

Heuristic Search

Computer Science cpSC322, Lecture 7

(Textbook Chpt 3.6)

May, 23, 2017



Course Announcements

Assignment 1: posted

If you are confused on basic search algorithm, different search strategies... Check **learning goals** at the end of lectures. Work on the **Practice Exercises** and **Please do come to office hours**

Giuseppe : Fri 830-930, my office CICS R 105

- Johnson, David davewj@cs.ubc.ca Office hour: ICCS X141, Wed 1-230pm
- Johnson, Jordon jordon@cs.ubc.ca Office hour: ICCS X141, Mon 11-1pm
- Kazemi, S. Mehran smkazemi@cs.ubc.ca Office hour: ICCS X141, Wed 230-4pm
- Rahman, MD Abed abed90@cs.ubc.ca Office hour: ICCS X141, Fri 3-430pm
- Wang, Wenyi wenyi.wang@alumni.ubc.ca Office hour: ICCS X141, Mon 1-230pm

Course Announcements

Inked Slides

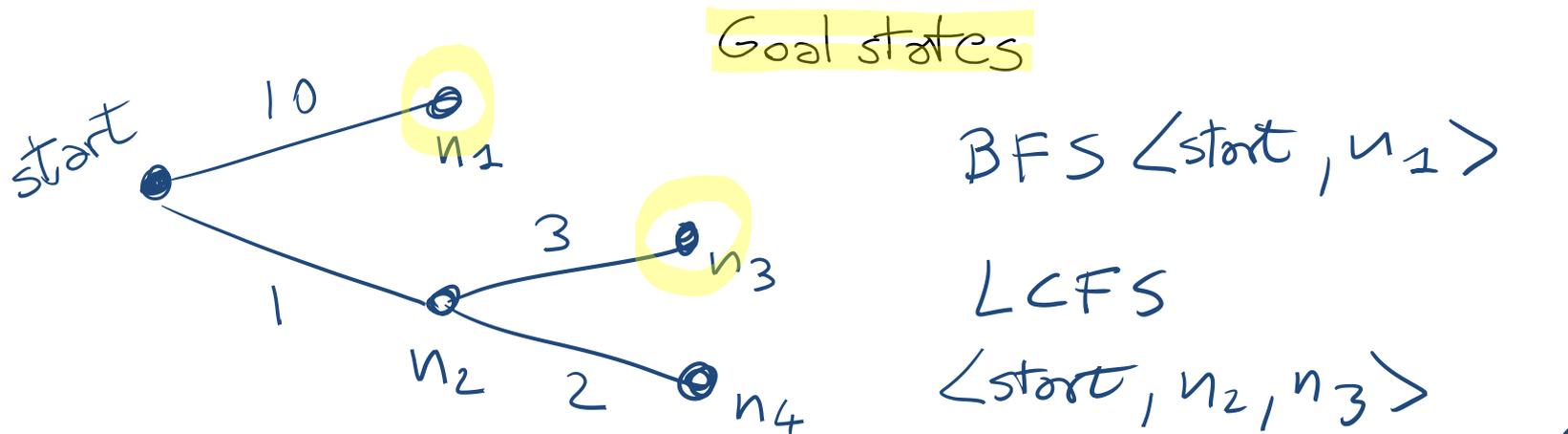
- **At the end of each lecture I revise/clean-up the slides. Adding comments, improving writing... make sure you check them out**

Lecture Overview

- **Recap**
 - **Search with Costs**
 - **Summary Uninformed Search**
- Heuristic Search

Recap: Search with Costs

- Sometimes there are **costs** associated with arcs.
 - The cost of a path is the sum of the costs of its arcs.

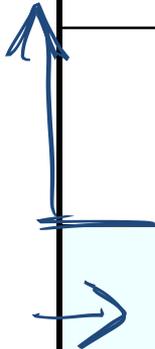


- **Optimal solution:** not the one that minimizes the *number of links*, but the one that minimizes *cost*
- **Lowest-Cost-First Search:** expand paths from the frontier in order of their costs.

