Logic: TD as search, Datalog (variables)

Computer Science cpsc322, Lecture 23

(Textbook Chpt 5.2 &

some basic concepts from Chpt 12)

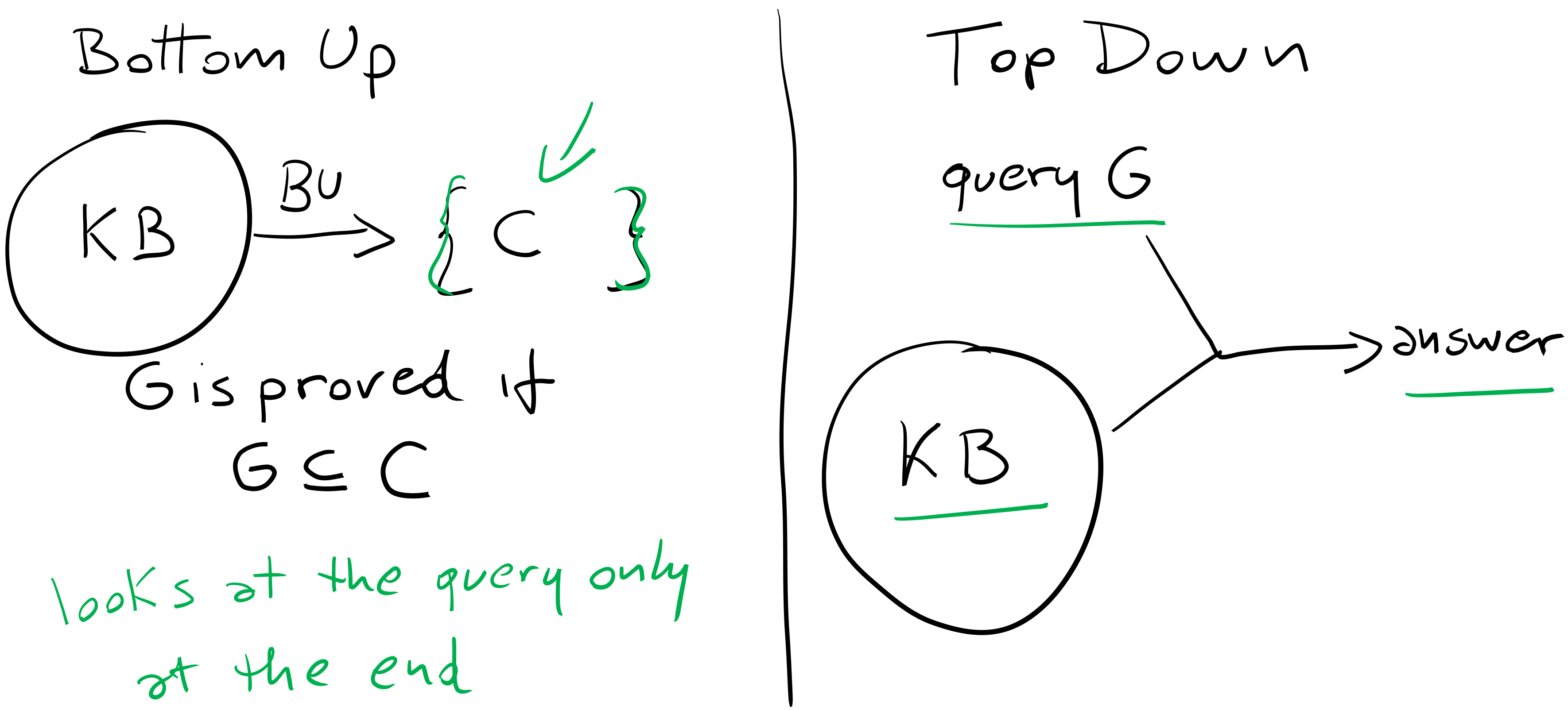
June, 8, 2017

Lecture Overview

- Recap Top Down
- TopDown Proofs as search
- Datalog

Top-down Ground Proof Procedure

Key Idea: search backward from a query *G* to determine if it can be derived from *KB*.



