Intelligent Systems (AI-2)

Computer Science cpsc422, Lecture 6

Sep, 21, 2016

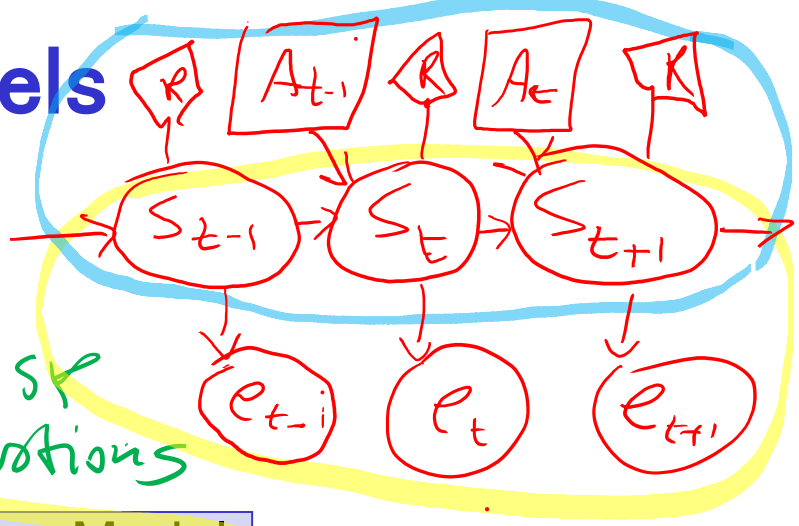
Slide credit POMDP: C. Conati and PViswanathan

Lecture Overview

Partially Observable Markov Decision Processes

- Summary
  - Belief State
  - Belief State Update
- Policies and Optimal Policy

Markov Models



Markov Chains

Noisy observations

Hidden Markov Model

Noisy Actions

Partially Observable Markov Decision Processes (POMDPs)

Markov Decision Processes (MDPs)

Rewards

Belief State and its Update

$$b(s)$$

$$b'(s') = \alpha P(e \mid s') \sum_{s} P(s' \mid s, a) b(s)$$

as

$$b' = Forward(b, a, e)$$

- To summarize: when the agent performs action ***a*** in belief state ***b***, and then receives observation ***e***, filtering gives a unique new probability distribution over state
  - ***deterministic transition from one belief state to another***

Belief Update: Example1

- Let's introduce a sensor that perceives the number of adjacent walls in a location with a 0.1 probability of error
  - P(2w|s) = 0.9 ; P(1w|s) = 0.1 if *s* is non-terminal and not in third column
  - P(1w|s) = 0.9 ; P(2w|s) = 0.1 if *s* is non-terminal and in third column
- Try to compute the new belief state if agent **moves *left*** and then **perceives 1 adjacent wall**

0.111	0.111	0.111	0.000
0.111		0.111	0.000
0.111	0.111	0.111	0.111

$$b'(s') = \alpha \quad P(e \mid s') \quad \sum_{s} P(s' \mid a, s) b(s)$$

$$b'(1,1) = \alpha \;\; X \;\; \left[P((1,1) \mid (1,1), left) b(1,1) + P((1,1) \mid (1,2), left) b(1,2) + P((1,1) \mid (2,1), left) b(2,1) \right]$$

X should be equal to ?

A. 0.1 **B.** 0.2 **C.** 0.9

Belief Update: Example 2

- Let's introduce a sensor that perceives the number of adjacent walls in a location with a 0.1 probability of error
  - P(2w|s) = 0.9 ; P(1w|s) = 0.1 if *s* is non-terminal and not in third column
  - P(1w|s) = 0.9 ; P(2w|s) = 0.1 if *s* is non-terminal and in third column
- Try to compute the new belief state if agent **moves *right*** and then **perceives 2 adjacent wall**

0.111	0.111	0.111	0.000
0.111		0.111	0.000
0.111	0.111	0.111	0.111

$$b'(s') = \alpha \quad P(e \mid s') \quad \sum_{s} P(s' \mid a, s) b(s)$$

$$b'(1,2) = \alpha \; P(2w \mid (1,2)) \times$$

$$\begin{bmatrix} P((1,2) \mid (1,1), right) b(1,1) + \\ P((1,2) \mid (1,2), right) b(1,2) + \\ P((1,2) \mid (1,3), right) b(1,3) \end{bmatrix}$$

Optimal Policies in POMDs ?

- Theorem (Astrom, 1965):
  - The optimal policy in a POMDP is a function $\pi^*(b)$ where b is the belief state (probability distribution over states)
- That is, $\pi^*(b)$ is a function from belief states (probability distributions) to actions
  - It does *not* depend on the actual state the agent is in
  - Good, because the agent does not know that, all it knows are its beliefs!
- Decision Cycle for a POMDP agent
  - Given current belief state *b*, execute $a = \pi^*(b)$
  - Receive observation *e*
  - compute: $b'(s') = \alpha P(e \mid s') \sum_s P(s' \mid s, a) b(s)$
  - Repeat

How to Find an Optimal Policy?

?

