Search: Advanced Topics

Computer Science cpsc322, Lecture 9

(Textbook Chpt 3.6)

May, 23, 2013

Lecture Overview ✓ ✓

$f = \underline{c + h}$

- **Recap A***
- Branch & Bound
- A* tricks
- Other Pruning

A* advantages

What is a key advantage of A* ?

A. Does not need to consider the cost of the paths
B. Has a linear space complexity
C. It is often optimal
D. None of the above

Branch-and-Bound Search

- Biggest advantages ofA*….

uses heuristics + optimal

- What is the biggest problem withA*?

space

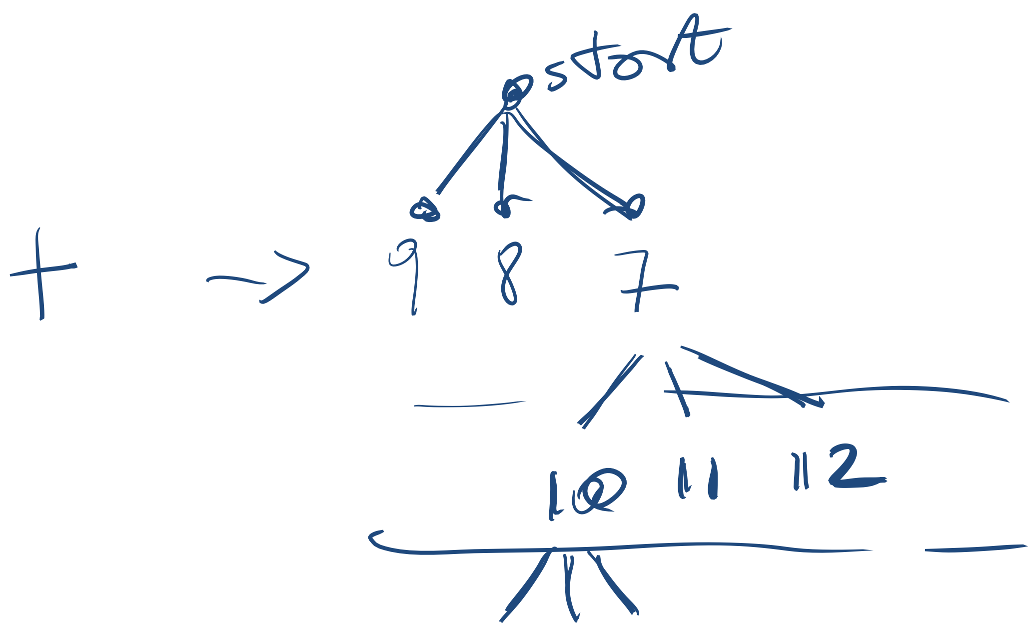
- Possible, preliminary Solution:

DFS + h

Branch-and-Bound Search Algorithm

- Follow exactly the same search path as **depth-first search**
  - **treat the frontier as a stack**: expand the most-recently added path first
  - the **order in which neighbors are expanded** can be governed by some arbitrary node-ordering heuristic

we can use $f = c + h$



Once this strategy has found a solution….

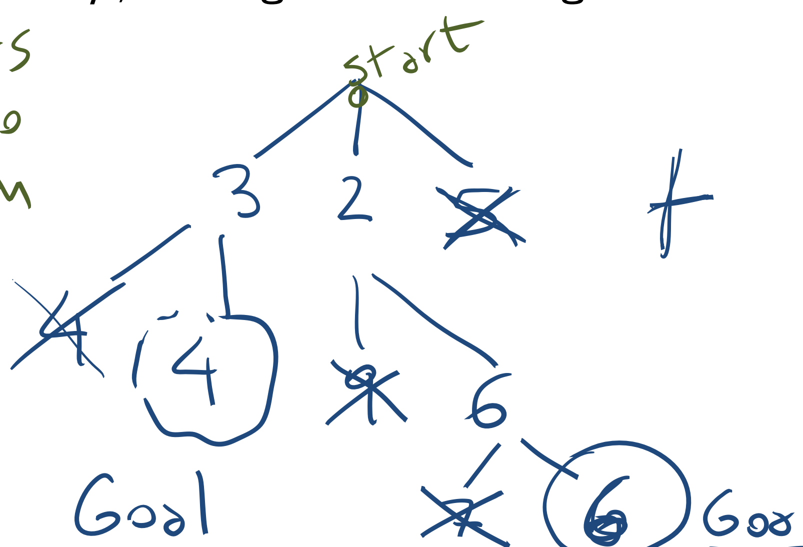
What should it do next ?