Top-down Proof Procedure: Basic elements

Notation: An answer clause is of the form:

$$yes \leftarrow a_1 \land a_2 \land \cdots \land a_m$$

Express query as an answer clause (e.g.,

query $a_1 \land a_2 \land \cdots \land a_m$)

$$yes \leftarrow a_1 \land \cdots \land a_m$$

Rule of inference (called SLD Resolution)

Given an answer clause of the form:

$$yes \leftarrow a_1 \land a_2 \land \cdots \land a_m$$

and the clause: in KB

$$a_i \leftarrow b_1 \land b_2 \land \cdots \land b_p$$

You can generate the answer clause

$$yes \leftarrow a_1 \land \cdots \land a_{i-1} \land b_1 \land b_2 \land \cdots \land b_p \land a_{i+1} \land \cdots \land a_m$$

- **Successful Derivation**: When by applying the inference rule you obtain the answer clause $yes \leftarrow .$

$a \leftarrow e \wedge f.$ $\quad a \leftarrow b \wedge c.$ $\quad b \leftarrow k \wedge f.$

$c \leftarrow e.$ $\quad d \leftarrow k.$ $\quad e.$

$f \leftarrow j \wedge e.$ $\quad f \leftarrow c.$ $\quad j \leftarrow c.$

Query: a `(two ways)`

$yes \leftarrow a.$ $\qquad yes \leftarrow a.$

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Systematic Search in different R&R systems

Constraint Satisfaction (Problems): ✓

- State: assignments of values to a subset of the variables
- Successor function: assign values to a "free" variable
- Goal test: set of constraints
- Solution: possible world that satisfies the constraints
- Heuristic function: *none (all solutions at the same distance from start)*

Planning (forward) : ✓

- State possible world
- Successor function states resulting from valid actions
- Goal test assignment to subset of vars
- Solution sequence of actions
- Heuristic function empty-delete-list (solve simplified problem)

start state: query as an answer clause

Logical Inference (top Down)

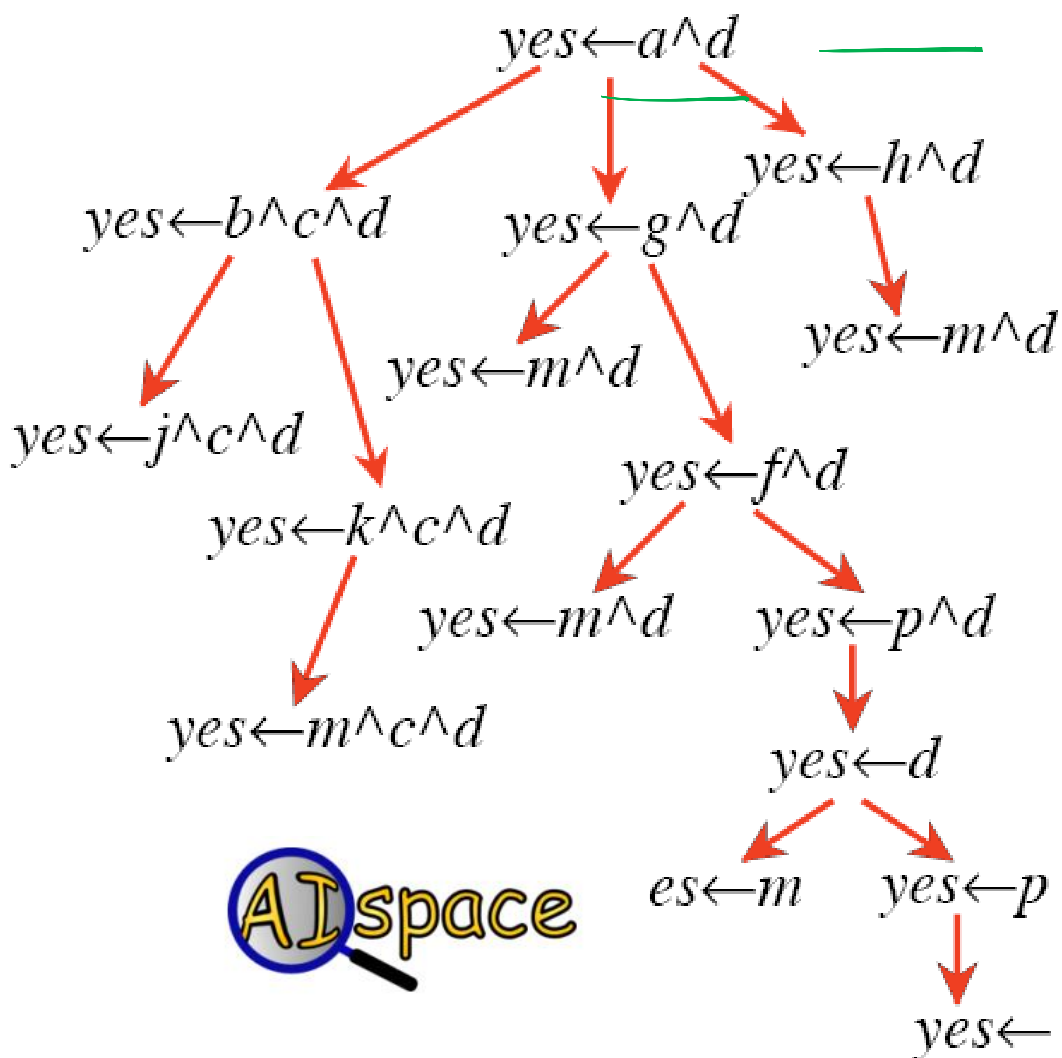
- State answer clause — yes ← □
- Successor function states resulting from substituting one atom with all the clauses of which it is the head
- Goal test empty answer clause — yes ←
- Solution start state
- Heuristic function …………………….. ✓ see next slide

