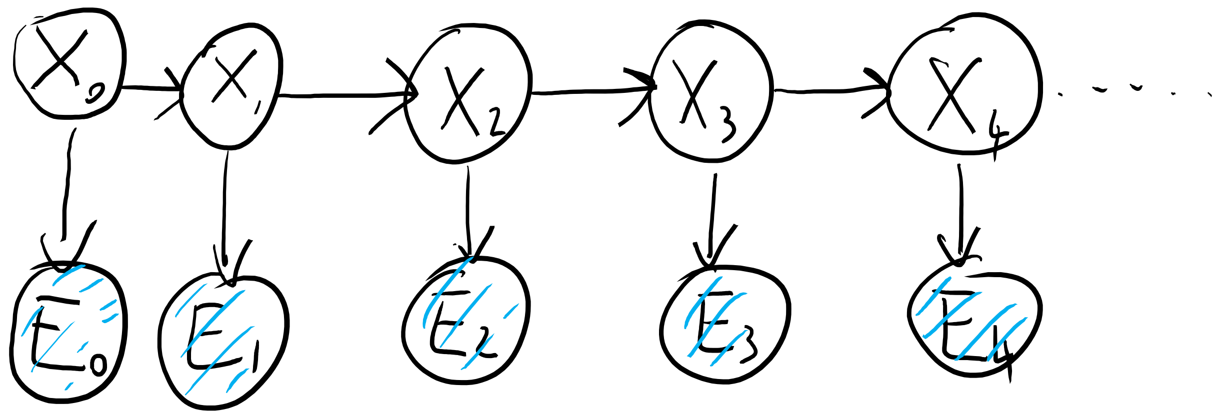
- $P(E_t|S_t)$ specifies the sensor model

Useful inference in HMMs

- **In general (Filtering):** compute the posterior distribution over the current state given all evidence to date

$$P(X_t \,/\, e_{0:t})$$

$P(X_4 | e_0 \dots e_4)$

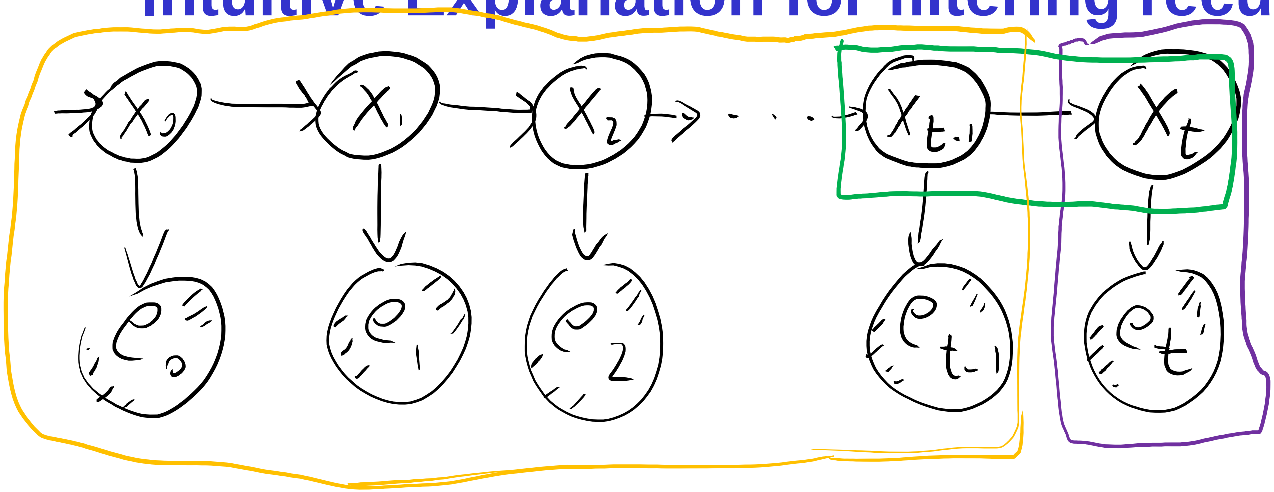


- observed
- value is known

$E_0 = e_0$ .. ..