Recap Uninformed Search



	Complete	Optimal	Time	Space
DFS	N <i>Y if no cycles and finite search space</i>	N	$O(b^m)$	$O(mb)$
BFS	Y	Y	$O(b^m)$	$O(b^m)$
IDS	Y	Y	$O(b^m)$	<u>$O(mb)$</u>
LCFS	Y Costs > 0	Y Costs <u>≥ 0</u>	$O(b^m)$	$O(b^m)$



Recap Uninformed Search

- Why are all these strategies called uninformed?

Because they **do not consider any information about the states (end nodes)** to decide which path to expand first on the frontier

eg

$(\langle n_0, n_2, n_3 \rangle 12)$, $(\langle n_0, n_3 \rangle 8)$, $(\langle n_0, n_1, n_4 \rangle 13)$

In other words, they are general they do not take into account the **specific nature of the problem**.

Lecture Overview

- **Recap**
 - Search with Costs
 - Summary Uninformed Search
- **Heuristic Search**

Beyond uninformed search...

iclicker.

What information we could use to better select paths from the frontier?

- A. an estimate of the distance from the last node on the path to the goal
- B. an estimate of the distance from the start state to the goal
- C. an estimate of the cost of the path
- D. None of the above

Heuristic Search

Uninformed/Blind search algorithms do not take into account the goal until they are at a goal node.

Often there is extra knowledge that can be used to guide the search: an *estimate* of the distance from node n to a goal node.

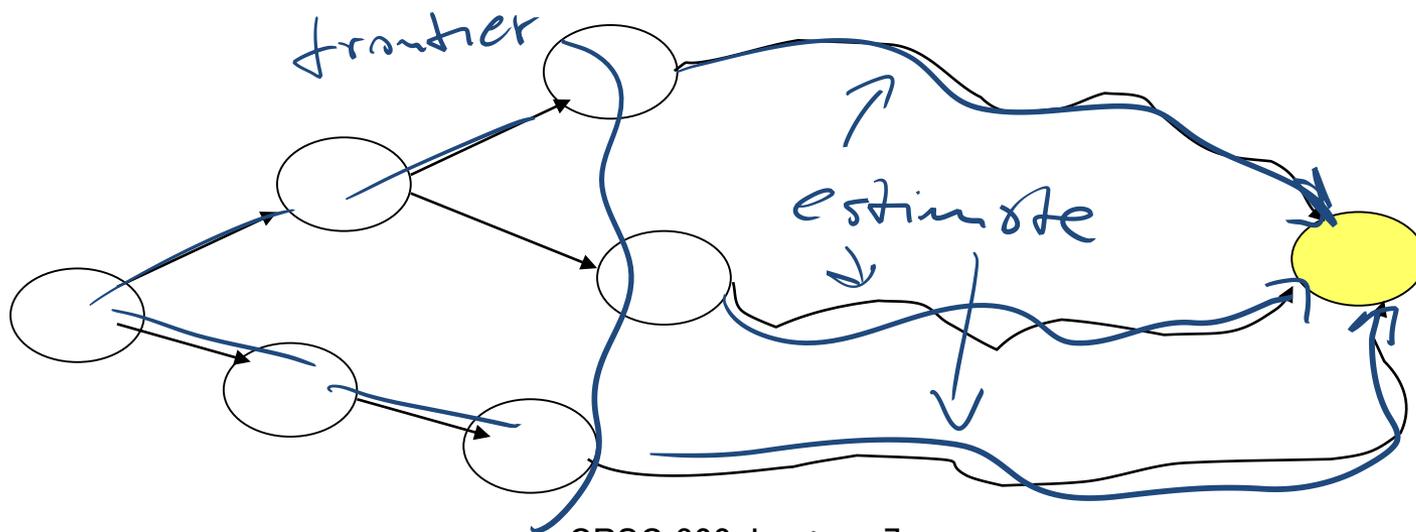
This is called a *heuristic*

More formally

Definition (search heuristic)

A search heuristic $h(n)$ is an estimate of the cost of the shortest path from node n to a goal node.