A. Keep running DFS, looking for deeper solutions?
B. Stop and return that solution
C. Keep searching, but only for shorter solutions
D. None of the above

Branch-and-Bound Search Algorithm

- Keep track of a lower bound and upper bound on solution cost at each path
  - lower bound: $LB(p) = \boxed{f(p)} = cost(p) + h(p)$
  - upper bound: UB = cost of the best solution found so far.
    - ✓ if no solution has been found yet, set the upper bound to ∞.
- When a path p is selected for expansion:
  - if $LB(p) \geq UB$, remove p from frontier without expanding it (pruning)
  - else expand p, adding all of its neighbors to the frontier

The numbers correspond to f for the path from start to that node



$UB = \infty$

6

4

Same for all paths at any given time

Branch-and-Bound Analysis

- Complete ?

yes | no | It depends

- Optimal ?

yes | no | It depends

- Space complexity?

$O(b^m)$ | $O(m^b)$ | $O(bm)$ | $O(b+m)$

- Time complexity?

Branch-and-Bound Analysis

- Completeness: no, for the same reasons that DFS isn't complete
  - however, for many problems of interest there are no infinite paths and no cycles
  - hence, for many problems B&B is complete
- Time complexity: $O(b^m)$
- Space complexity:……. $O(bm)$
  - Branch & Bound has the same space complexity as…. DFS
  - this is a big improvement over …………..! A^*
- Optimality: ……. yes

Lecture Overview

- **Recap A***
- Branch & Bound
- A* tricks
- Pruning Cycles and Repeated States

Other *A** Enhancements

The main problem with *A** is that it uses exponential space. Branch and bound was one way around this problem. Are there others?

- ...... Iterative Deepening A* IDA*
- Memory-bounded *A**

(Heuristic) Iterative Deepening – IDA*

B & B can still get stuck in infinite (extremely long) paths

- Search depth-first, but to a fixed depth /bound
  - if you don't find a solution, increase the depth tolerance and try again
  - depth is measured in………f………

    start node $f(start) = h(start)$

  - Then update with the …lowest…f….. that passed the previous bound

Analysis of Iterative Deepening A* (IDA*)

- Complete and optimal:

yes | no | It depends

- Space complexity:

$O(b^m)$ | $O(m^b)$ | $O(bm)$ | $O(b+m)$

- Time complexity:

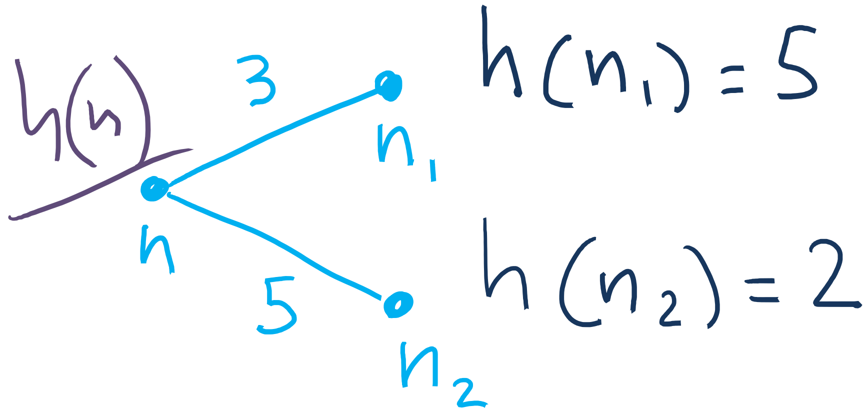
$O(b^m)$ | $O(m^b)$ | $O(bm)$ | $O(b+m)$

(Heuristic) Iterative Deepening – IDA*

- Counter-intuitively, the asymptotic complexity is not changed, even though we visit paths multiple times (*go back to slides on uninformed ID*)

$$\left(\frac{b}{b-1}\right)^2$$

Heuristic value by look ahead



What is the most accurate admissible heuristic value for n, given only this info ?