e.g. 1w 1w 2w 1w 2w

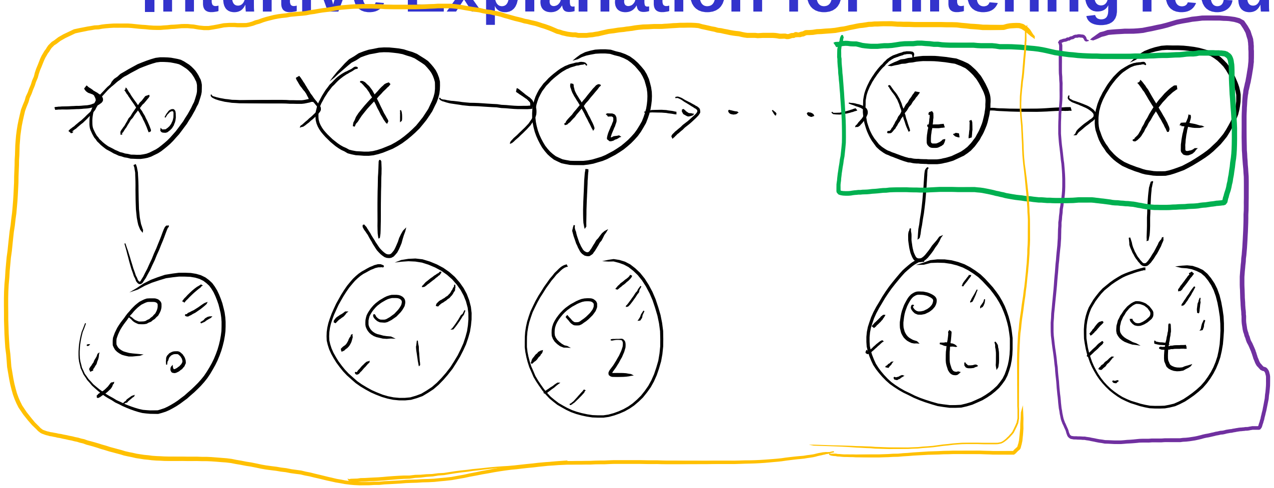
Intuitive Explanation for filtering recursive formula



sequence of evidences $e_0 : e_t$

$$P(X_t / e_{0:t}) =$$

Intuitive Explanation for filtering recursive formula



sequence of evidences $e_0:e_t$

$$P(X_t / e_{0:t}) = \alpha\, P(e_t|X_t) * \sum_{X_{t-1}} P(X_t|x_{t-1}) * P(x_{t-1}|e_0:e_{t-1})$$

X_t generated evidence e_t

whatever x_{t-1} was, X_t was reached from there

and evidence $e_0:e_{t-1}$ must have been generated before getting to x_{t-1}

Lecture Overview

Filtering for HMM (more when we will do temporal models)

Partially Observable MDPs

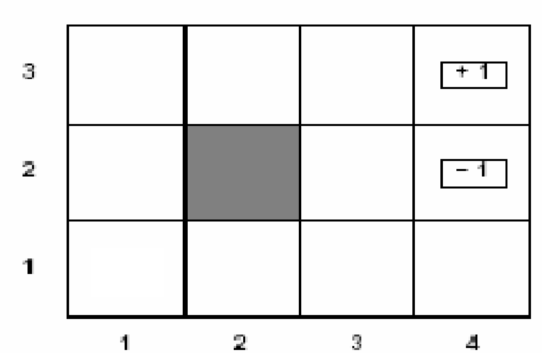
- Formal Specification and example
    - Belief State
    - Belief State Update

POMDP: Intro

- The MDPs we looked at so far were *fully observable*
  - The agent always knows which state it is in
  - The uncertainty is in …………..?
  - Policy only depends on………….?

Belief States

- In POMDPs, the agent cannot tell for sure where it is in the space state, all it can have are *beliefs* on that
  - ***probability distribution over states***
  - This is usually called ***belief state b***
  - *b(s)* is the probability assigned by *b* to the agent being in state *s*
- **Example**: Suppose we are in our usual grid world, but
  - the agent has no information at all about its position in non-terminal states
  - It knows only when it is in a terminal state (because the game ends)



- What is the initial belief state, if the agent knows that it is not in a terminal state?