Search Graph

KB

$a \leftarrow b \wedge c.$	$a \leftarrow g.$
$a \leftarrow h.$	$b \leftarrow j.$
$b \leftarrow k.$	$d \leftarrow m.$
$d \leftarrow p.$	$f \leftarrow m.$
$f \leftarrow p.$	$g \leftarrow m.$
$g \leftarrow f.$	$k \leftarrow m.$
$h \leftarrow m.$	$p.$

Prove: $? \leftarrow a \wedge d.$



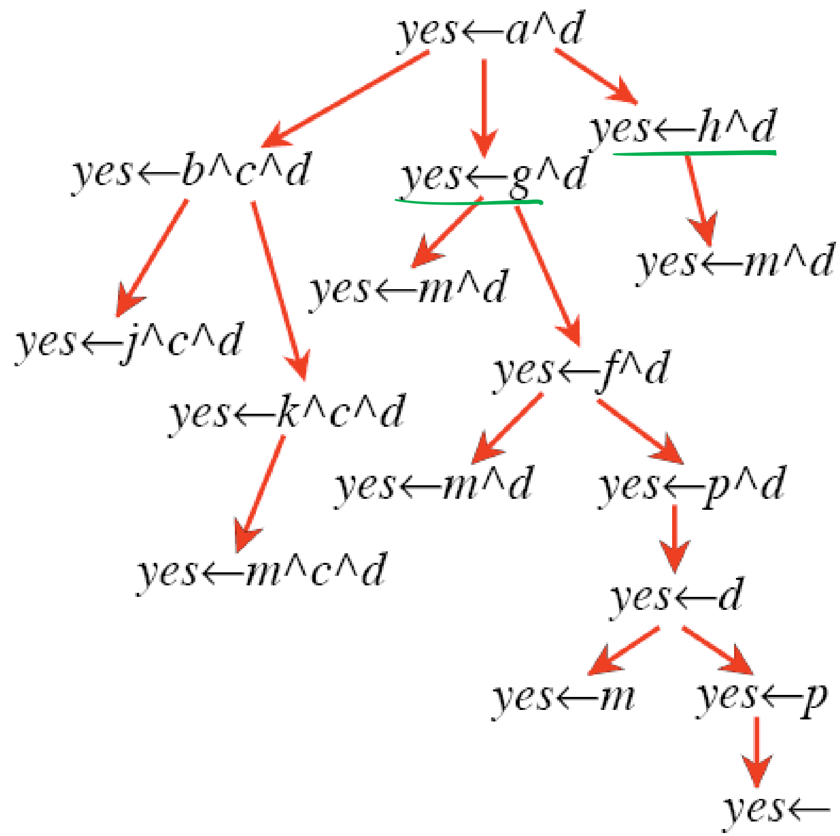
Heuristics?