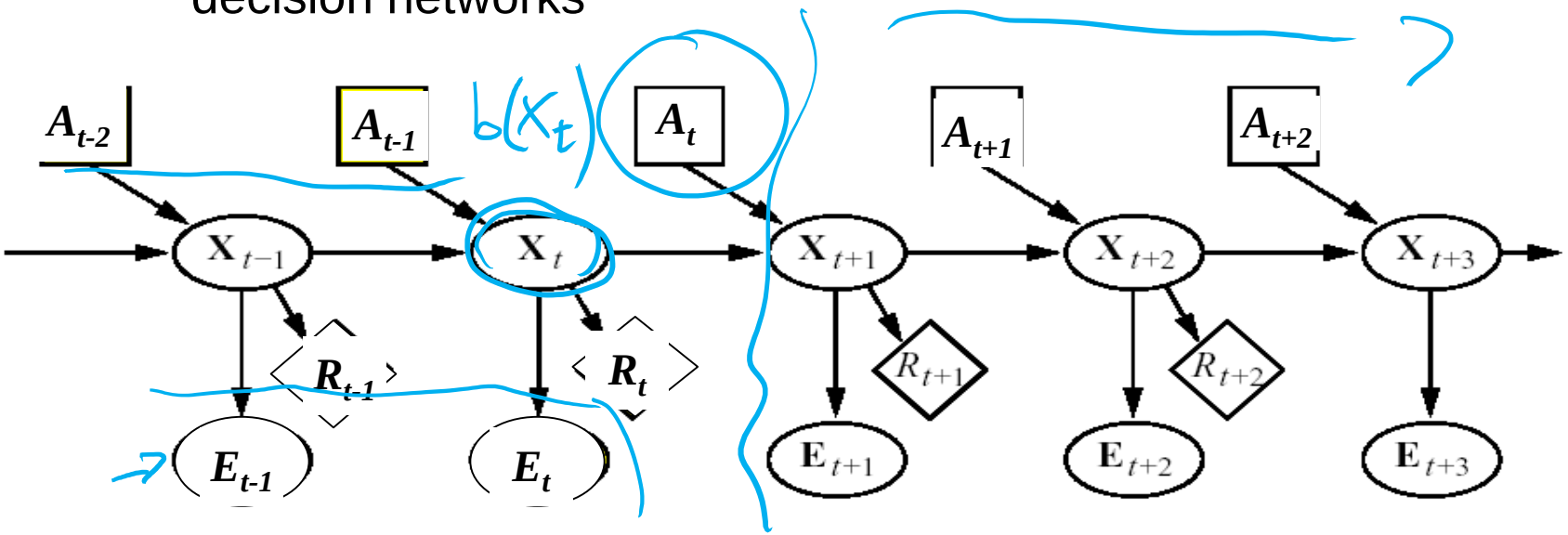
- Turn a POMDP into a corresponding MDP and then solve that MDP
- Generalize VI to work on POMDPs
- Develop Approx. Methods
  - Point-Based VI
  - Look Ahead

Finding the Optimal Policy: State of the Art

- Turn a POMDP into a corresponding MDP and then apply VI: only small models
- Generalize VI to work on POMDPs
  - 10 states in1998
  - 200,000 states in 2008-09
- Develop Approx. Methods 2009 - now
  - Point-Based VI and Look Ahead
  - Even 50,000,000 states
    http://www.cs.uwaterloo.ca/~ppoupart/software.html

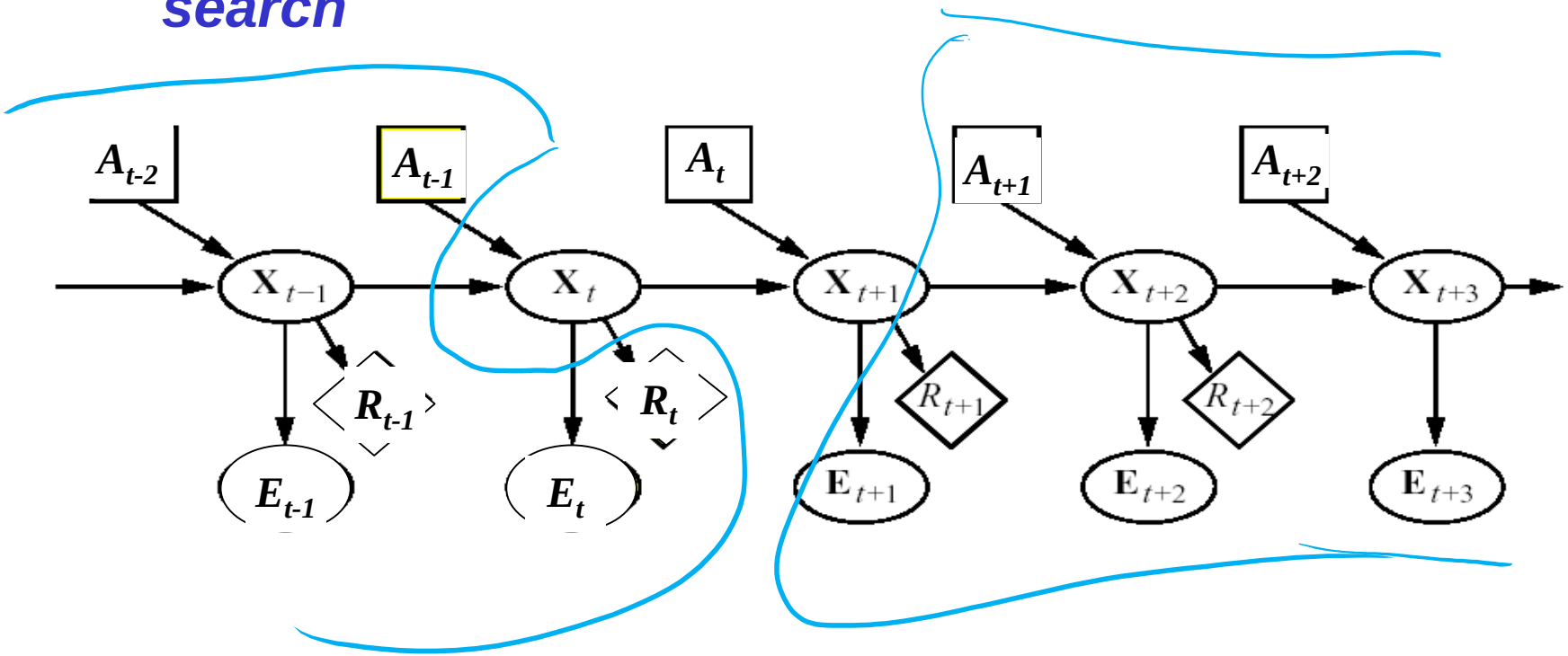
Dynamic Decision Networks (DDN)