A. 7

B. 5

C. 2

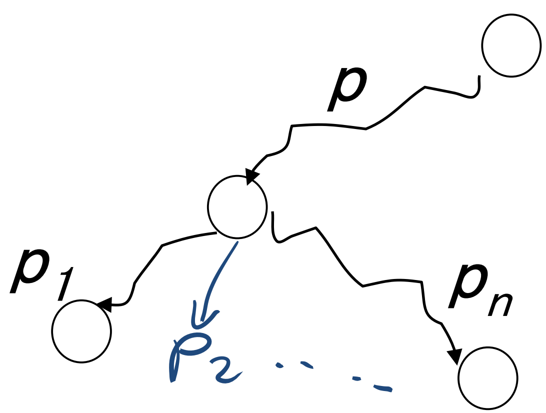
D. 8

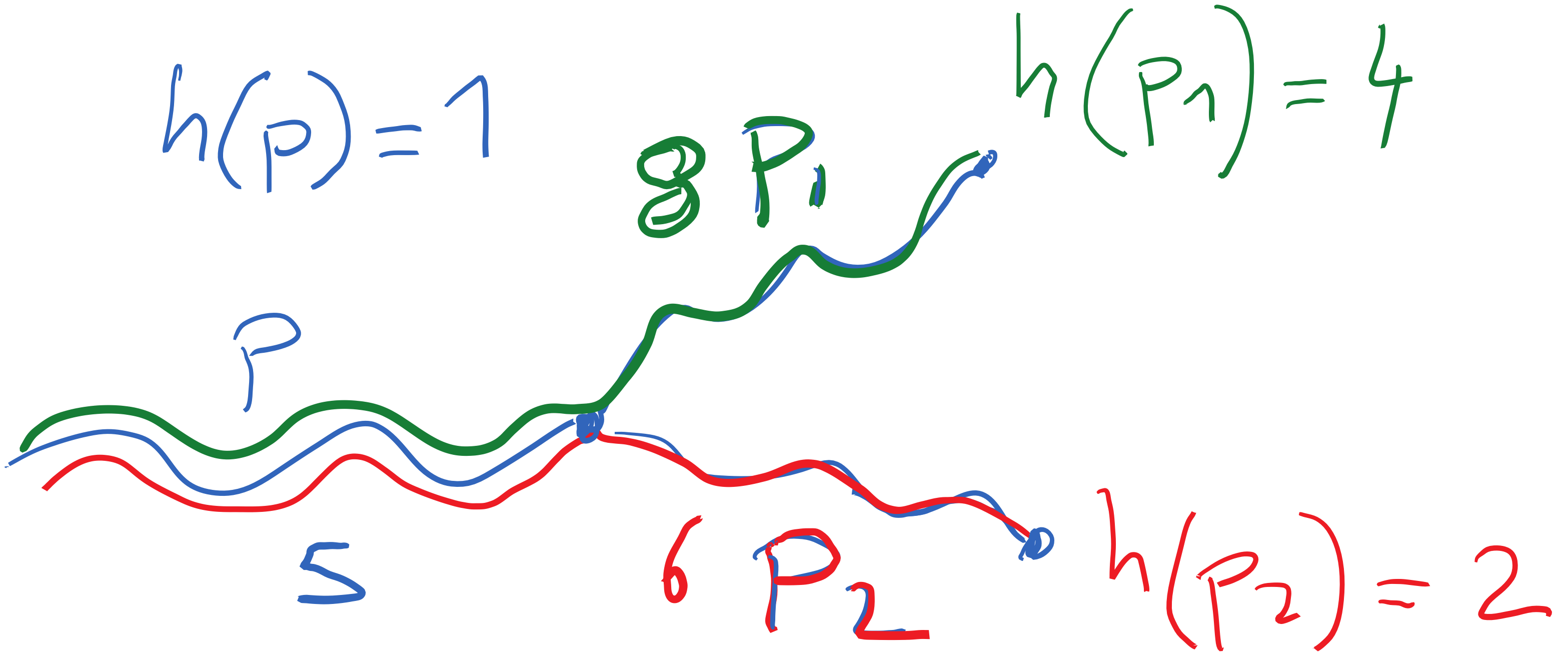
because

$$\min_i \left[cost(n, n_i) + h(n_i) \right]$$

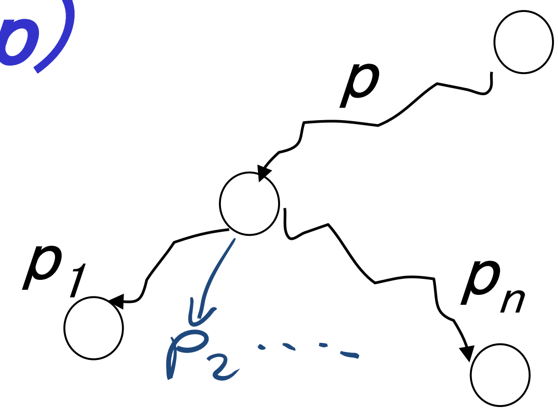
Memory-bounded *A**

- Iterative deepening A* and B & B use a tiny amount of memory
- **what if we've got more memory to use?**
- keep as much of the fringe in memory as we can
- if we have to delete something:
  - delete the worst paths (with highest f.)