Search Graph

KB

$a \leftarrow b \wedge c.$	$a \leftarrow g.$
$a \leftarrow h.$	$b \leftarrow j.$
$b \leftarrow k.$	$d \leftarrow m.$
$d \leftarrow p.$	$f \leftarrow m.$
$f \leftarrow p.$	$g \leftarrow m.$
$g \leftarrow f.$	$k \leftarrow m.$
$h \leftarrow m.$	$p.$

Prove: $? \leftarrow a \wedge d.$



Possible Heuristic?

Number of atoms in the answer clause

Admissible?

A. Yes B. No C. It Depends

Search Graph

Prove: $? \leftarrow a \wedge d$

KB

$a \leftarrow b \wedge c.$

$a \leftarrow g.$

$a \leftarrow h.$ $b \leftarrow j.$

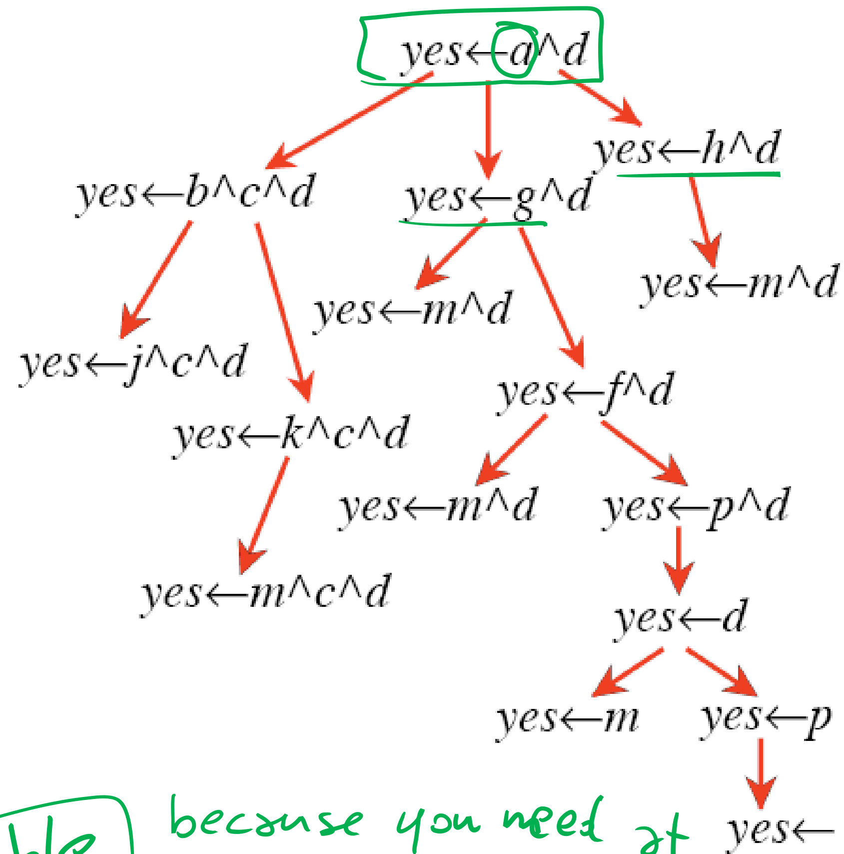
$b \leftarrow k.$ $d \leftarrow m.$

$d \leftarrow p.$ $f \leftarrow m.$

$f \leftarrow p.$ $g \leftarrow m.$

$g \leftarrow f.$ $k \leftarrow m.$

$h \leftarrow m.$ $p.$



Heuristics?

of atoms in answer clause

Admissible

because you need at least that number of resolution steps to obtain yes← i.e. the goal state

Better Heuristics?

iclicker

If the body of an answer clause contains a symbol that does not match the head of any clause in the KB what should the most informative heuristic value for that answer clause be ?

