Intelligent Systems (AI-2)

Computer Science cpsc422, Lecture 21

Oct, 30, 2017

Slide credit: some slides adapted from Stuart Russell (Berkeley),
some from Prof. Carla PGomes (Cornell)

David Buchman and Professor David Poole are the recipients of the **UAI 2017**
Best Student Paper Award, *"Why Rules are Complex: Real-Valued Probabilistic Logic Programs are not Fully Expressive*". This paper proves some surprising results about what can and what cannot be represented by **a popular method that combines logic and probability**. Such models are important as they let us go beyond features in machine learning to reason about objects and relationships with uncertainty.

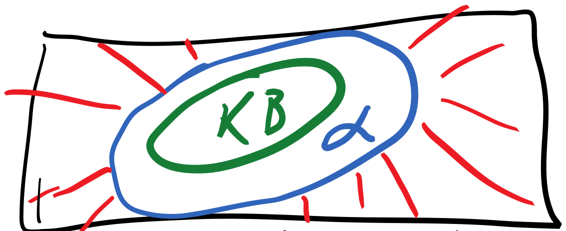
Lecture Overview

- Finish Resolution in Propositional logics
- Satisfiability problems
- WalkSAT
- Start Encoding Example

Proof by resolution

$KB \models \alpha$

~~equivalent to~~ : $KB \wedge \neg\alpha$ *unsatisfiable*



All Interpretations

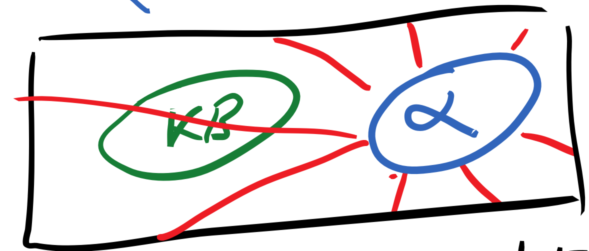
Models of KB

Models of α

Models of ¬α

$KB \models \alpha$

~~equivalent to~~ : $KB \wedge \neg\alpha$ *unsatisfiable*



All Interpretations

Key ideas

proof

$KB \models \alpha$

show

equivalent to : $KB \wedge \neg\alpha$ *unsatisfiable*

- Simple Representation for $KB \wedge \neg\alpha$ Conjunctive Normal Form
- Simple Rule of Derivation

Resolution

Conjunctive Normal Form (CNF)

Rewrite $KB \wedge \neg\alpha$ into **conjunction of disjunctions**

literals

$$\underbrace{(A \vee \neg B)}_{\text{Clause}} \wedge \underbrace{(B \vee \neg C \vee \neg D)}_{\text{Clause}}$$

- Any KB can be converted into CNF !

Example: Conversion to CNF

$A \Leftrightarrow (B \vee C)$

1. Eliminate $\Leftrightarrow$, replacing $\alpha \Leftrightarrow \beta$ with $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$.
   $(A \Rightarrow (B \vee C)) \wedge ((B \vee C) \Rightarrow A)$

2. Eliminate $\Rightarrow$, replacing $\alpha \Rightarrow \beta$ with $\neg \alpha \vee \beta$.
   $(\neg A \vee B \vee C) \wedge (\neg(B \vee C) \vee A)$

3. Using de Morgan's rule replace $\neg(\alpha \vee \beta)$ with $(\neg \alpha \wedge \neg \beta)$:
   $(\neg A \vee B \vee C) \wedge ((\neg B \wedge \neg C) \vee A)$

4. Apply distributive law ($\vee$ over $\wedge$) and flatten:
   $(\neg A \vee B \vee C) \wedge (\neg B \vee A) \wedge (\neg C \vee A)$

Example: Conversion to CNF

$A \Leftrightarrow (B \vee C)$

5. KB is the conjunction of all of its sentences (all are true), so write each clause (disjunct) as a sentence in KB:

…

$(\neg A \vee B \vee C)$
$(\neg B \vee A)$
$(\neg C \vee A)$
…

Full Propositional Logics

DEFs.