- Comprehensive approach to agent design in partially observable, stochastic environments
- Basic elements of the approach
  - Transition and observation models are represented via a Dynamic Bayesian Network (DBN).
  - The network is extended with decision and utility nodes, as done in decision networks

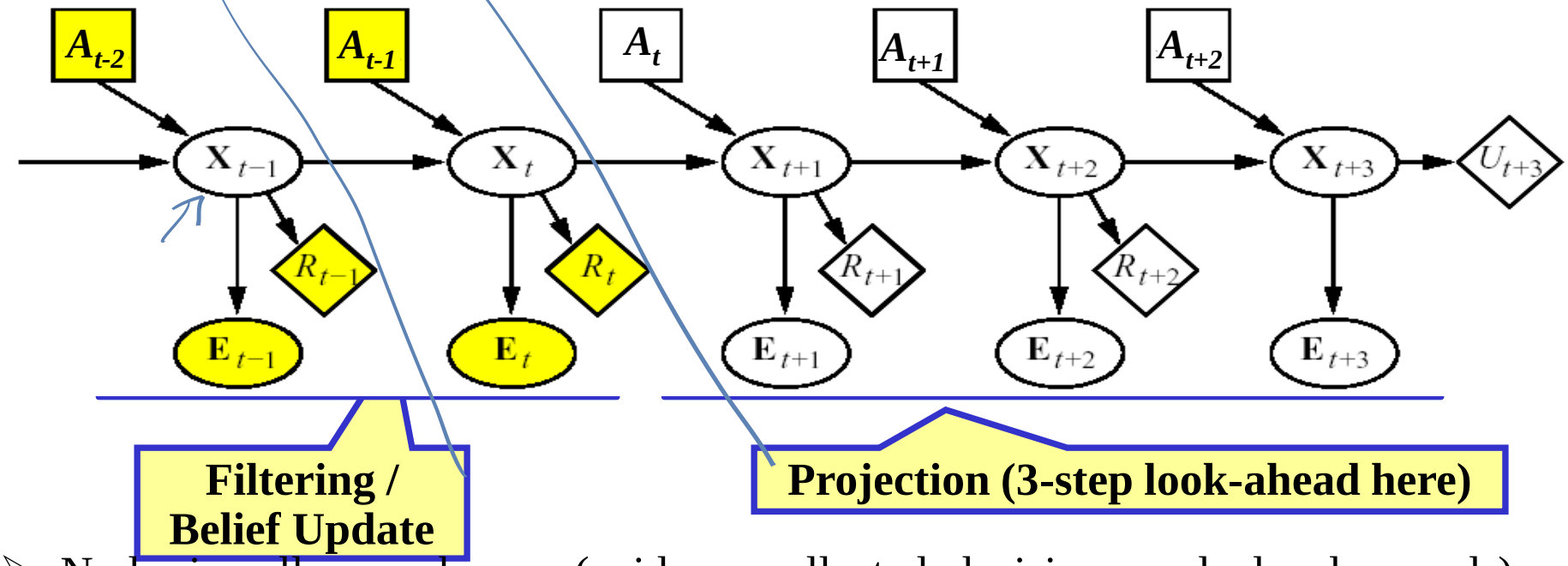


Dynamic Decision Networks (DDN)

- A filtering algorithm is used to incorporate each new percept and the action to update the belief state $\mathbf{X_t}$
- Decisions are made by projecting forward possible action sequences and choosing the best one: ***look ahead search***



Dynamic Decision Networks (DDN)