  - ``back them up'' to a common ancestor




h(p) = 1
8 p1
h(p1) = 4
p
5
6 p2
h(p2) = 2

MBA*: Compute New *h(p)*



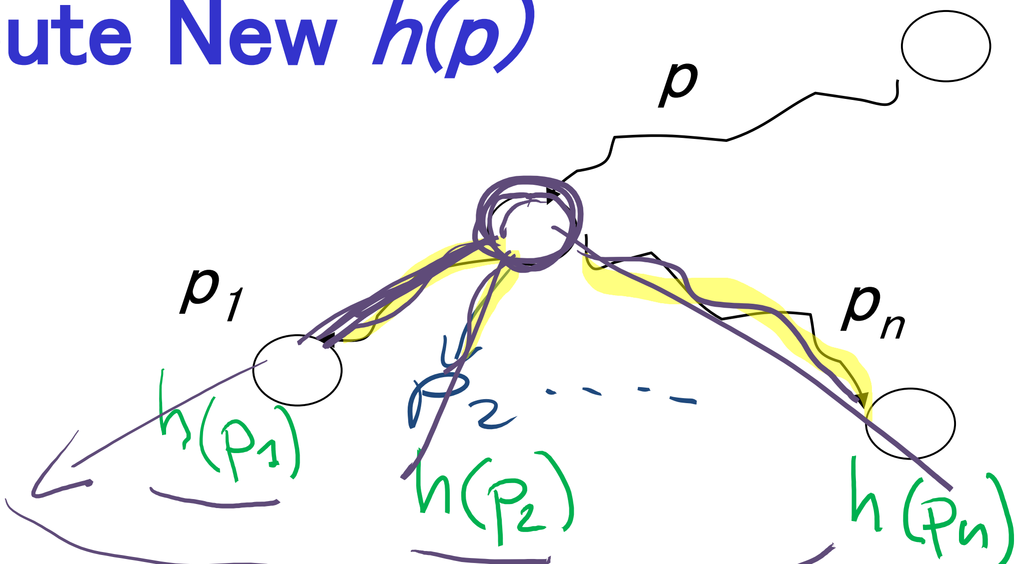
A $$\text{New } h(p) = \min\left(\max_i\left[\left(\text{cost}(p_i) - \text{cost}(p)\right) + h(p_i)\right], \text{Old } h(p) \right)$$

B $$\text{New } h(p) = \max\left(\min_i\left[\left(\text{cost}(p_i) - \text{cost}(p)\right) + h(p_i)\right], \text{Old } h(p) \right)$$

