Intelligent Systems (AI-2)

Computer Science cpsc422, Lecture 22

Nov, 1, 2017

Slide credit: some from Prof. Carla PGomes (Cornell)
some slides adapted from Stuart Russell (Berkeley), some from Prof.
Jim Martin (Univ.of Colorado)

Lecture Overview

- **SAT : example**
- **First Order Logics**
  - Language and Semantics
  - Inference

Satisfiability problems (SAT)

Consider a CNF sentence, e.g.,

$$(\neg D \lor \neg B \lor C) \land (B \lor \neg A \lor \neg C) \land (\neg C \lor \neg B \lor E) \land (E \lor \neg D \lor B) \land (B \lor E \lor \neg C)$$

Is there an interpretation in which this sentence is true (i.e., that is a model of this sentence)?

Many combinatorial problems can be reduced to checking the satisfiability of propositional sentences ……and returning a model

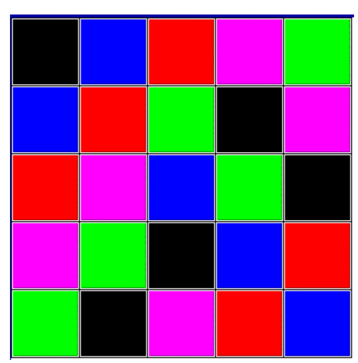
Encoding the Latin Square Problem in Propositional Logic

In **combinatorics** and in experimental design, a **Latin square** is

- an ***n* × *n* array**
- filled with ***n* different sym**bols,
- each occurring exactly once in each row and exactly once in each column.
- Here is an example:

A	B	C
C	A	B
B	C	A

Here is another one:

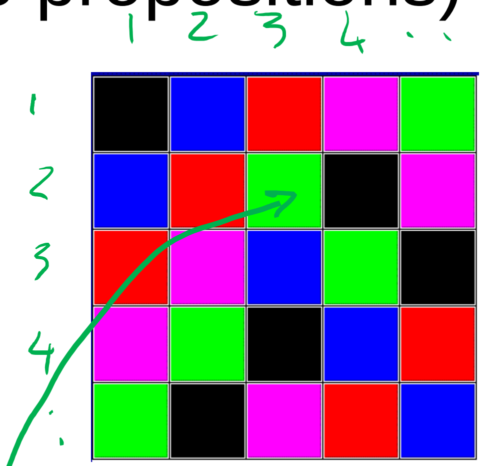


Encoding Latin Square in Propositional Logic: Propositions

Variables must be binary! (They must be propositions)

Each variables represents a color assigned to a cell ***i j***.

Assume colors are encoded as an integer ***k***



$$x_{ijk} \in \{0,1\}$$

Assuming colors are encoded as follows

(black, 1) (red, 2) (blue, 3) (green, 4) (purple, 5)

$x_{234} = 1$

$x_{233} = 0$ True or false, ie. 1 or 0 with respect to the interpretation represented by the picture?

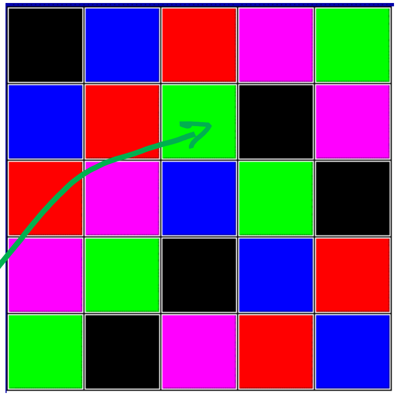
How many vars/propositions overall? n^3

Encoding Latin Square in Propositional Logic

Variables must be binary! (They must be propositions)

Each variables represents a color assigned to a cell.

Assume colors are encoded as integers



$$x_{ijk} \in \{0,1\}$$

Assuming colors are encoded as follows

(black, 1) (red, 2) (blue, 3) (green, 4) (purple, 5)

$x_{234} = 1$

$x_{233} = 0$ True or false, ie. 0 or 1 with respect to the interpretation represented by the picture?