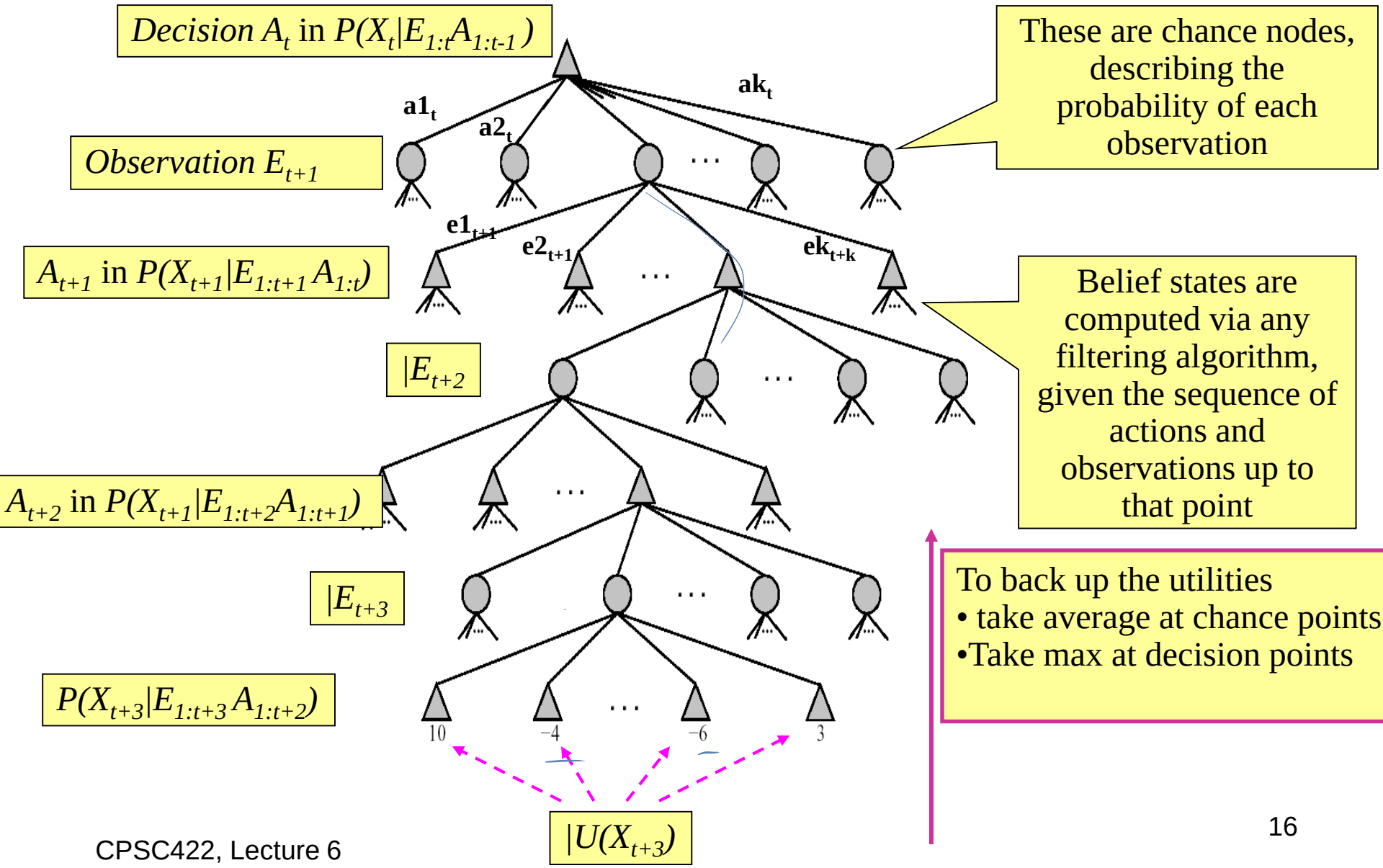
- Nodes in yellow are known (evidence collected, decisions made, local rewards)
- Agent needs to make a decision at time t (A_t node)
- Network unrolled into the future for 3 steps
- Node U_{t+3} represents the utility (or expected optimal reward V^*) in state X_{t+3}
  - i.e., the reward in that state and all subsequent rewards
  - Available only in approximate form (from another approx. method)