A. Zero
B. Infinity
C. Twice the number of clauses in the KB
D. None of the above

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Representation and Reasoning in Complex domains

- In complex domains expressing knowledge with **propositions** can be quite limiting

up_s_2
up_s_3
ok_cb_1
ok_cb_2
$live\_w_1$
$connected\_w_1\_w_2$

- It is often natural to consider **individuals** and their **properties**

$up(s_2)$
$up(s_3)$
$ok(cb_1)$
$ok(cb_2)$
$live(w_1)$
$connected(w_1, w_2)$

There is no notion that

up_s_2
up_s_3

$live\_w_1$
$connected\_w_1\_w_2$

What do we gain….

By breaking propositions into relations applied to individuals?

- Express **knowledge** that **holds for set of individuals** (by introducing variables)

  live(W) <- connected_to(W,W1) ∧ live(W1) ∧ wire(W) ∧ wire(W1).

- We can **ask generic queries** (i.e., containing vars)

  variabless

  ? connected_to(W, w_1)

Datalog vs PDCL (better with colors)

First Order Logic

$\forall X \exists Y p(X, Y) \Leftrightarrow \neg q(Y)$

$p(a_1, a_2)$

$\neg q(a_5)$

Datalog

$p(X) \leftarrow q(X) \wedge r(X, Y)$

$r(X, Y) \leftarrow s(Y)$

$s(a_1), q(a_2)$

Propositional Logic

$\neg(p \vee q) \rightarrow (r \wedge s \wedge f),$

p, r

PDCL

$p \leftarrow s \wedge f$

$r \leftarrow s \wedge q \wedge p$

r

p

Datalog: a relational rule language

Datalog expands the syntax of PDCL….

A variable is a symbol starting with an upper case letter

Examples: X, Y

A constant is a symbol starting with lower-case letter or a sequence of digits.

Examples: alan, w1

A term is either a variable or a constant.

Examples: X, Y, alan, w1

A predicate symbol is a symbol starting with a lower-case letter.

Examples: live, connected, part-of, in

Datalog Syntax (cont'd)

An atom is a symbol of the form p or $p(t_1 \ldots. t_n)$ where p is a predicate symbol and t_i are terms

Examples: sunny,   in(alan,X)

A definite clause is either an atom (a fact) or of the form:

$$h \leftarrow b_1 \wedge \ldots \wedge b_m$$

where h and the b_i are atoms (Read this as ``h if b.")

Example: in(X,Z) ← in(X,Y) ∧ part-of(Y,Z)

A knowledge base is a set of definite clauses

Datalog: Top Down Proof Procedure

in(alan, r123).
part_of(r123,cs_building).
in(X,Y) ← part_of(Z,Y) ∧ in(X,Z).

- Extension of Top-Down procedure for PDCL.
  How do we deal with variables?
  - Idea:
    - Find a clause with head that matches the query
    - Substitute variables in the clause with their matching constants
  - Example:

Query: yes ← in(alan, cs_building).

↓

in(X,Y) ← part_of(Z,Y) ∧ in(X,Z).
with Y = cs_building
X = alan

yes ← part_of(Z,cs_building) ∧ in(alan, Z).

Example proof of a Datalog query

in(alan, r123).
part_of(r123,cs_building).
in(X,Y) ← part_of(Z,Y) ∧ in(X,Z).

Query: yes ← in(alan, cs_building).