How many vars/propositions overall? n^3

Encoding Latin Square in Propositional Logic: Clauses

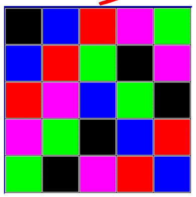
- Some color must be assigned to each cell (clause of length n);

$$\forall_{ij}(x_{ij1} \vee x_{ij2} \dots x_{ijn}) \qquad \forall_{ik}(x_{i1k} \vee x_{i2k} \dots x_{ink})$$

A. B.

J

i



K = color

- No color is repeated in the same row (sets of negative binary clauses);

$$\forall_{ik}(\neg x_{i1k} \vee \neg x_{i2k}) \wedge (\neg x_{i1k} \vee \neg x_{i3k}) \dots (\neg x_{i1k} \vee \neg x_{ink}) \cdots (\neg x_{i(n-1)k} \vee \neg x_{ink})$$

How many clauses?

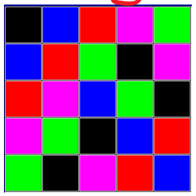
Encoding Latin Square in Propositional Logic: Clauses

- Some color must be assigned to each cell (clause of length n);

$$\forall_{ij}(x_{ij1} \vee x_{ij2} \ldots x_{ijn})$$

i

j



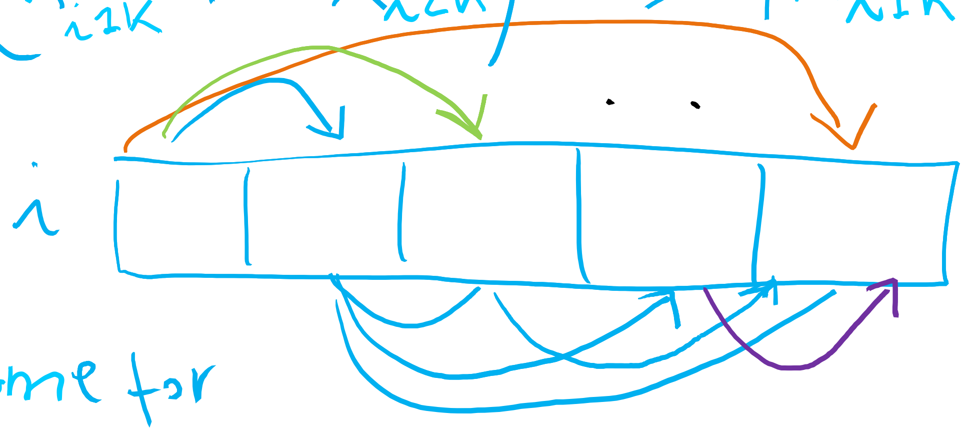
k = color

- No color repeated in the same row (sets of negative binary clauses);

$$\forall_{ik}(\neg x_{i1k} \vee \neg x_{i2k}) \wedge (\neg x_{i1k} \vee \neg x_{i3k}) \ldots (\neg x_{i1k} \vee \neg x_{ink}) \cdots (\neg x_{i(n-1)k} \vee \neg x_{ink})$$

$n * n = n^2$

$\neg(x_{i1k} \wedge x_{i2k}) \Rightarrow \neg x_{i1k} \vee \neg x_{i2k}$



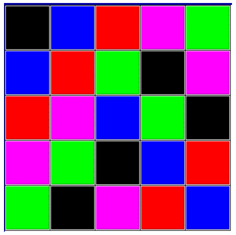
$\frac{n*(n-1)}{2}$

same for

How many clauses?

$n^2 * \frac{n*(n-1)}{2}$ $O(n^4)$

Encoding Latin Square Problems in Propositional Logic: FULL MODEL



n^3

Variables: x_{ijk} $cell\ i,j\ has\ color\ k;\quad i,j,k = 1,2,\ \ldots, n.\qquad x_{ijk} \in \{0,1\}$

Each variables represents a color assigned to a cell.