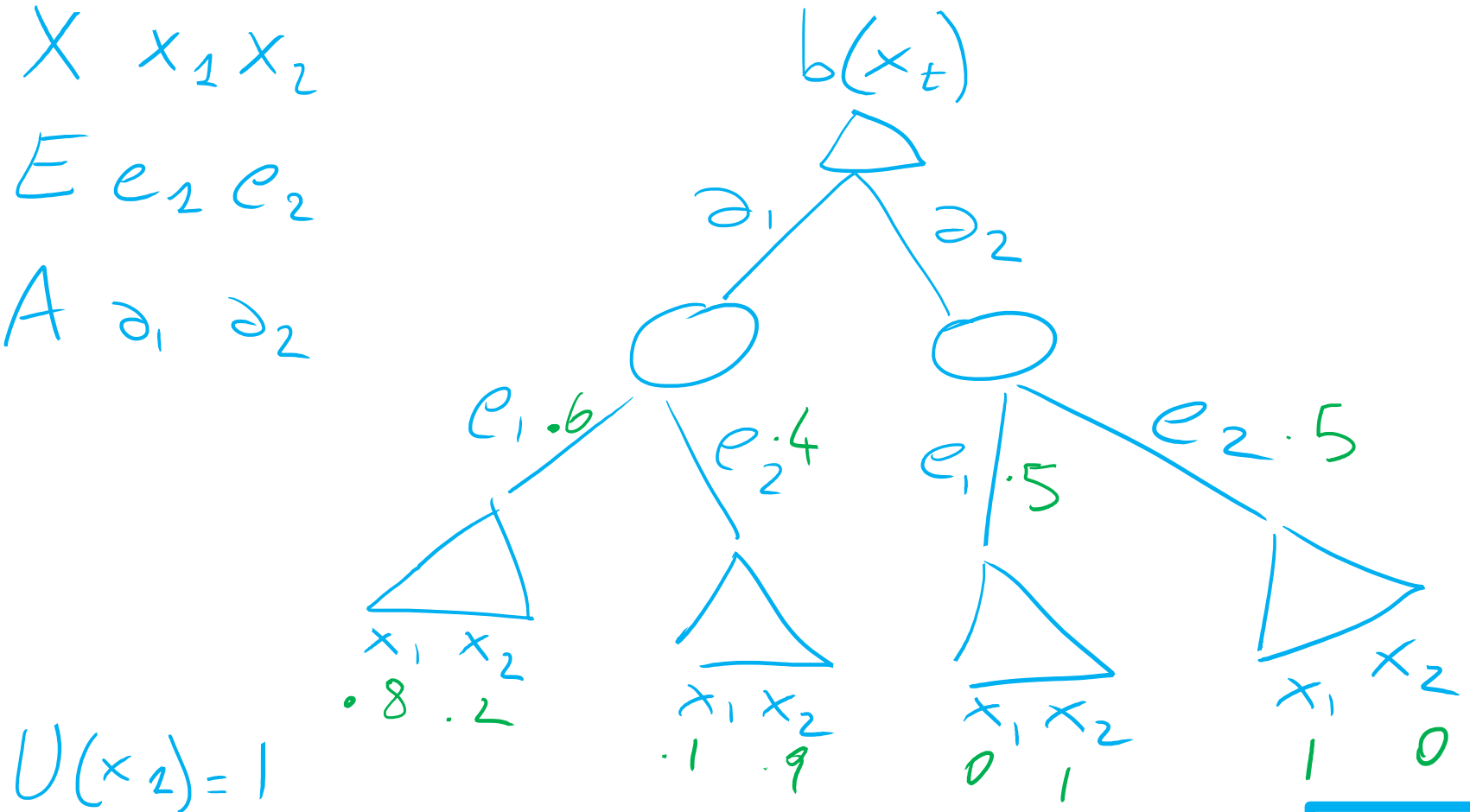
Look Ahead Search for Optimal Policy

General Idea:

- **Expand the decision process for n steps into the future, that is**
  - "Try" all actions at every decision point
  - Assume receiving all possible observations at observation points
- **Result: tree of depth *2n+1*** where
  - every branch represents one of the possible sequences of *n* actions and n observations available to the agent, and the corresponding belief states
  - The leaf at the end of each branch corresponds to the *belief state* reachable via that sequence of actions and observations – use filtering/belief-update to compute it
- **"Back Up" the utility values of the leaf nodes** along their corresponding branches, **combining it with the rewards** along that path
- **Pick the branch with the highest expected value**

Look Ahead Search for Optimal Policy





- Best action at time t?

A. a_1

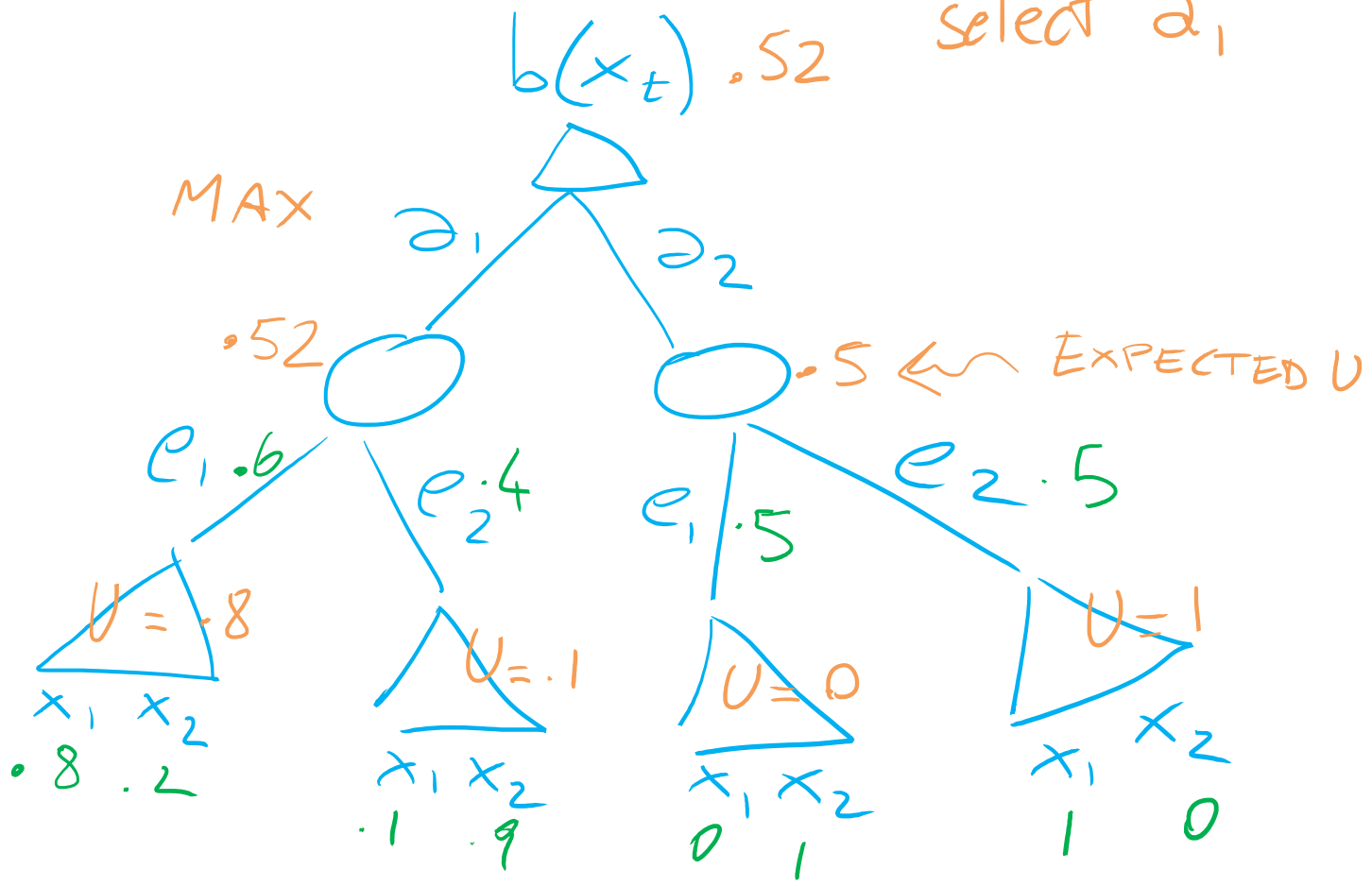
B. a_2

C. *indifferent*

$X \quad x_1 x_2$

$E \quad e_1 e_2$

$A \quad a_1 \quad a_2$



$U(x_1) = 1$

$U(x_2) = 0$

Look Ahead Search for Optimal Policy

- What is the time complexity for exhaustive search at depth *d*, with |A| available actions and |E| possible observations?