- h can be extended to paths: $h(\langle n_0, \dots, n_k \rangle) = h(n_k)$
- For now think of $h(n)$ as only using readily obtainable information (that is easy to compute) about a node.



More formally (cont.)

Definition (**admissible heuristic**)

A search heuristic $h(n)$ is **admissible** if it is never an overestimate of the cost from n to a goal. ←

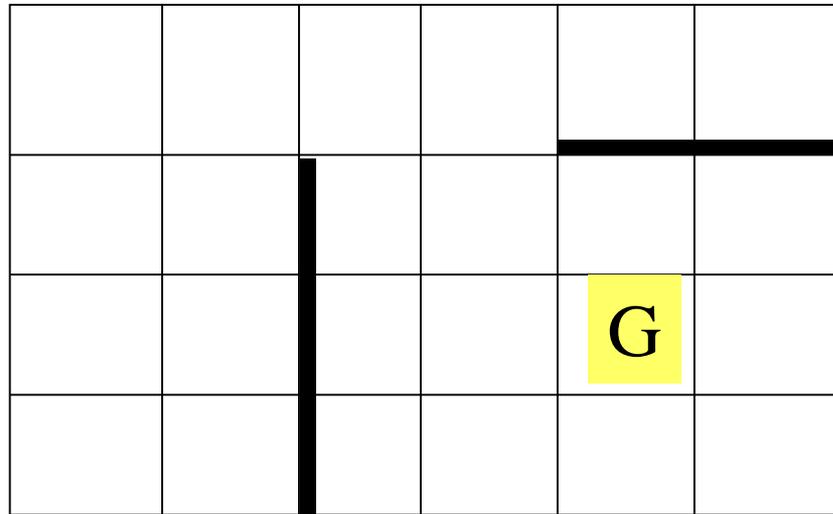
- There is never a path from n to a goal that has path cost less than $h(n)$.
- another way of saying this: $h(n)$ is a **lower bound** on the cost of getting from n to the nearest goal.



Example Admissible Heuristic Functions

Search problem: robot has to find a route from start location to goal location on a grid (discrete space with obstacles)

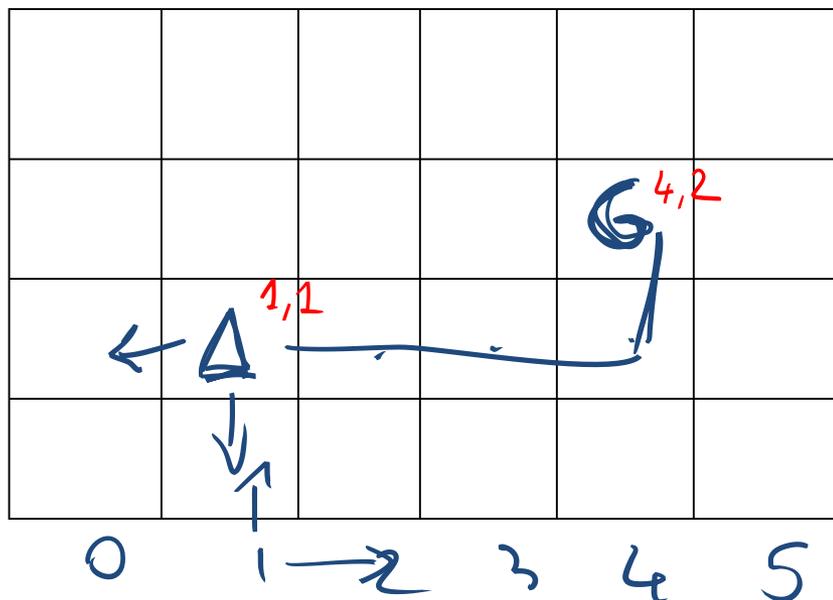
Final cost (quality of the solution) is the number of steps