Clauses: $O(n^4)$

- Some color must be assigned to each cell (clause of length n);

$$\forall_{ij}(x_{ij1} \vee x_{ij2} \ldots x_{ijn})$$

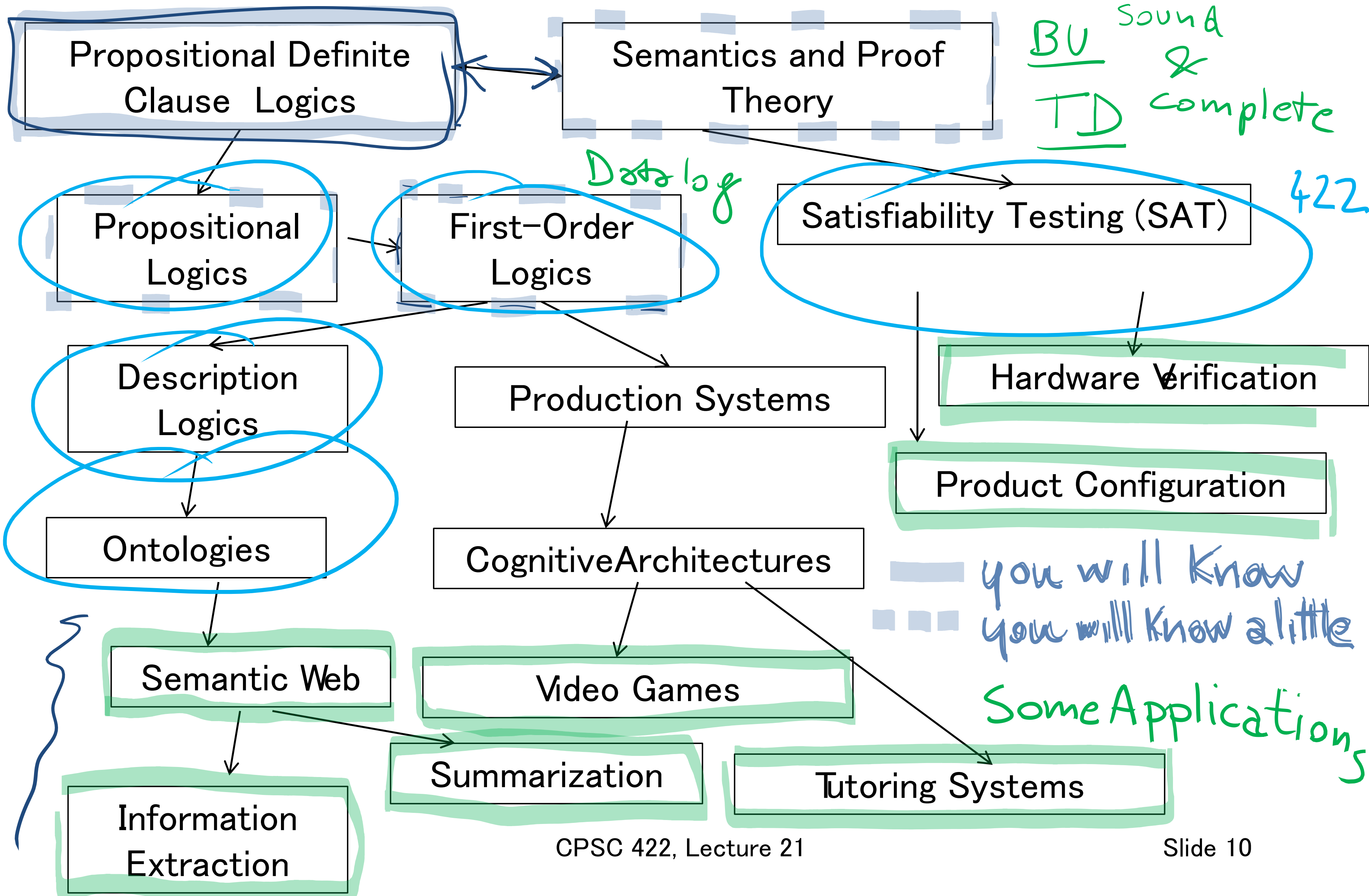
- No color repeated in the same row (sets of negative binary clauses);

$$\forall_{ik}(\neg x_{i1k} \vee \neg x_{i2k}) \wedge (\neg x_{i1k} \vee \neg x_{i3k}) \ldots (\neg x_{i1k} \vee \neg x_{ink}) \cdots (\neg x_{i(n-1)k} \vee \neg x_{ink})$$