Using clause: in(X,Y) ← part_of(Z,Y) ∧ in(X,Z), with Y = cs_building X = alan

yes ← part_of(Z,cs_building) ∧ in(alan, Z).

Using clause: part_of(r123,cs_building) with Z = r123

??????

A. yes ← part_of(Z, r123) ∧ in(alan, Z).
B. yes ← in(alan, r123).
C. yes ←.
D. None of the above

Example proof of a Datalog query

in(alan, r123).

part_of(r123,cs_building).

in(X,Y) ← part_of(Z,Y) ∧ in(X,Z).

Query: yes ← in(alan, cs_building).

Using clause: in(X,Y) ← part_of(Z,Y) ∧ in(X,Z), with Y = cs_building, X = alan

yes ← part_of(Z,cs_building) ∧ in(alan, Z).

Using clause: part_of(r123,cs_building) with Z = r123

yes ← in(alan, r123).

Using clause: in(alan, r123).

Using clause: in(X,Y) ← part_of(Z,Y) ∧ in(X,Z). With X = alan, Y = r123

yes ←.

yes ← part_of(Z, r123), in(alan, Z).

No clause with matching head: part_of(Z,r123).

fail

Tracing Datalog proofs in AIspace

- You can trace the example from the last slide in the AIspace Deduction Applet at http://aispace.org/deduction/ using file *ex-Datalog* available in course schedule

- Question 4 of assignment 3 will ask you to use this applet

Datalog: queries with variables

in(alan, r123).
part_of(r123,cs_building).
in(X,Y) ← part_of(Z,Y) & in(X,Z).

Query: in(alan, X1).

yes(X1) ← in(alan, X1).

What would the answer(s) be?

Datalog: queries with variables

in(alan, r123).
part_of(r123,cs_building).
in(X,Y) $\leftarrow$ part_of(Z,Y) & in(X,Z).

Query: in(alan, X1).

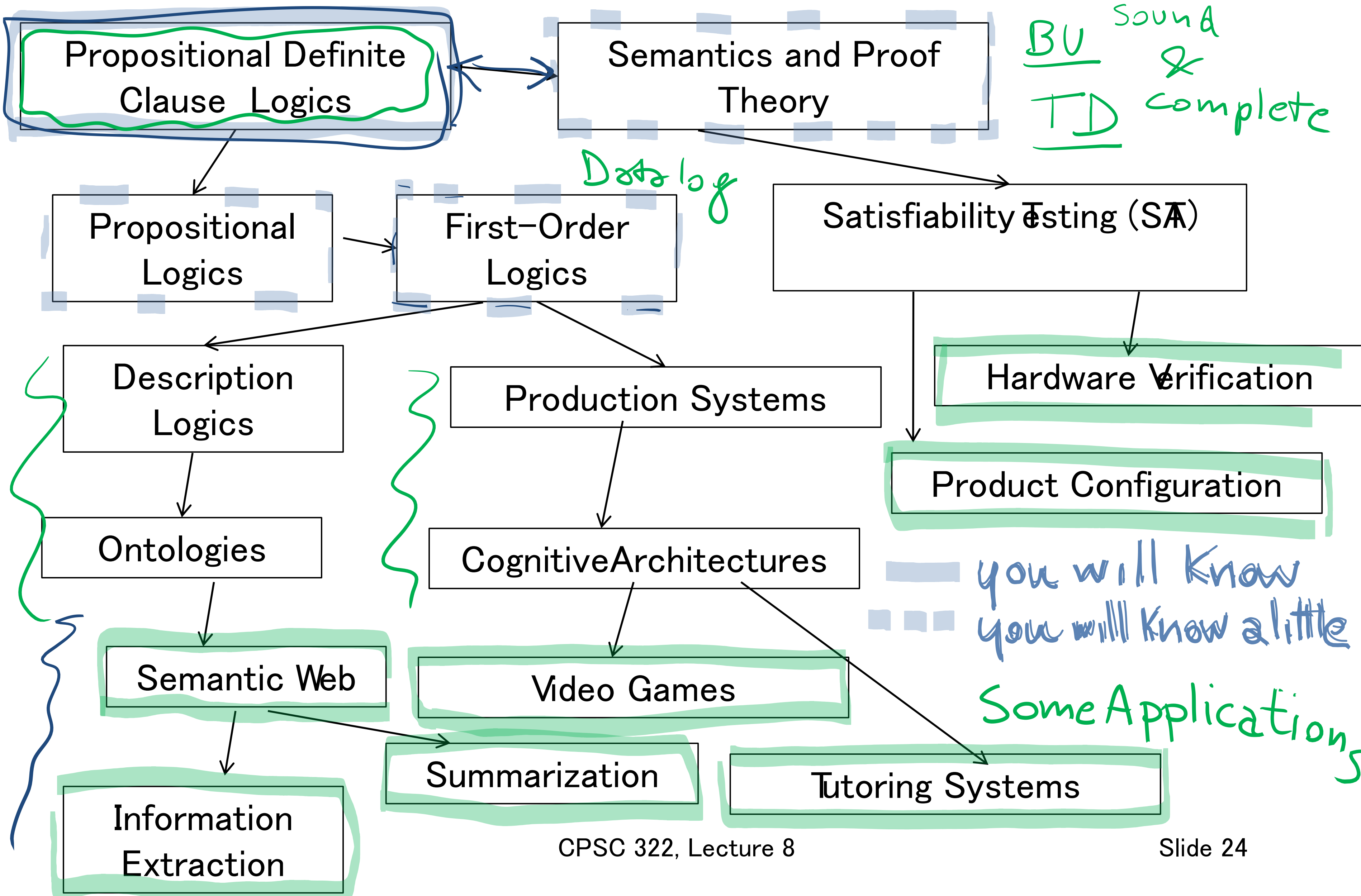
yes(X1) $\leftarrow$ in(alan, X1).