Example Admissible Heuristic Functions

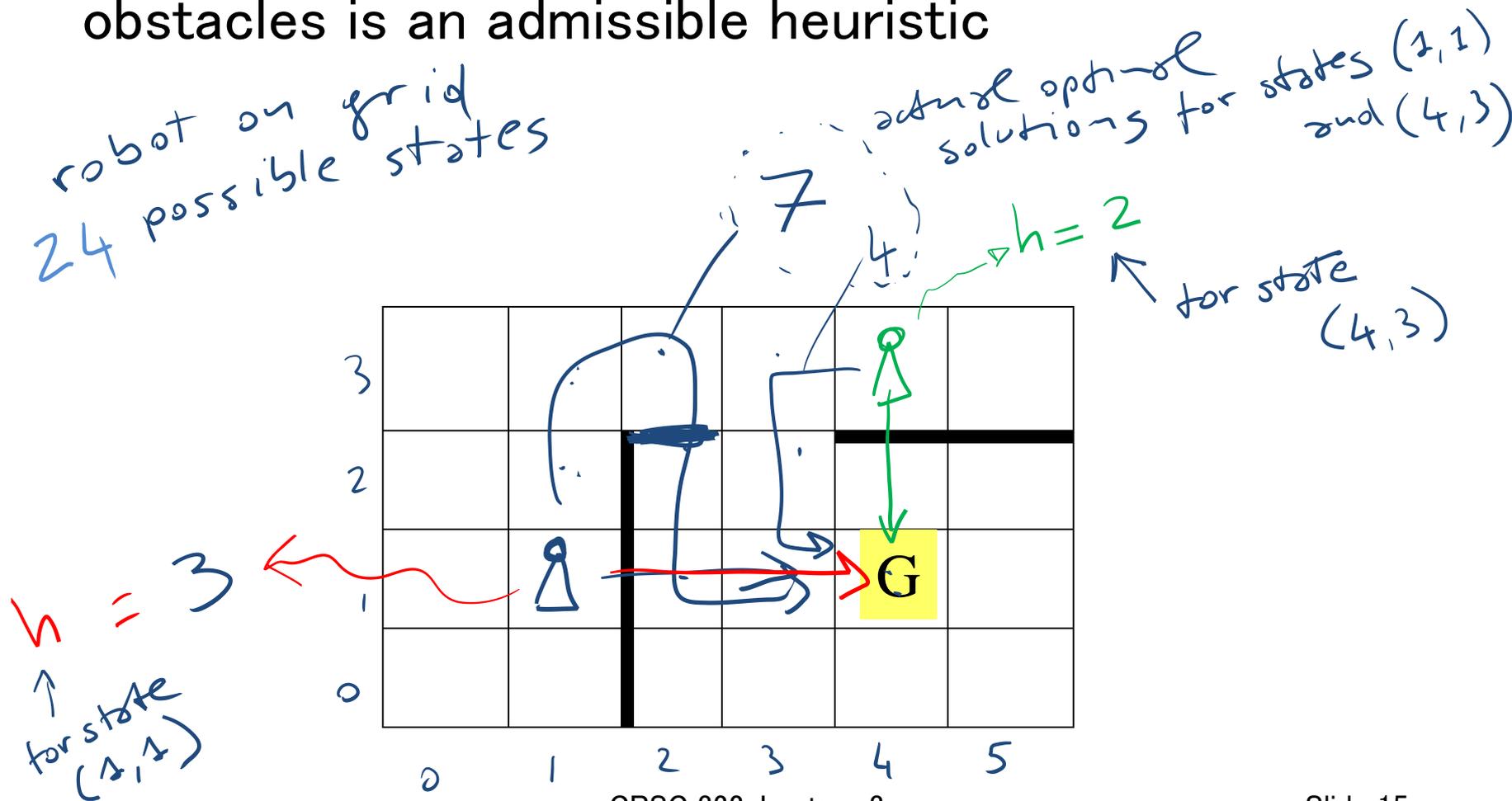
If no obstacles, cost of optimal solution is...