- No color repeated in the same column (sets of negative binary clauses);

$$\forall_{jk}(\neg x_{1jk} \vee \neg x_{2jk}) \wedge (\neg x_{1jk} \vee \neg x_{3jk}) \ldots (\neg x_{1jk} \vee \neg x_{njk}) \cdots (\neg x_{(n-1)jk} \vee \neg x_{njk})$$

Logics in AI: Similar slide to the one for planning



Relationships between different Logics

(better with colors)

First Order Logic

$\forall X \exists Y p(X, Y) \Longleftrightarrow \neg q(Y)$

$p(a_1, a_2)$

$-q(a_5)$

...

Datalog

$p(X) \leftarrow q(X) \wedge r(X, Y)$

$r(X, Y) \leftarrow s(Y)$

$s(a_1), q(a_2)$

Propositional Logic

$\neg(p \vee q) \rightarrow (r \wedge s \wedge f),$

p, r

PDCL

$p \leftarrow s \wedge f$

$r \leftarrow s \wedge q \wedge p$

r

p

Lecture Overview

- Finish SAT (example)
- **First Order Logics**
  - Language and Semantics
  - Inference

Representation and Reasoning in Complex domains (from 322)

- In complex domains expressing knowledge with **propositions** can be quite limiting

p
q
r
s

$\boxed{up}\_s_2$
$\boxed{up}\_s_3$
$\boxed{ok}\_cb_1$
$\boxed{ok}\_cb_2$
$live\_w_1$
$connected\_w_1\_w_2$

- It is often natural to consider **individuals** and their **properties**

$up(s_2)$
$up(s_3)$
$ok(cb_1)$
$ok(cb_2)$
$live(w_1)$
$connected(w_1, w_2)$

There is no notion that the system can reason about

$\boxed{up\_s_2 \quad up\_s_3}$

up are about the same property

$live\_w_1$
$connected\_w_1\_w_2$

w_1 are about the same individual

(from 322) What do we gain….

By breaking propositions into relations applied to individuals?

- Express **knowledge** that **holds for set of individuals** (by introducing variables)

$$live(W) \leftarrow connected\_to(W,W1) \land live(W1) \land wire(W) \land wire(W1).$$

- We can **ask generic queries** (i.e., containing vars)

variabless

$$?\ connected\_to(W, w_1)$$

"Full" First Order Logics (FOL)

LIKE DATALOG: Whereas propositional logic assumes the world contains facts, FOL (like natural language) assumes the world contains

- Objects: people, houses, numbers, colors, baseball games, wars, …
- Relations: red, round, prime, brother of, bigger than, part of, comes between, …
- Functions: father of, best friend, one more than, plus, …

FURTHERMORE WE HAVE

- More Logical Operators:….
- Equality: coreference (two terms refer to the same object)
- Quantifiers
  - ✓ Statements about unknown objects
  - ✓ Statements about classes of objects

Syntax of FOL

Constants	KingJohn, 2, ,...
Predicates	Brother, $>$,...
Functions	Sqrt, LeftLegOf,...
Variables	x, y, a, b,...
Connectives	$\neg$, $\Rightarrow$, $\wedge$, $\vee$, $\Leftrightarrow$
Equality	$=$
Quantifiers	$\forall$, $\exists$

Atomic sentences

Term is a *function* ($term_1$,...,$term_n$) or *constant* or *variable*

Atomic sentence is *predicate* ($term_1$,...,$term_n$)
or $term_1$ = $term_2$

E.g.,

predicate (constant , constant)

- *Brother*(*KingJohn*, *RichardTheLionheart*)

predicate (function (function (constant)), (function (function (constant)))

- > (*Length*(*LeftLegOf*(*Richard*)), *Length*(*LeftLegOf*(*KingJohn*)))

Complex sentences

Complex sentences are made from atomic sentences using connectives

$$\neg S, \quad S_1 \wedge S_2, \quad S_1 \vee S_2, \quad S_1 \Rightarrow S_2, \quad S_1 \Leftrightarrow S_2,$$

E.g.