What would the answer(s) be?

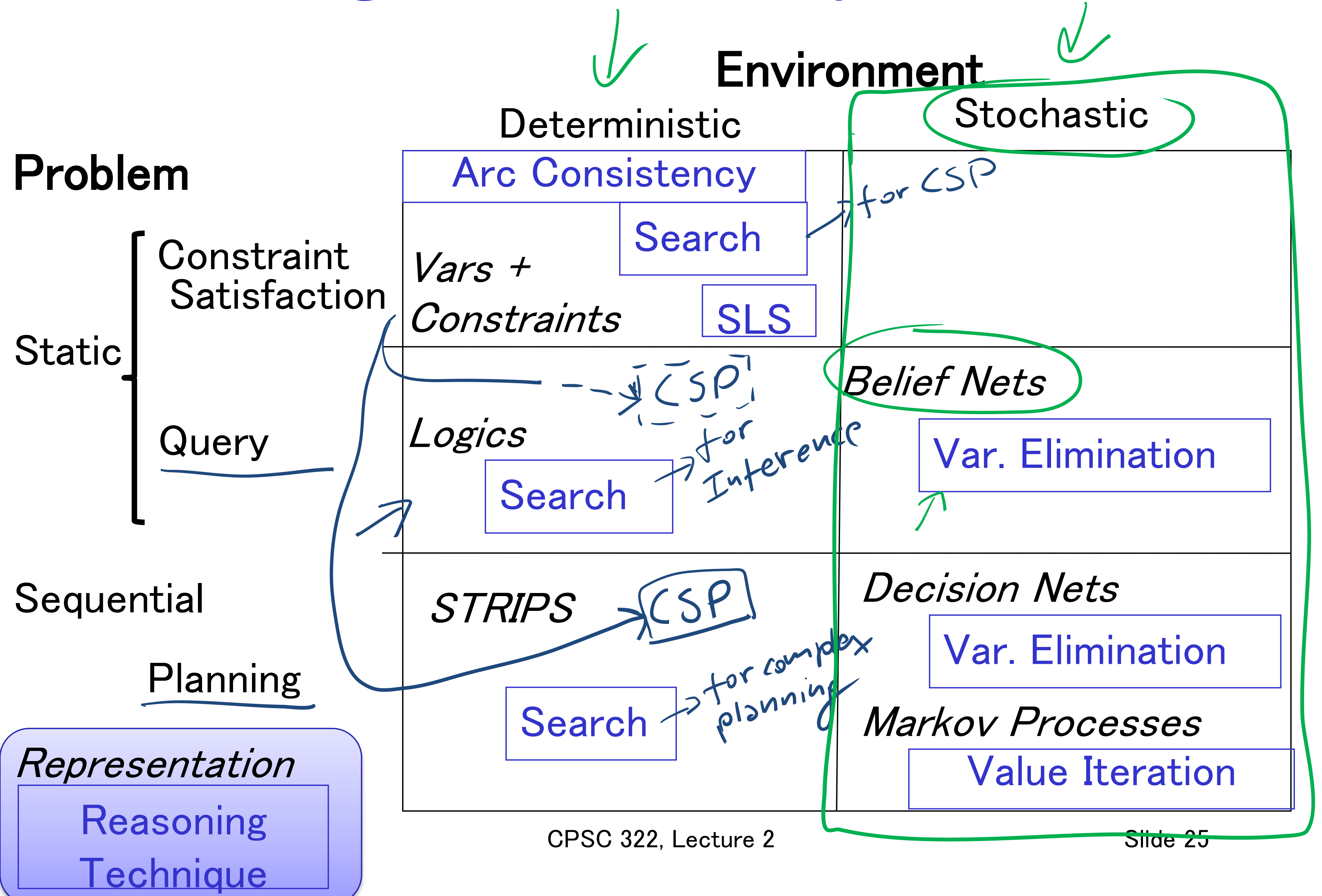
yes(r123).
yes(cs_building).