Belief States

- Initial belief state:
  - <1/9,1/9, 1/9,1/9,1/9,1/9, 1/9,1/9,1/9,0,0>

0.111	0.111	0.111	0.000
0.111		0.111	0.000
0.111	0.111	0.111	0.111

Observation Model

- As in HMM, the agent can learn something about its actual state by *sensing* the environment:
  - **Sensor Model *P(e|s):*** probability of observing the evidence e in state ***s***
- A POMDP is fully specified by
  - Reward function: *R(s)* (we'll forget about *a* and *s'* for simplicity)
  - Transition Model: *P(s' |a,s)*
  - Observation model: *P(e|s)*
- Agent's belief state is updated by computing the conditional probability distribution over all the states given the sequence of observations and actions so far

State Belief Update

- We just saw *filtering* for HMM?
  - Compute conditional probability distribution over states at time t given all observations so far

$$\mathbf{P}(\mathbf{X}_t \mid \mathbf{e}_{0:t}) = \alpha\, \mathbf{P}(\mathbf{e}_t \mid \mathbf{X}_t) \sum_{x_{t-1}} \mathbf{P}(\mathbf{X}_t \mid \mathbf{x}_{t-1})\, P(\mathbf{x}_{t-1} \mid \mathbf{e}_{0:t-1})$$

Filtering at time *t-1*

Inclusion of new evidence (sensor model)

Propagation to time *t*

- State belief update is similar but includes actions
  - If the agent has current belief state ***b(s)***, performs action ***a*** and then perceives evidence ***e***, the new belief state ***b'(s')*** is

$$b'(s') = \alpha P(e \mid s') \sum_{s} P(s' \mid a, s)\, b(s)$$

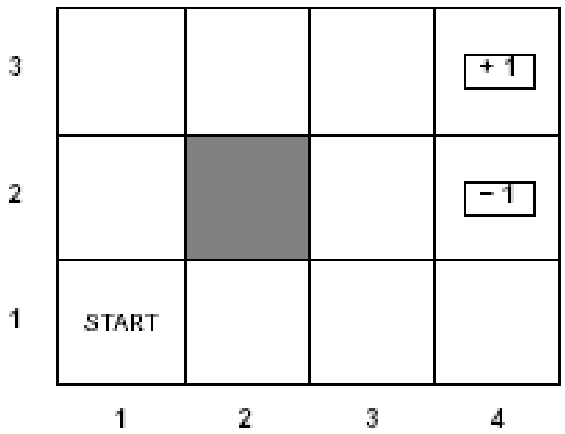
Inclusion of new evidence: Probability of perceiving *e* in *s'*

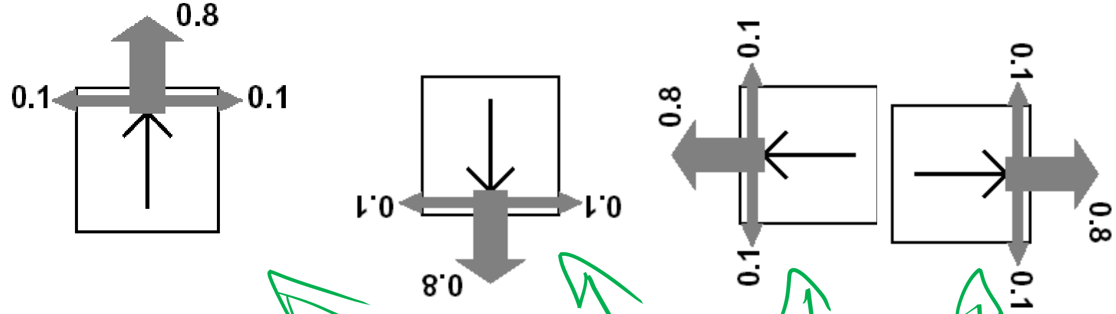
Sum over all the states that can take to s' after performing \a

Filtering at time t-1: State belief based on all observations and actions up to t-1

Propagation at time t: Probability of transition to s' given s and a

Grid World Actions Reminder





Agent moves in the above grid via actions *Up, Down, Left, Right*

Each action has:

- 0.8 probability to reach its intended effect
- 0.1 probability to move at right angles of the intended direction
- If the agents bumps into a wall, it says there

Example (no observation)

(column, row)

- Back to the grid world, what is the belief state after agent performs action *left* in the initial situation?
- The agent has no information about its position
  - Only one fictitious observation: *no observation*
  - *P(no observation | s) = 1* for every *s*

0.111	0.111	0.111	$\boxed{0.000}$	3
0.111		0.111	$\boxed{0.000}$	2
0.111	0.111	0.111	0.111	1
1	2	3	4	

- Let's instantiate $b'(s') = \alpha P(e \mid s') \sum_{s} P(s' \mid a, s) b(s)$
- For state (1,1) (action a = *left*)

$$b'(1,1) = \alpha\left[P((1,1) \mid (1,1), left)b(1,1) + P((1,1) \mid (2,1), left)b(2,1) + \ldots\right]$$

What is missing to get the correct answer?