A. $O(d * |A| * |E|)$

B. $O(|A|^d * |E|^d)$

C. $O(|A|^d * |E|)$

i>clicker

- Would Look ahead work better when the discount factor is?

A. *Close to 1*

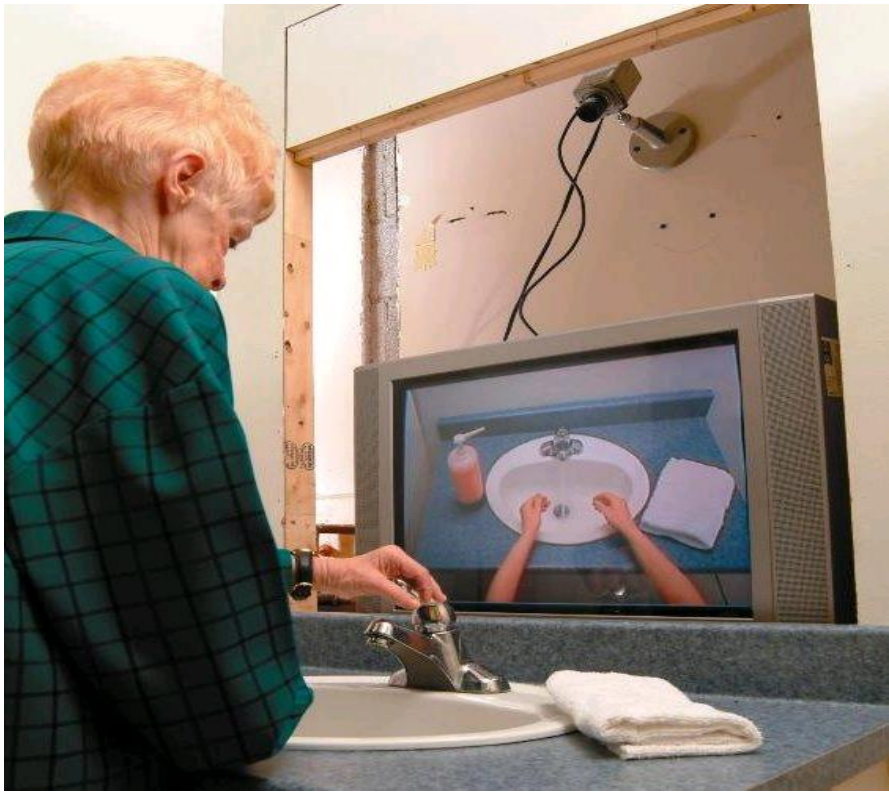
B. *Not too close to 1*

Some Applications of POMDPs……

- Jesse Hoey, Tobias Schröder, Areej Alhothali (**2015**), Affect control processes: Intelligent affective interaction using a POMDP, *AI Journal*
- S Young, M Gasic, B Thomson, J Williams (**2013**) POMDP-based Statistical Spoken Dialogue Systems: a Review, *Proc IEEE*,
- J. D. Williams and S. Young. Partially observable Markov decision processes for spoken dialog systems. *Computer Speech & Language*, 21(2):393–422, **2007**.
- S. Thrun, et al. Probabilistic algorithms and the interactive museum tour-guide robot Minerva. *International Journal of Robotic Research*, 19(11):972–999, **2000**.
- A. N.Rafferty,E. Brunskill,Ts L. Griffiths, and Patrick Shafto. Faster teaching by POMDP planning. In *Proc. of Ai in Education*, pages 280–287, **2011**
- P. Dai, Mausam, and D. S.Weld. Artificial intelligence for artificial artificial intelligence. In *Proc. of the 25th AAAI Conference on AI* , **2011**. [intelligent control of workflows]

Another "famous" Application

Learning and Using POMDP models of Patient–Caregiver Interactions During Activities of Daily Living