C $$\text{New } h(p) = \max\left(\max_i\left[\left(\text{cost}(p_i) - \text{cost}(p)\right) + h(p_i)\right], \text{Old } h(p) \right)$$

MBA*: Compute New *h(p)*

iclicker



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4 5

C $$\text{New } h(p) = \max\left(\max_i\left[\left(\text{cost}(p_i) - \text{cost}(p)\right) + h(p_i)\right], \text{Old } h(p)\right)$$

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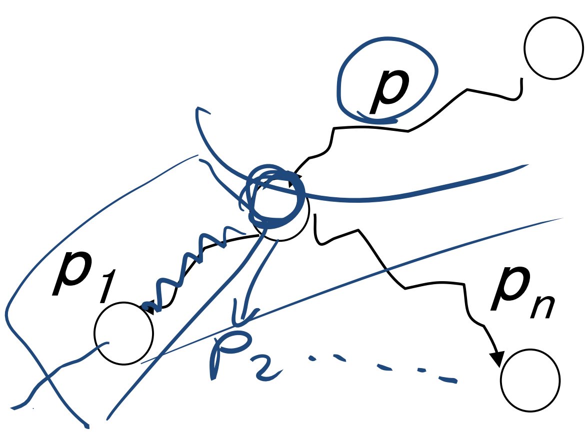
min max

max min



$$h(p) = \max\left(\min \left[cost(p_i) - cost(p) \right] + h(p_i) \right)$$

and old $h(p)$

Lecture Overview

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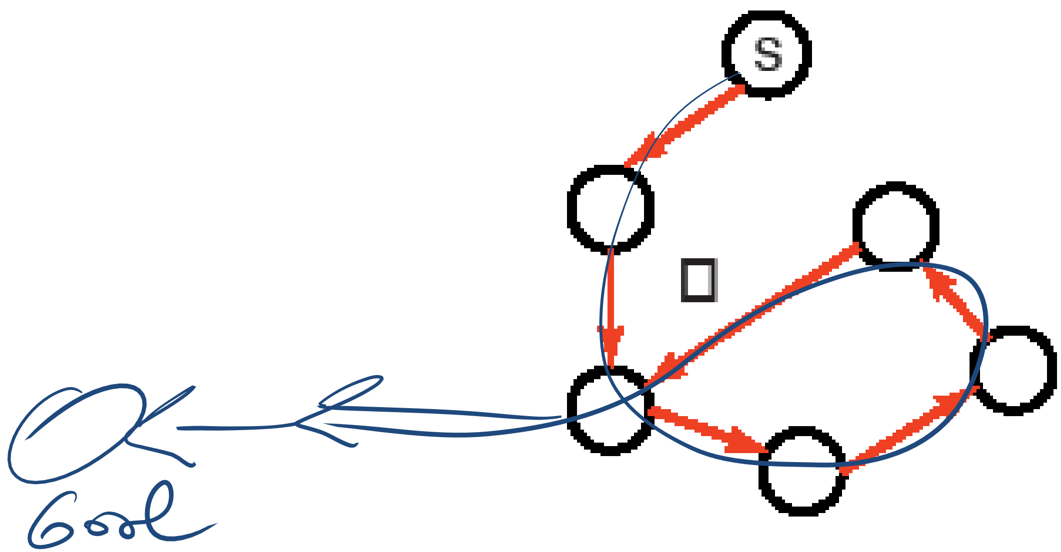
Cycle Checking



You can prune a path that ends in a node already on the path. This pruning cannot remove an optimal solution.

- The time is ……linear…… in path length.