A. $P((1,1) \mid (1,2), down)b(1,2)$

B. $P((1,1) \mid (1,3), left)b(1,3)$

C. $P((1,1) \mid (1,2), left)b(1,2)$

Example

(column, row)

- Back to the grid world, what is the belief state after agent performs action *left* in the initial situation?

0.111	0.111	0.111	0.000	3
0.111		0.111	0.000	2
0.111	0.111	0.111	0.111	1
1	2	3	4	

- The agent has no information about its position
  - Only one fictitious observation: *no observation*
  - *P(no observation | s) = 1* for every *s*
- Let's instantiate $b'(s') = \alpha P(e \mid s') \sum_s P(s' \mid a, s) b(s)$

$$b'(1,1) = \alpha\left[P((1,1) \mid (1,1), left) b(1,1) + P((1,1) \mid (1,2), left) b(1,2) + P((1,1) \mid (2,1), left) b(2,1)\right]$$

.9 .111 .1 .111 .8 .111

$$b'(1,2) = \alpha\left[P((1,2) \mid (1,1), left) b(1,1) + P((1,2) \mid (1,2), left) b(1,2) + P((1,2) \mid (1,3), left) b(1,3)\right]$$

....

- Do the above for every state to get the new belief state

After five *Left* actions

0.111	0.111	0.111	0.000
0.111		0.111	0.000
0.111	0.111	0.111	0.111

0.300	0.010	0.008	0.000
0.221		0.059	0.012
0.371	0.012	0.008	0.000

?

tiny but not 0.

Example

- Let's introduce a sensor that perceives the number of adjacent walls in a location with a 0.1 probability of error
  - P(2w|s) = 0.9 ; P(1w|s) = 0.1 if *s* is non-terminal and not in third column
  - P(1w|s) = 0.9 ; P(2w|s) = 0.1 if *s* is non-terminal and in third column
- Try to compute the new belief state if agent **moves *left*** and then **perceives 1 adjacent wall**

0.111	0.111	~~0.111~~	0.000
0.111		~~0.111~~	0.000
0.111	0.111	~~0.111~~	0.111

$$b'(s') = \alpha \; P(e \mid s') \sum_{s} P(s' \mid a, s) b(s)$$

$$b'(1{,}1) = \alpha \; X \; [P((1{,}1) \mid (1{,}1), left) b(1{,}1) + P((1{,}1) \mid (1{,}2), left) b(1{,}2) + P((1{,}1) \mid (2{,}1), left) b(2{,}1)]$$

X should be equal to ?

A. 0.1 **B.** 0.2 **C.** 0.9

Learning Goals for today's class

You can:

- Define and compute **filtering** on an HMM
- Define a **POMDP**
- Define and compute a **state belief update** for a **POMDP**
- Define a **Policy** for a POMDP

TODO for Wed

Read Textbook 9.5.6 Partially Observable MDPs

Check what to do with readings (details on course webpage)

- Carefully read the paper before class
- Send by email
  - (at least 3) questions on the assigned paper
  - a brief summary of the paper (no more than half a page)
  - First Wed 28

Assignment 1 will be out on Wed

- **Partially Observable Markov Decision Process (POMDP):** As the name suggests, POMDPs model scenarios where the agent cannot observe the world state fully [123]. A POMDP agent needs to execute actions for two reasons: for changing the world state (as in an MDP) and for obtaining additional information about the current world state. As Section 7.1.1 explains, a POMDP is a large Continuous MDP, in which a state-variable is the world state, and its value denotes the agent's belief (probability) that it is in that state. Straightforward implementations of MDP algorithms do not scale up to POMDPs and, over the years, a large number of specialized POMDP techniques have been developed, with successes in scaling the algorithms to millions of states [214]. POMDPs have also seen several applications, e.g., dialog management [241], intelligent control of workflows [65], intelligent tutoring [200], and several robotic planning applications [233].