$Sibling(KingJohn, Richard) \Rightarrow Sibling(Richard, KingJohn)$

$\forall$x $P(x)$ is true in an interpretation $\boldsymbol{I}$ iff P is true with x being each possible object in $\boldsymbol{I}$

$\exists$x P(x) is true in an interpretation I iff P is true with x being some possible object in I

Truth in first-order logic

Like in Prop. Logic a sentences is true with respect to an interpretation

$\neg A \wedge (B \Rightarrow C)$,

2^3

A	B	C	
T	F	F	×
F	T	T	✓

In FOL interpretations are **much more complex** but still same idea: possible configuration of the world

symbols: CONSTANTS → objects; Predicates → relations; Functions → functions

2 objects △ □

c_1 → △, c_2 → □

2 CONSTANT SYMBOLS $\{c_1, c_2\}$ → △ □

1 unary Predicate P → {△}

1 binary Predicate Q → {{△, △}}

IS $\forall x\, P(x)$ TRUE?

A. yes
B. no

Truth in first-order logic

Like in Prop. Logic a sentences is true with respect to an interpretation

$\neg A \wedge B \Rightarrow C,$ 2^3

A	B	C	
T	F	F	×
T	T	T	✓

In FOL interpretations are much more complex but still same idea: possible configuration of the world

symbols $\left\{ \begin{array}{l} \text{CONSTANTS} \\ \text{Predicates} \\ \text{Functions} \end{array} \right.$ $\begin{array}{l} \rightarrow \text{objects} \\ \rightarrow \text{relations} \\ \rightarrow \text{functions} \end{array}$

2 objects △ □

2 CONSTANT SYMBOLS $\{c_1, c_2\}$ → △ (c_1) □ (c_2)

1 unary Predicate P → {△}

1 binary Predicate Q → {{△, △}}

IS $\forall x\, P(x)$ TRUE? NO

but if P → {△, □} Yes!