Literal: an atom or a negation of an atom $p \quad \neg q \quad r$

Clause: is a disjunction of literals $p \vee \neg r \vee q$

Conjunctive Normal Form (CNF): a conjunction of clauses

$(p) \wedge (q \vee \neg r) \wedge (\neg q \vee p)$

INFERENCE: $KB \overset{?}{\models} \alpha$ formula

- **Convert all formulas in KB and** $\neg\alpha$ **in CNF**
- **Apply Resolution Procedure**

$p \vee q \quad r \vee \neg q \rightarrow p \vee r$

$KB \nvdash \alpha$

$KB \vdash \alpha$

Resolution Deduction step

Resolution: inference rule for CNF: sound and complete! *

$(A \vee B \vee C)$

$(\neg A)$

"If A or B or C is true, but not A, then B or C must be true."

$\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

$\therefore (B \vee C)$

$(A \vee B \vee C)$

$(\neg A \vee D \vee E)$

"If A is false then B or C must be true, or if A is true then D or E must be true, hence since A is either true or false, B or C or D or E must be true."

$\_\_\_\_\_\_\_\_\_\_\_\_\_$

$\therefore (B \vee C \vee D \vee E)$

$(A \vee B)$

$(\neg A \vee B)$

Simplification

$\_\_\_\_\_\_\_\_\_\_$

$\therefore (B \vee B) \equiv B$

Resolution Algorithm

but this is equivalent to prove that $KB \wedge \neg\alpha$ is unsatisfiable

- The resolution algorithm tries to prove: $KB \models \alpha$
- $KB \wedge \neg\alpha$ is converted in CNF
- Resolution is applied to each pair of clauses with complementary literals $\neg q \vee r \vee s \quad p \vee q \rightarrow r \vee s \vee p$
- Resulting clauses are added to the set (if not already there)

- Process continues until one of two things can happen:

assuming Resol is sound

1. Two clauses resolve in the empty clause. i.e. query is entailed

$P \quad \neg P \rightarrow \emptyset \quad > KB \vdash_R \alpha \Rightarrow KB \models \alpha$