$\langle n_0, n_1, n_2 \ldots n_k \rangle$

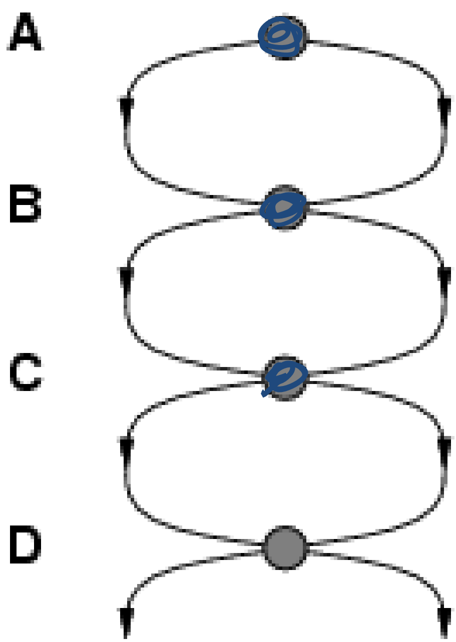
n_2

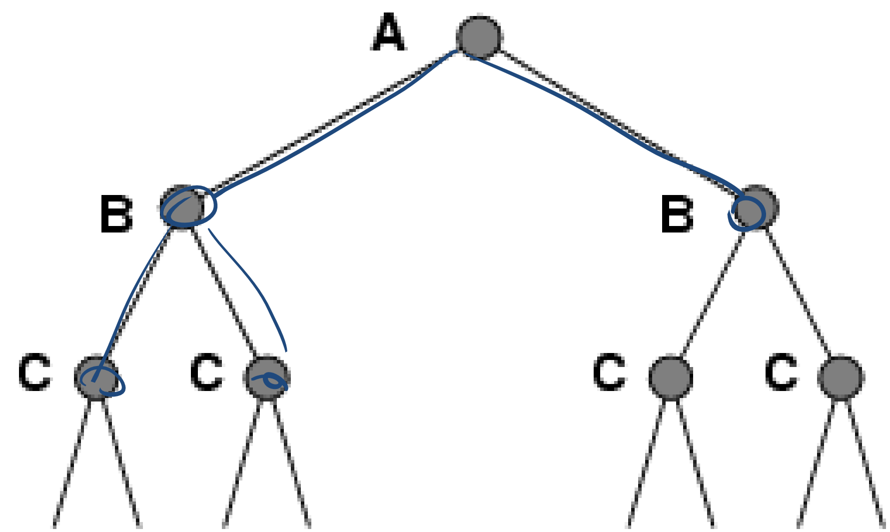
n_i

n_j

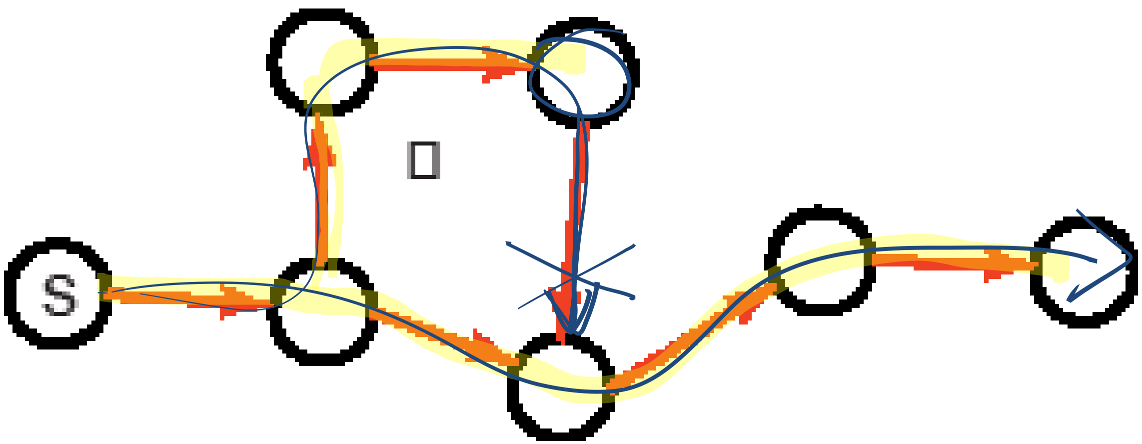
Repeated States / Multiple Paths

Failure to detect repeated states can turn a linear problem into an exponential one!