Again, you can trace the SLD derivation for this query
in the AIspace Deduction Applet

Logics in AI: Similar slide to the one for planning



Big Picture: R&R systems



Next Class on Tue

Intro to probability

- Random Variable
- Prob. Distribution
- Marginalization
- Conditional Probability
- Chain Rule
- Bayes' Rule
- Marginal and Conditional Independence

Assignment-3: will be posted before Tue

Full Propositional Logics (not for 322)

DEFs.

Literal: an atom or a negation of an atom $p \quad \neg q \quad r$

Clause: is a disjunction of literals $p \vee \neg r \vee q$

Conjunctive Normal Form (CNF): a conjunction of clauses $(p) \wedge (q \vee \neg r) \wedge (\neg q \vee p)$

INFERENCE:

$KB \overset{?}{\models} \alpha$ ← formula

- Convert all formulas in KB and $\neg\alpha$ in CNF
- Apply Resolution Procedure (at each step combine two clauses containing complementary literals into a new one)
- Termination $p \vee q \quad r \vee \neg q \rightarrow p \vee r$
  - No new clause can be added $KB \nvdash \alpha$
  - Two clause resolve into an empty clause $KB \vdash \alpha$

Propositional Logics: Satisfiability (SAT problem)

Does a set of formulas have a model? Is there an interpretation in which all the formulas are true?

(Stochastic) Local Search Algorithms can be used for this task!

Evaluation Function: number of unsatisfied clauses

WalkSat: One of the simplest and most effective algorithms:

Start from a randomly generated interpretation

- Pick an unsatisfied clause
- Pick an proposition to flip (randomly 1 or 2)
  1. To minimize # of unsatisfied clauses
  2. Randomly

Full First-Order Logics (FOLs)

We have constant symbols, predicate symbols and function symbols

So interpretations are much more complex (but the same basic idea – one possible configuration of the world)

- constant symbols => individuals, entities
- predicate symbols => relations
- function symbols => functions

INFERENCE:

- **Semidecidable:** algorithms exists that says yes for every entailed formulas, but no algorithm exists that also says no for every non-entailed sentence
- Resolution Procedure can be generalized to FOL