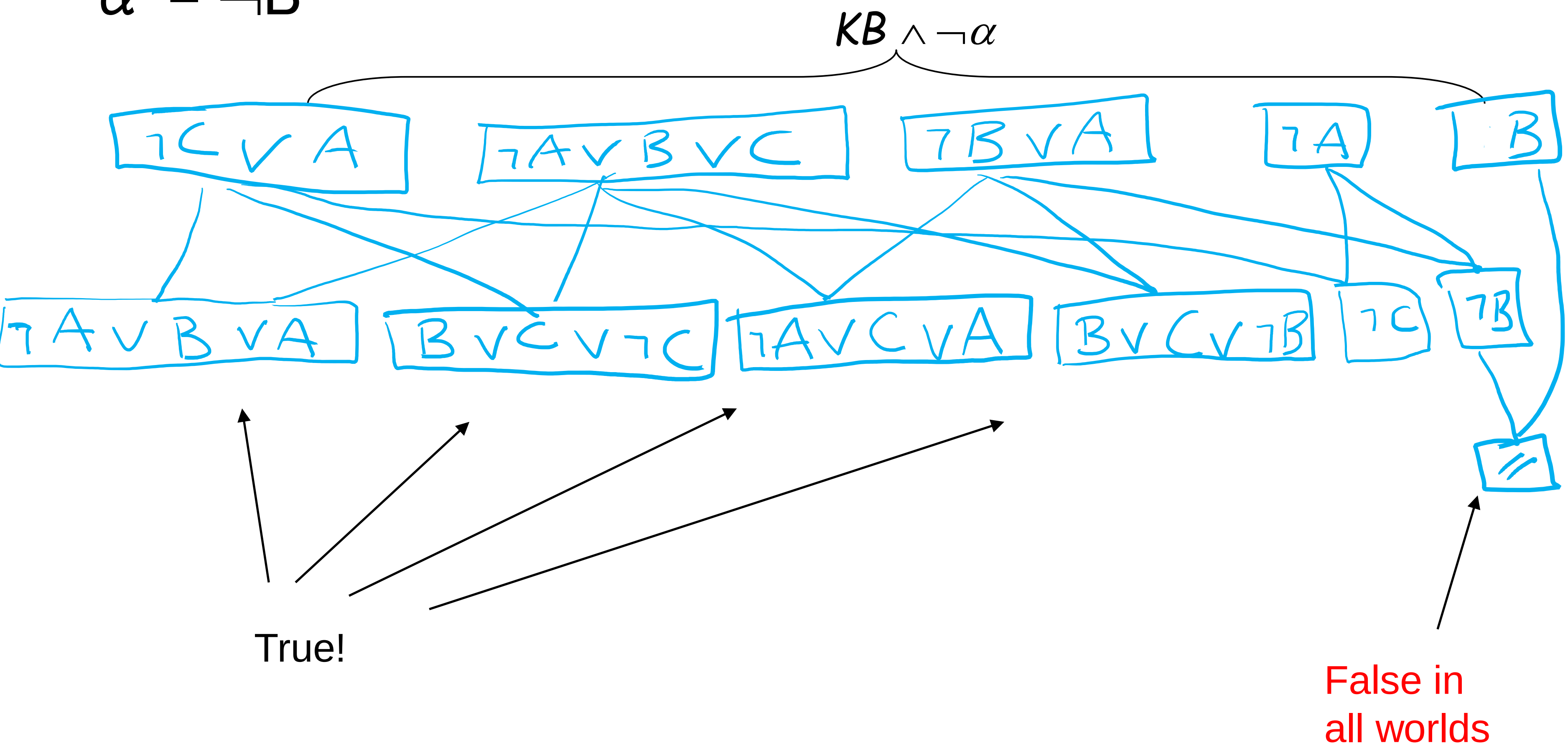
2. No new clauses can be added: We find no contradiction, there is a model that satisfies the sentence and hence we cannot entail the query.

$KB \nvdash_R \alpha \Rightarrow KB \not\models \alpha$ assuming Resol. is complete

Resolution example

$KB = (A \Leftrightarrow (B \vee C)) \wedge \neg A$

$\alpha = \neg B$



Resolution algorithm

Proof by contradiction, i.e., show $KB \wedge \neg\alpha$ unsatisfiable

```
function PL-RESOLUTION(KB, α) returns true or false
    inputs: KB, the knowledge base, a sentence in propositional logic
            α, the query,

    clauses ← the set of clauses in the CNF representation of KB ∧ ¬α
    new ← { }
    loop do
        for each Ci, Cj in clauses do
            resolvents ← PL-RESOLVE(Ci, Cj)
            if resolvents contains the empty clause then return true
            new ← new ∪ resolvents
        if new ⊆ clauses then return false     ; no new clauses were created
        clauses ← clauses ∪ new
```

Lecture Overview

- Finish Resolution in Propositional logics
- **Satisfiability problems**
- **WalkSAT**
- **Hardness of SAT**
- Start Encoding Example

Satisfiability problems

Consider a CNF sentence, e.g.,

$(\neg D \lor \neg B \lor C) \land (B \lor \neg A \lor \neg C) \land (\neg C \lor \neg B \lor E) \land (E \lor \neg D \lor B) \land (B \lor E \lor \neg C)$

Is there an interpretation in which this sentence is true (i.e., that is a model of this sentence)?

Many **combinatorial problems** can be reduced to checking the satisfiability of propositional sentences (example later)… and returning the model

How can we solve a SAT problem?