$$\begin{array}{l} \text{Goal state } (X_G, Y_G) \\ \text{Current state } (X_c, Y_c) \\ \text{Manhattan distance} = |X_G - X_c| + |Y_G - Y_c| \\ \text{In example: } |4 - 1| + |2 - 1| = 4 \end{array}$$



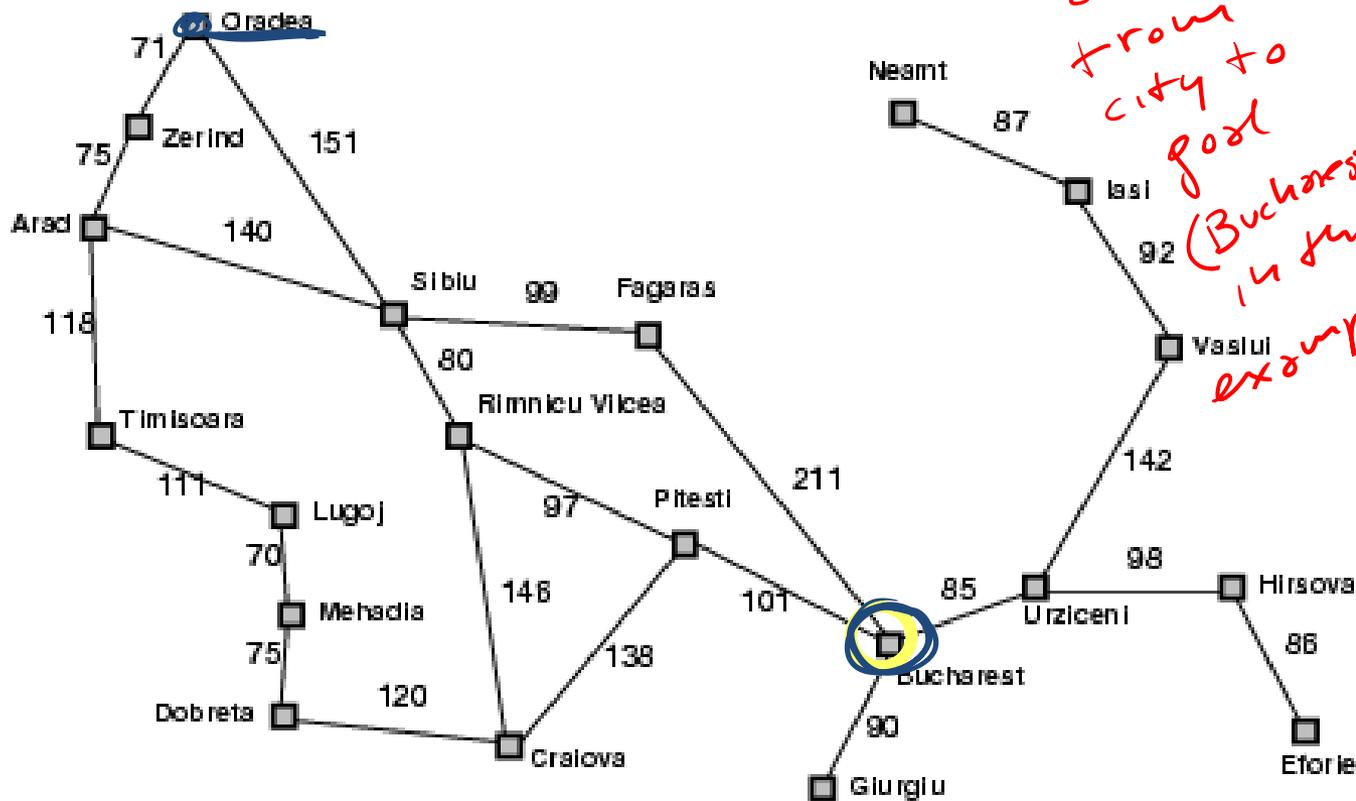
Example Admissible Heuristic Functions

If there are obstacles, the optimal solution without obstacles is an admissible heuristic



Example Admissible Heuristic Functions

- Similarly, If the nodes are **points on a Euclidean plane** and the cost is the distance, we can use the straight-line distance from n to the closest goal as the value of $h(n)$.