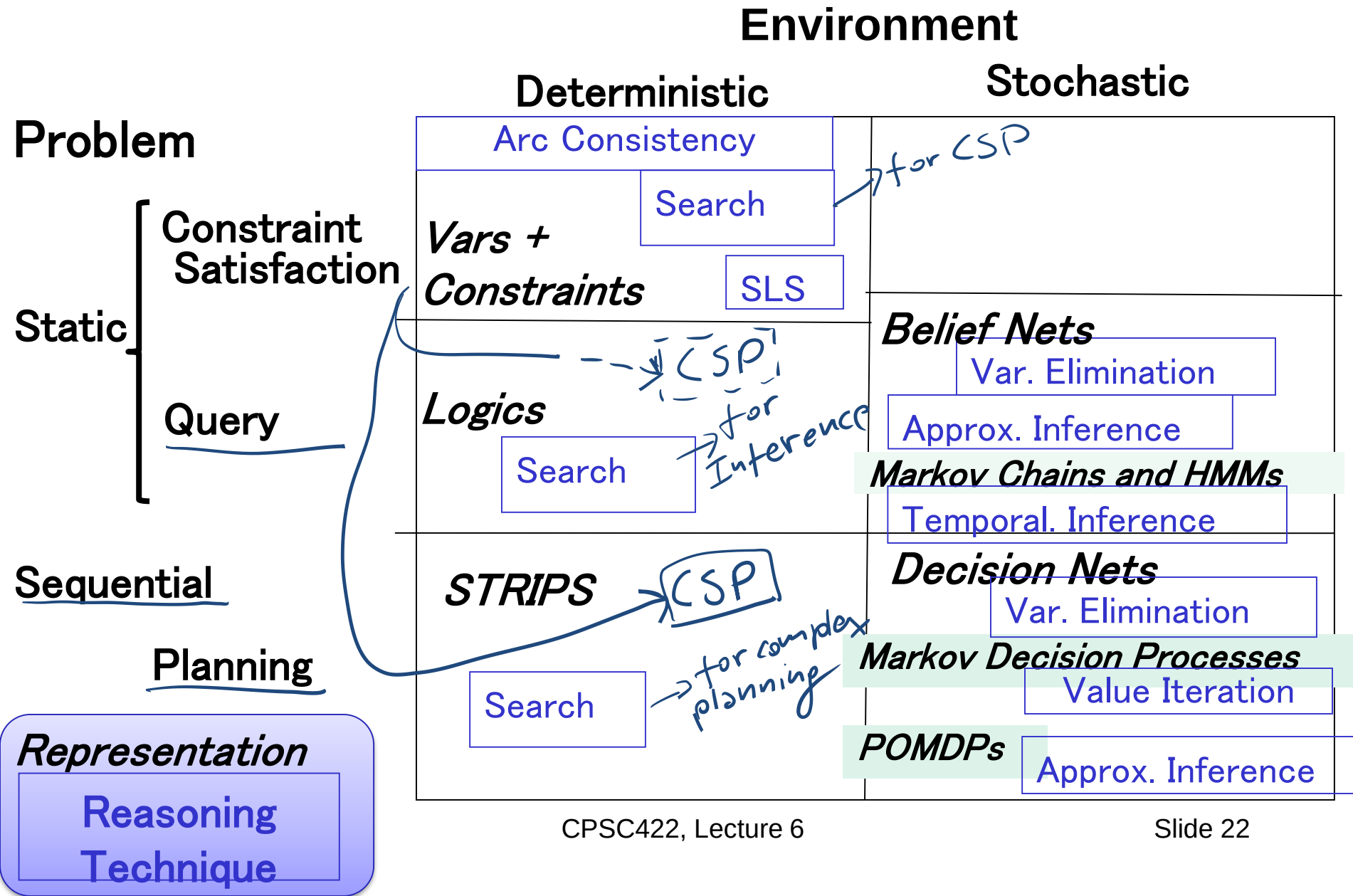


Goal: Help Older adults living with cognitive disabilities (such as Alzheimer's) when they:

- **forget the proper sequence of tasks** that need to be completed
- **they lose track of the steps** that they have already completed.