Consider a CNF sentence, e.g.,

$(\neg D \vee \neg B \vee C) \wedge (A \vee C) \wedge (\neg C \vee \neg B \vee E) \wedge (E \vee \neg D \vee B) \wedge (B \vee E \vee \neg C)$

Each clause can be seen as a ***constraint*** *that reduces the number of interpretations that can be models*

Eg $(A \vee C)$ eliminates interpretations in which A=F and C=F

So SAT is a **Constraint Satisfaction Problem**: Find a possible world that is satisfying all the constraints (here all the clauses)

WalkSAT algorithm

(Stochastic) Local Search Algorithms can be used for this task!

Evaluation Function: number of unsatisfied clauses

WalkSat: One of the simplest and most effective algorithms:

Start from a randomly generated interpretation

- Pick randomly an unsatisfied clause
- Pick a proposition/atom to flip (randomly 1 or 2)
  1. Randomly
  2. To minimize # of unsatisfied clauses

WalkSAT: Example

unsatisfied clauses

D = 0 ✓ ✗ ✓ ✓ ✗ 2

E = 1 ✗ ✗ ✓ ✓ ✗ 3

B = 1 ✓ ✗ ✓ ✓ ✓ 1

$$(\neg D \vee B \vee C) \wedge (A \vee C) \wedge (\neg C \vee \neg B) \wedge (E \vee \neg D \vee B) \wedge (B \vee C)$$

A	B	C	D	E
0	0	0	1	0
0	1	0	1	0

flip B

Look at algo on previous slide

pick randomly unsatisfied clause

assume $(E \vee \neg D \vee B)$

pick randomly 1 or 2

assume 2

flip B → B=1

Because by flipping B we are left with only 1 unsatisfied clause, while by flipping E with 3 and by flipping D with 2 (see above)

Pseudocode for WalkSAT

```
function WALKSAT(clauses, p, max-flips) returns a satisfying model or failure
   inputs: clauses, a set of clauses in propositional logic
           p, the probability of choosing to do a "random walk" move
           max-flips, number of flips allowed before giving up

    pw ← a random assignment of true/false to the symbols in clauses
   for i = 1 to max-flips do
       if  pw satisfies clauses then return  pw
       clause ← a randomly selected clause from clauses that is false in  pw
 1     with probability p flip the value in  pw  of a randomly selected symbol
              from clause
 2    else flip whichever symbol in clause maximizes the number of satisfied clauses
   return failure
```

pw = possible world / interpretation

The `WalkSAT` algorithm

If it returns failure after it tries *max–flips* times, what can we say?

A. The sentence is unsatisfiable

B. Nothing

C. The sentence is satisfiable

Typically most useful when we expect a solution to exist

Hard satisfiability problems

Consider *random* 3-CNF sentences. e.g.,

$(\neg D \lor \neg B \lor C) \land (B \lor \neg A \lor \neg C) \land (\neg C \lor \neg B \lor E) \land (E \lor \neg D \lor B) \land (B \lor E \lor \neg C)$

m = number of clauses (5)

n = number of symbols (5)

- Under constrained problems:
  - ✓ Relatively few clauses constraining the variables
  - ✓ Tend to be easy

  E.g. For the above problem16 of 32 possible assignments are solutions
  - (so 2 random guesses will work on average)