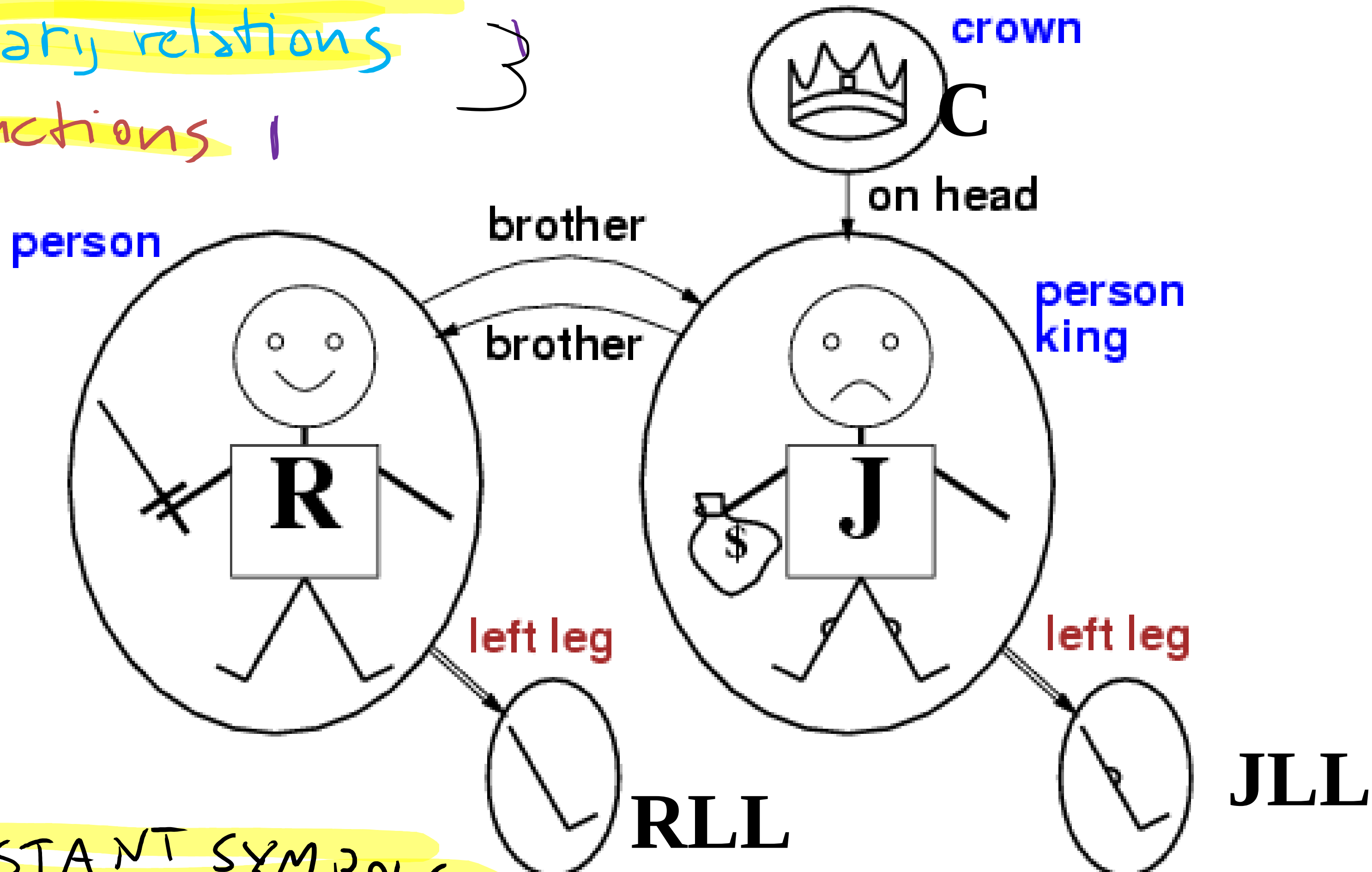
Interpretations for FOL: Example

5 objects

binary relations 2

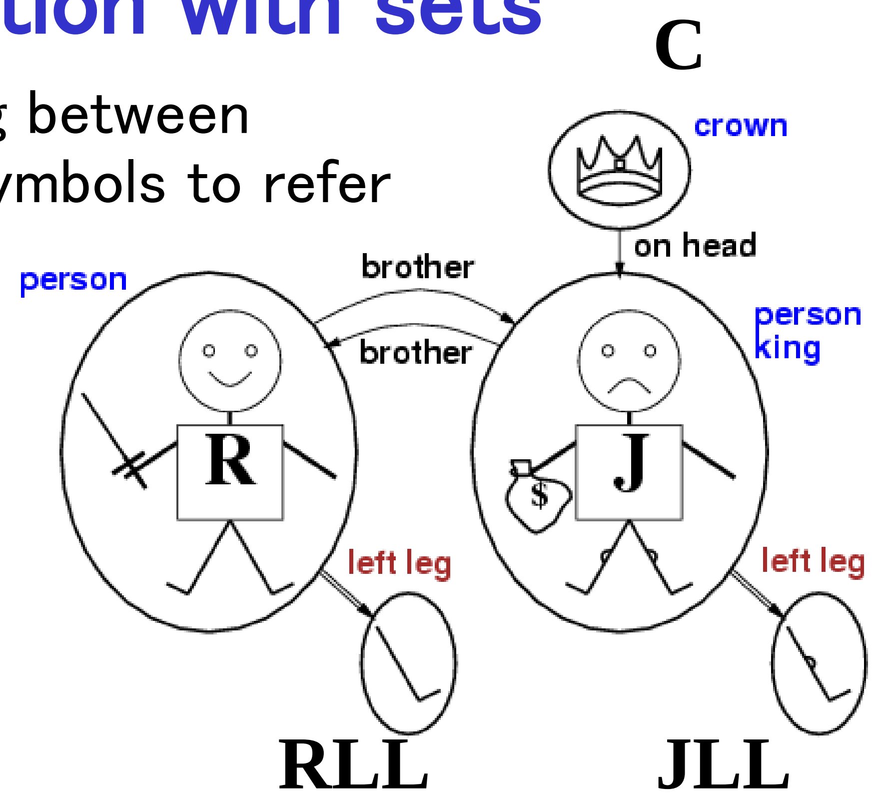
unary relations 3

functions 1



CONSTANT SYMBOLS 5

Same interpretation with sets



Since we have a one to one mapping between symbols and object we can use symbols to refer to objects

- {R, J, RLL, JLL, C}

Property Predicates

- Person = {R, J}
- Crown = {C}
- King = {J}

Relational Predicates

- Brother = { <R,J>, <J,R>}
- OnHead = {<C,J>}

Functions

- LeftLeg = {<R, RLL>, <J, JLL>}

How many Interpretations with….

assuming unique names

5 Objects and 5 symbols $5!$ J R C RLL JLL