Source: *Jesse Hoey UofT 2007*

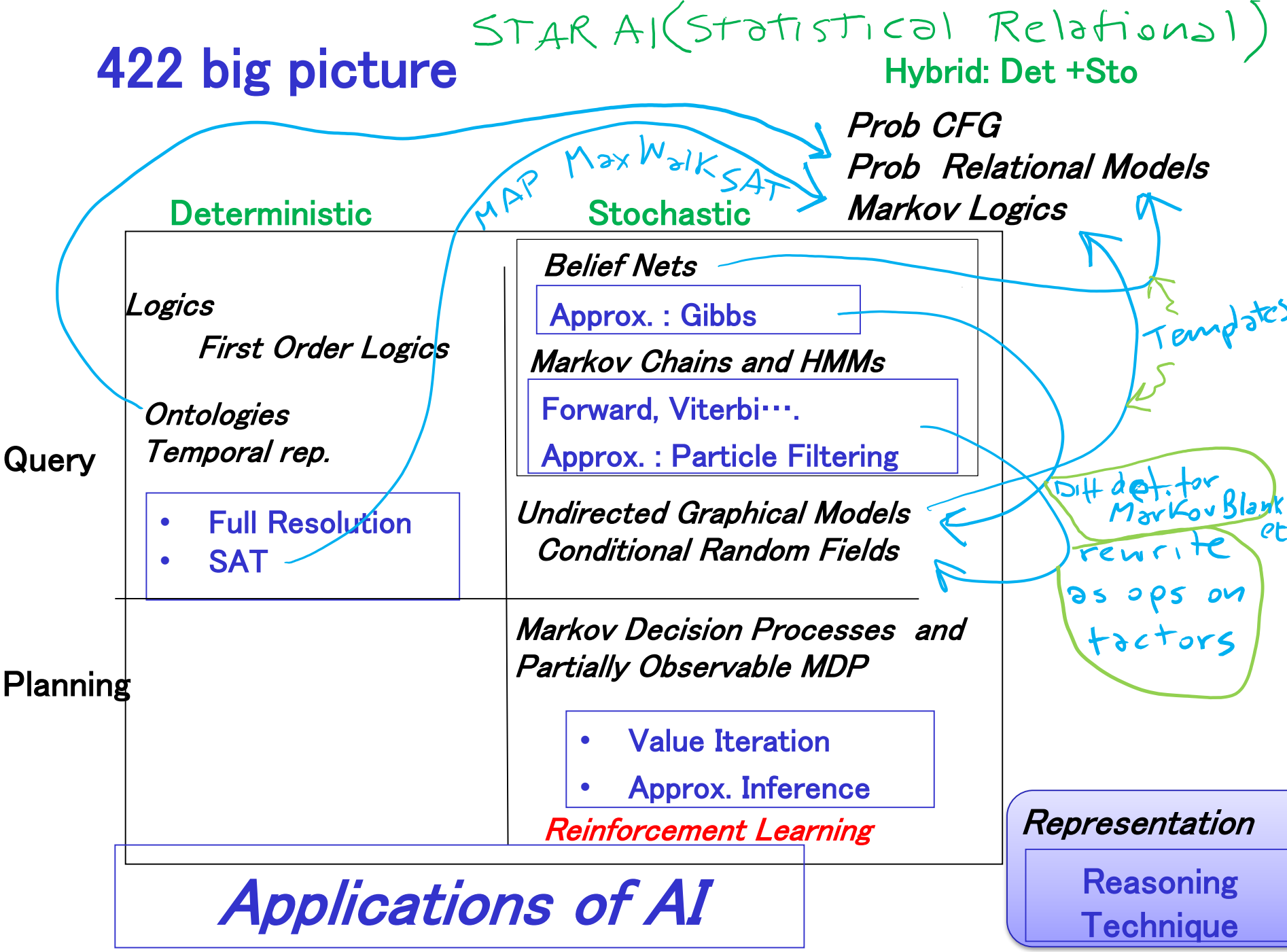
R&R systems BIG PICTURE



422 big picture

STAR AI (Statistical Relational)

Hybrid: Det +Sto



Learning Goals for today's class

You can:

- Define a **Policy** for a POMDP
- Describe space of possible methods for computing optimal policy for a given POMDP
- Define and trace Look Ahead Search for finding an (approximate) Optimal Policy
- Compute Complexity of Look Ahead Search

TODO for next Fri

- **Read textbook 11.3 (Reinforcement Learning)**
  - 11.3.1 Evolutionary Algorithms
  - 11.3.2 Temporal Differences
  - 11.3.3 Q-learning

- Assignment 1 will be posted on Connect today
  - VInfo and VControl
  - MDPs (Value Iteration)
  - POMDPs

- In practice, the hardness of POMDPs arises from the complexity of policy spaces and the potentially large number of states.

- Nervertheless, real-world POMDPs tend to exhibit a significant amount of structure, which can often be exploited to improve the scalability of solution algorithms.

  - Many POMDPs have simple policies of high quality. Hence, it is often possible to quickly find those policies by restricting the search to some class of compactly representable policies.

  - When states correspond to the joint instantiation of some random variables (features), it is often possible to exploit various forms of probabilistic independence (e.g., conditional independence and context-specic independence), decomposability (e.g., additive separability) and sparsity in the POMDP dynamics to mitigate the impact of large state spaces.

Symbolic Perseus

- Symbolic Perseus - point-based value iteration algorithm that uses Algebraic Decision Diagrams (ADDs) as the underlying data structure to tackle large factored POMDPs
- Flat methods: 10 states at 1998, 200,000 states at 2008
- Factored methods: 50,000,000 states
- http://www.cs.uwaterloo.ca/~ppoupart/software.html

POMDP as MPD

- By applying simple rules of probability we can derive a:
  Transition model P(b'\|a,b)

$$P(b'|a,b) = \sum_e P(b'|e,a,b) \sum_{s'} P(e|s') \sum_s P(s'|s,a) b(s)$$

where $P(b'|e,a,b) = 1$ if $b' = Forward(e,a,b)$

$= 0$ otherwise

When the agent performs a given action *a* in belief state *b*, and then receives observation *e*, filtering gives a unique new probability distribution over state

deterministic transition from one belief state to the next

- We can also define a ***reward function*** for belief states

?

$$\rho(b) = \sum_s b(s) R(s)$$

Solving POMDP as MPD

- So we have defined a POMD as an MDP over the belief states
  - Why bother?
- Because it can be shown that an optimal policy $\pi^*(b)$ for this MDP is also an optimal policy for the original POMDP
  - i.e., solving a POMDP in its physical space is equivalent to solving the corresponding MDP in the belief state
- **Great, we are done!**

POMDP as MDP

- But how does one find the optimal policy *π*(b)?*
  - One way is to restate the POMDP as an MPD in belief state space
- ***State space*** :
  - space of probability distributions over original states
  - For our grid world the belief state space is?
  - initial distribution <1/9,1/9, 1/9,1/9,1/9,1/9, 1/9,1/9,1/9,0,0> is a point in this space
- What does the transition model need to specify?

?

$P(b' \mid a, b)$

Does not work in practice

- Although a transition model can be effectively computed from the POMDP specification
- Finding (approximate) policies for continuous, multidimensional MDPs is PSPACE-hard
  - Problems with a few dozen states are often unfeasible
- Alternative approaches….

How to Find an Optimal Policy?

- Turn a POMDP into a corresponding MDP and then solve the MDP (☹)
- Generalize VI to work on POMDPs (also ☹)
- Develop Approx. Methods (☺)
  - **Point-Based Value Iteration**
  - Look Ahead

Recent Method: Point-based Value Iteration

- Find a solution **for a sub-set of all states**
- Not all states are necessarily reachable
- Generalize the solution to all states
- Methods include: PERSEUS, PBVI, and HSVI and other similar approaches (FSVI, PEGASUS)

How to Find an Optimal Policy?

- Turn a POMDP into a corresponding MDP and then solve the MDP
- Generalize VI to work on POMDPs **(also ☹)**
- Develop Approx. Methods (☺)
  - Point-Based VI
  - **Look Ahead**