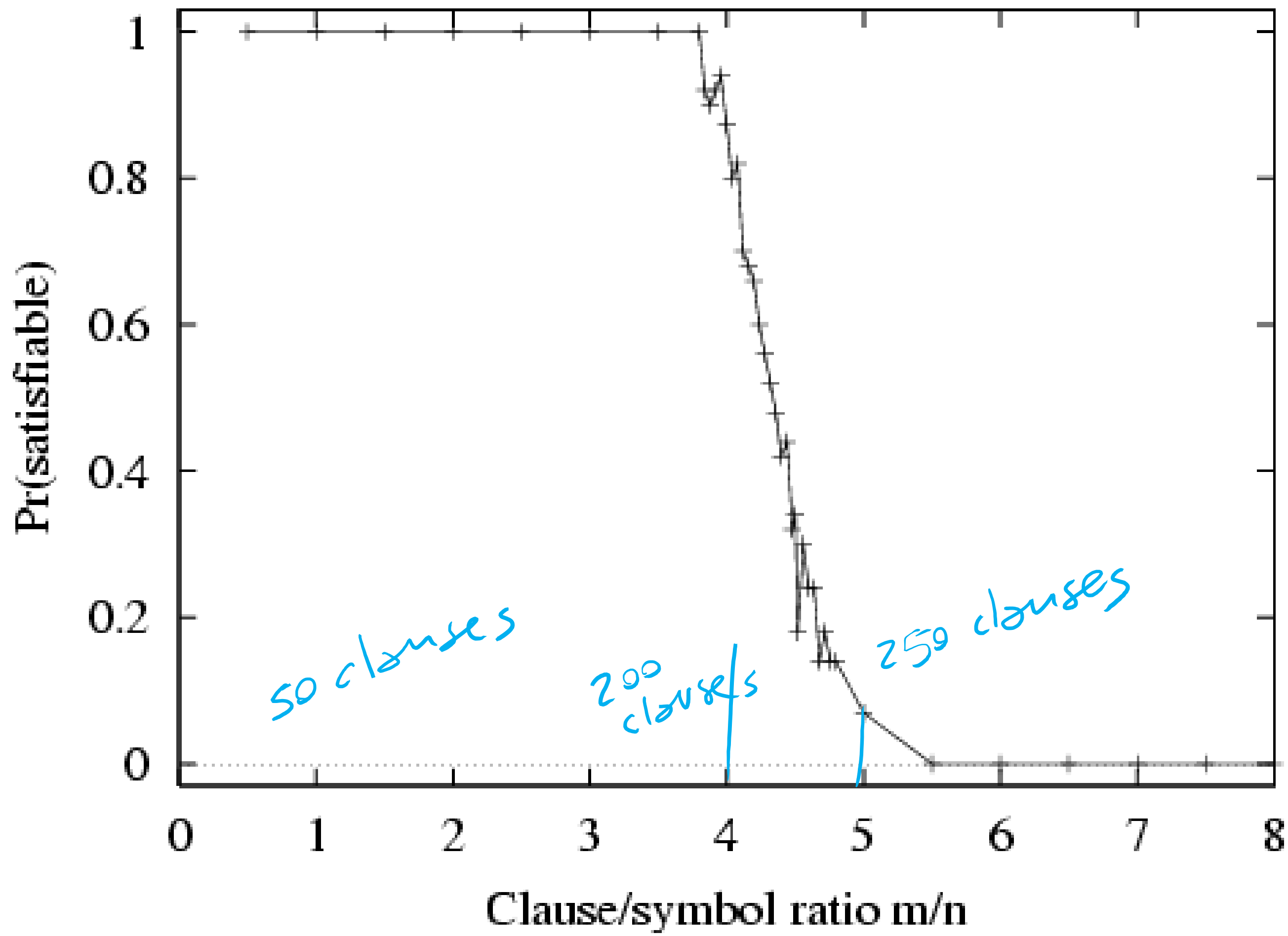
Hard satisfiability problems

What makes a problem hard?

- Increase the number of clauses while keeping the number of symbols fixed
- Problem is more constrained, fewer solutions

- You can investigate this experimentally….