- {R, J, RLL, JLL, C}

3 Property Predicates (Unary Relations)

R J RLL JLL C

- Person 0/1 0/1 0/1 0/1 0/1 2^5
- Crown
- King

A. 2^5 B. 2^{25} C. 25^2

2 Relational Predicates

- Brother 25 possibilities; each one can be 0/1 so 2^{25}
- OnHead

1 Function

- LeftLeg 5^5

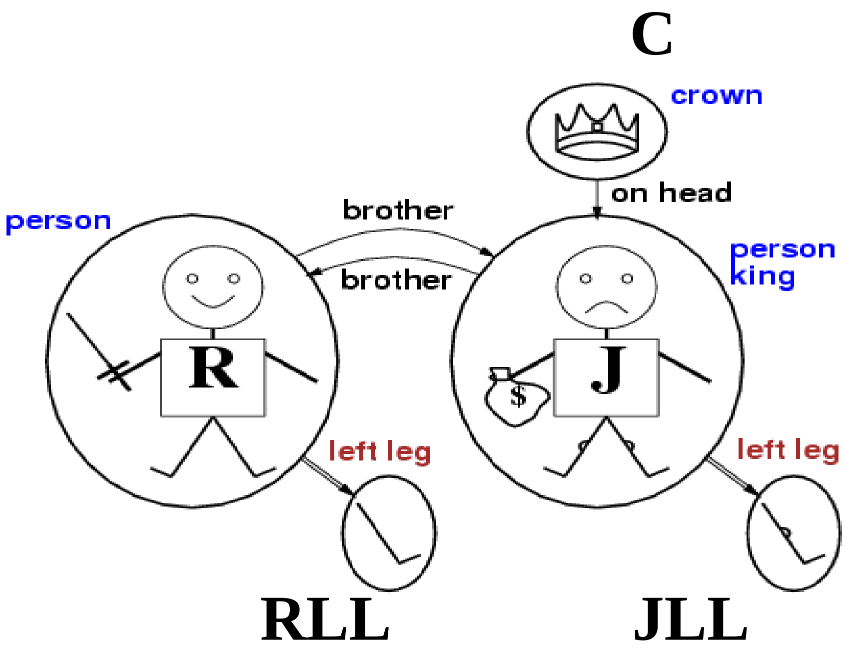
TOTAL $5! * (2^5)^3 * (2^{25})^2 * 5^5$

To summarize: Truth in first-order logic

- Sentences are true with respect to an **interpretation**
- World contains objects (**domain elements**)
- Interpretation specifies referents for

constant symbols	→	**objects**
predicate symbols	→	**relations**
function symbols	→	**functional relations**