Multiple-Path Pruning

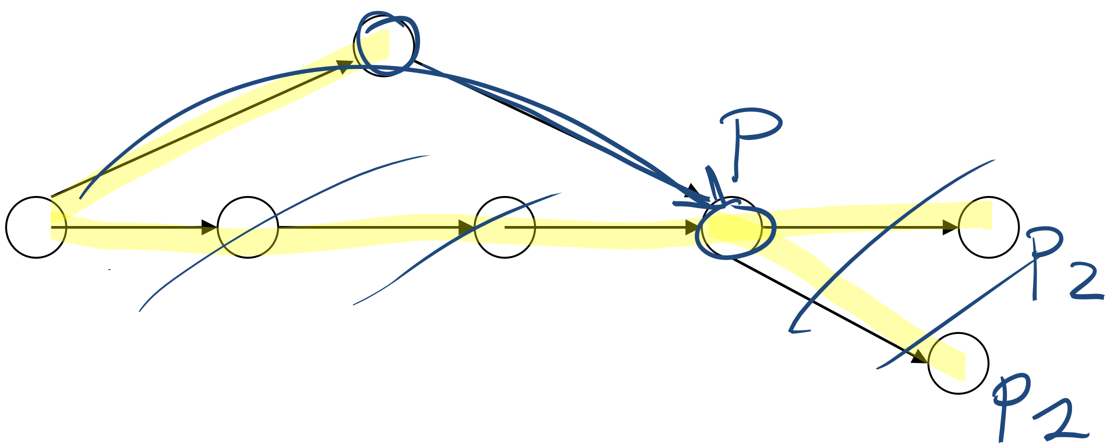


- You can prune a path to node *n* that you have already found a path to
- (if the new path is longer – more costly).

Multiple-Path Pruning & Optimal Solutions

Problem: what if a subsequent path to *n* is shorter than the first path to *n* ?

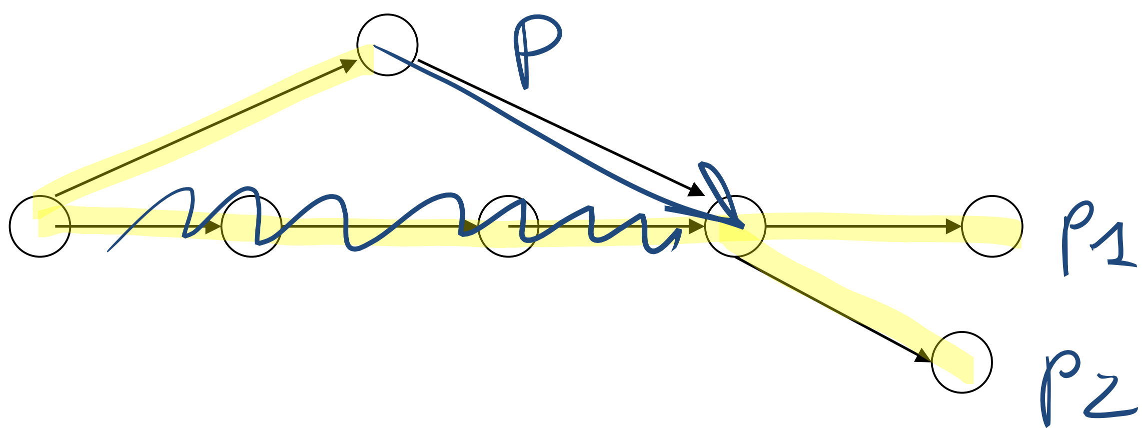
- You can remove all paths from the frontier that use the longer path. (as these can't be optimal)



Multiple-Path Pruning & Optimal Solutions

Problem: what if a subsequent path to *n* is shorter than the first path to *n* ?

- You can change the initial segment of the paths on the frontier to use the shorter path.



Learning Goals for today's class

- Define/read/write/trace/debug different search algorithms
    - With / Without cost
    - Informed / Uninformed

- Pruning cycles and Repeated States

Next class: Thurs

- Dynamic Programming
- Recap Search
- Start Constraint Satisfaction Problems (CSP)
- Chp 4.

- Start working on assignment-1 !