P(satisfiable) for random 3-CNF sentences, n = 50



m = number of clauses

n = number of symbols

- Hard problems seem to cluster near $m/n = 4.3$ (critical point)

Lecture Overview

- Finish Resolution in Propositional logics
- Satisfiability problems
- WalkSAT
- **Start Encoding Example**

Encoding the Latin Square Problem in Propositional Logic

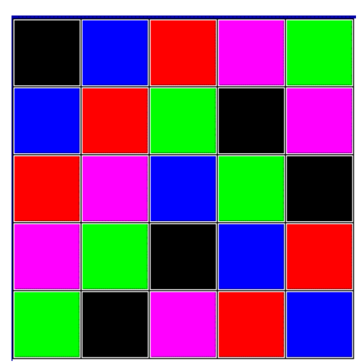
In combinatorics and in experimental design, a **Latin square** is

- an $n \times n$ array
- filled with n different symbols,
- each occurring exactly once in each row and exactly once in each column.

Here is an example:

A	B	C
C	A	B
B	C	A

Here is another one:

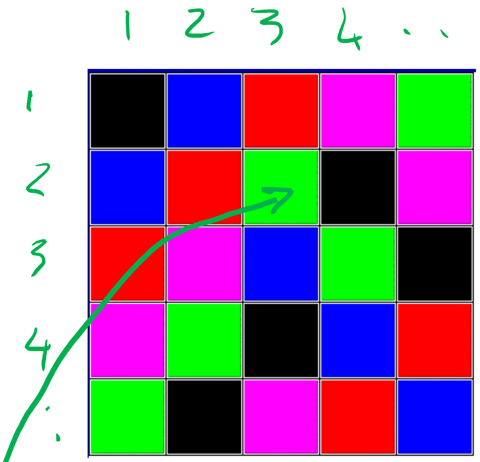


Encoding Latin Square in Propositional Logic: Propositions

Variables must be binary! (They must be propositions)

Each variables represents a color assigned to a cell.

Assume colors are encoded as integers



$$x_{ijk} \in \{0,1\}$$

Assuming colors are encoded as follows

(black, 1) (red, 2) (blue, 3) (green, 4) (purple, 5)

$$x_{234} = 1$$

$$x_{233} = 0$$

True or false, ie. 1 or 0 with respect to the interpretation represented by the picture?

How many vars/propositions overall? n^3

Encoding Latin Square in Propositional Logic: Clauses

- Some color must be assigned to each cell (clause of length n);

$$\forall_{ij}(x_{ij1} \vee x_{ij2} \ldots x_{ijn})$$

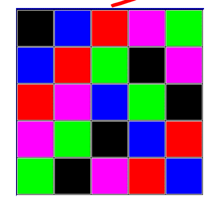
A.

$$\forall_{ik}(x_{i1k} \vee x_{i2k} \ldots x_{ink})$$

B.