- An atomic sentence ***predicate(term$_1$,...,term$_n$)*** is true iff the **objects** referred to by ***term$_1$,...,term$_n$*** are in the **relation** referred to by ***predicate***



Quantifiers

Allows us to express

- **Properties of collections of objects** instead of enumerating objects by name
- **Properties of an unspecified object**

Universal: "for all" $\forall$

Existential: "there exists" $\exists$

Universal quantification

$\forall$<*variables*> <*sentence*>

Everyone at UBC is smart:

$$\forall x\ At(x, UBC) \Rightarrow Smart(x)$$

$\forall x$ *P* is true in an interpretation I iff *P* is true with *x* being each possible object in I

Equivalent to the conjunction of instantiations of *P*

$$\begin{array}{l} \quad At(KingJohn, UBC) \Rightarrow Smart(KingJohn) \\ \land At(Richard, UBC) \Rightarrow Smart(Richard) \\ \land At(Ralphie, UBC) \Rightarrow Smart(Ralphie) \\ \land \ldots \end{array}$$

Existential quantification

$\exists$<*variables*> <*sentence*>

Someone at UBC is smart:

$\exists x$ At(x, UBC) $\wedge$ Smart(x)

$\exists x$ *P* is true in an interpretation ***I*** iff *P* is true with *x* being some possible object in ***I***

Equivalent to the disjunction of instantiations of *P*

At(KingJohn, UBC) $\wedge$ Smart(KingJohn)

$\vee$ At(Richard, UBC) $\wedge$ Smart(Richard)

$\vee$ At(Ralphie, UBC) $\wedge$ Smart(Ralphie)

$\vee$ ...

Properties of quantifiers

$\exists x\ \forall y$ is not the same as $\forall y\ \exists x$

$\exists x\ \forall y\ Loves(x,y)$

- "There is a person who loves everyone in the world"

$\forall y\ \exists x\ Loves(x,y)$

- "Everyone in the world is loved by at least one person"

Quantifier duality: each can be expressed using the other

$\forall x\ Likes(x,IceCream) \qquad \neg\exists x\ \neg Likes(x,IceCream)$

$\exists x\ Likes(x,Broccoli) \qquad \neg\forall x\ \neg Likes(x,Broccoli)$

Lecture Overview

- Finish SAT (example)
- **First Order Logics**
  - Language and Semantics
  - **Inference**

FOL: Inference

Resolution Procedure can be generalized to FOL

- **Every formula** can be rewritten in **logically equivalent CNF**
  - Additional rewriting rules for quantifiers
- **Similar Resolution step,** but variables need to be unified (like in DATALOG)

$In(x, y) \vee \neg Charged(x)$ $\quad\quad \theta = \{z/x, v/y\}$

$\neg In(z, v) \vee Connected(z)$

$\boxed{\neg Charged(x) \vee Connected(x)}$

NLP Practical Goal for FOL: the ultimate Web question-answering system?

Map NL queries into FOPC so that answers can be effectively computed

What African countries are not on the Mediterranean Sea?

$$\exists c\ Country(c) \wedge \neg Borders(c, Med.Sea) \wedge In(c, Africa)$$

- *Was 2007 the first El Nino year after 2001?*

$$ElNino(2007) \wedge \neg \exists y\ Year(y) \wedge After(y, 2001) \wedge Before(y, 2007) \wedge ElNino(y)$$

NL query $\xrightarrow{\text{parsing}}$ synt. tree $\xrightarrow{\lambda\text{ calculus}}$ "FOPC" query $\xrightarrow{\text{applied}}$ KB

KB ← IExtraction into ← Web

→NER

Relation identification & Extraction

Learning Goals for today's class

You can:

- Explain differences between Proposition Logic and First Order Logic
- Compute number of interpretations for FOL
- Explain the meaning of quantifiers
- Describe application of FOL to NLP: Web question answering

Next class Fri

- Ontologies (e.g., Wordnet, Probase), Description Logics…
- Midterm will be returned (sorry for the delay)

Assignment-3 will be out today