J

i



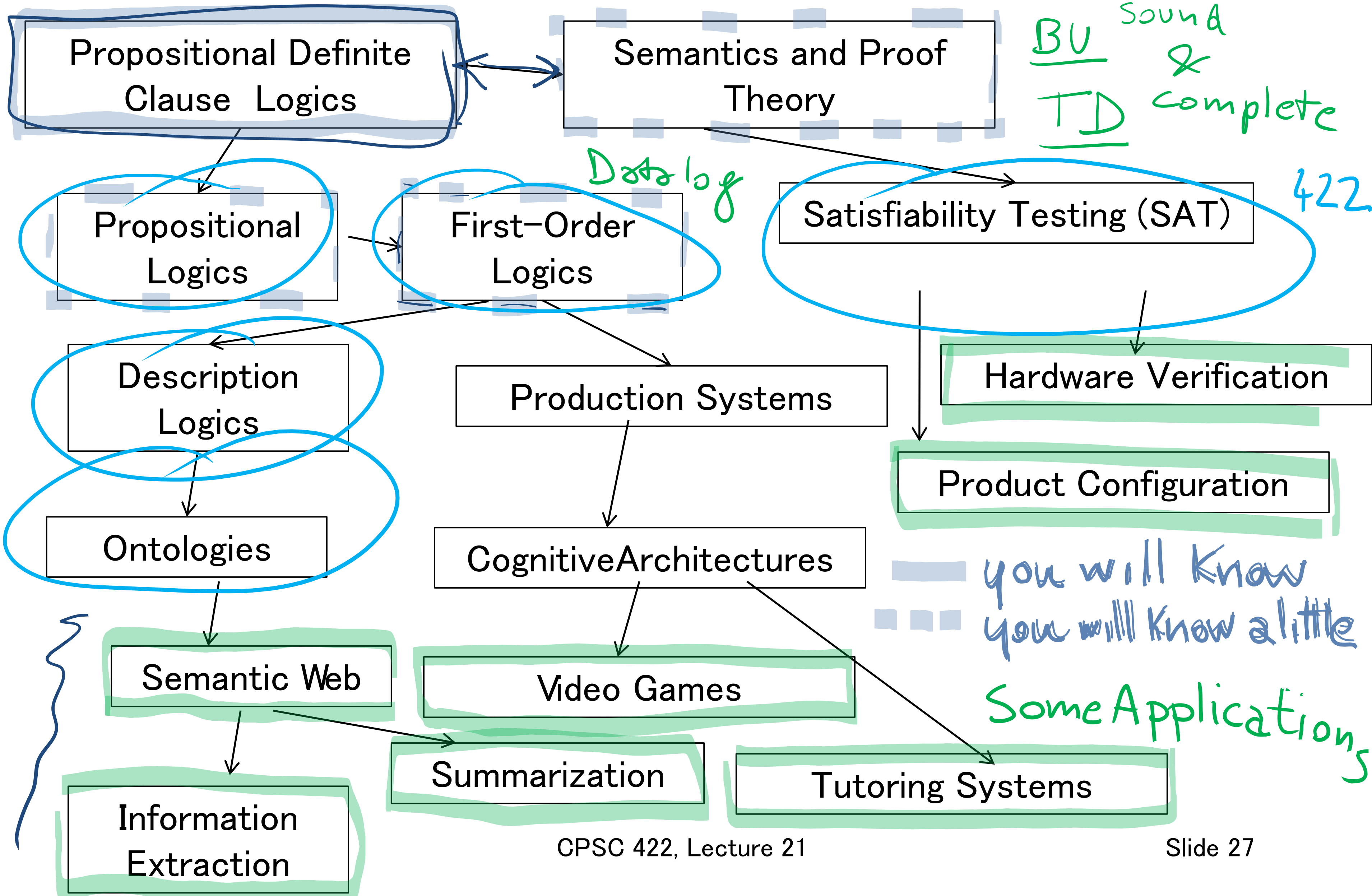
k = color

- No color is repeated in the same row (sets of negative binary clauses);

$$\forall_{ik}(\neg x_{i1k} \vee \neg x_{i2k}) \wedge (\neg x_{i1k} \vee \neg x_{i3k}) \ldots (\neg x_{i1k} \vee \neg x_{ink}) \cdots (\neg x_{ink} \vee \neg x_{i(n-1)k})$$

How many clauses?

Logics in AI: Similar slide to the one for planning



Relationships between different Logics

(better with colors)

First Order Logic

$\forall X \exists Y p(X,Y) \Longleftrightarrow \neg q(Y)$

$p(a_1, a_2)$

$\neg q(a_5)$

...

Datalog

$p(X) \leftarrow q(X) \wedge r(X,Y)$

$r(X,Y) \leftarrow s(Y)$

$s(a_1), q(a_2)$

Propositional Logic

$\neg(p \vee q) \rightarrow (r \wedge s \wedge f),$

p, r

PDCL

$p \leftarrow s \wedge f$

$r \leftarrow s \wedge q \wedge p$

r

p

Learning Goals for today's class

You can:

- Specify, Trace and Debug the resolution proof procedure for propositional logics
- Specify, Trace and Debug WalkSat
- Encode the Latin square problem in propositional logics (basic ideas)

Next class Wed

- First Order Logic
- Extensions of FOL

- Assignment-3 will be posted on Wed!