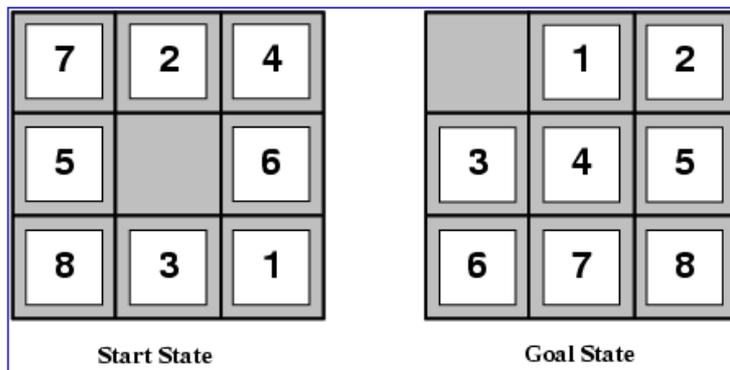


always \leq than any road from city to goal (Bucharest) in this example

Straight-line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	10
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Admissible Heuristic Function for 8-puzzle



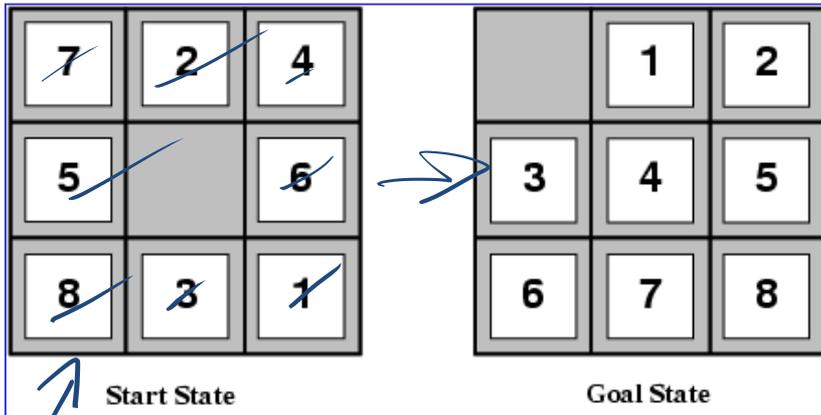
A reasonable admissible heuristics for the 8-puzzle is?

- A. Number of misplaced tiles plus number of correctly place tiles
- B. Number of misplaced tiles
- C. Number of correctly placed tiles
- D. None of the above

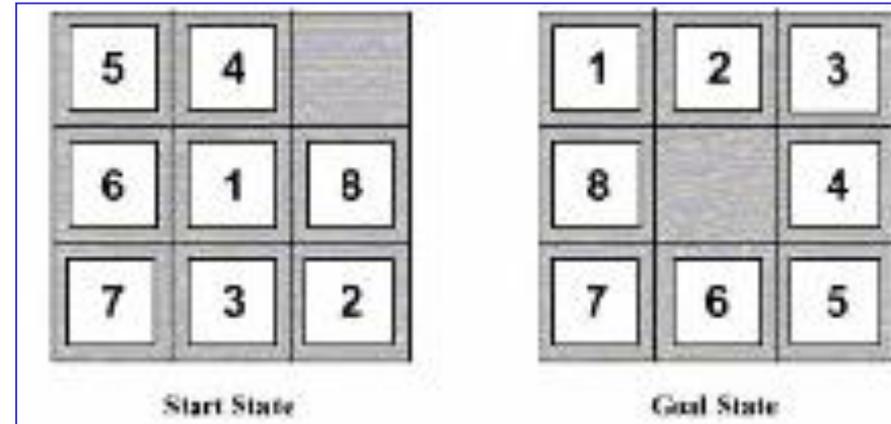
Admissible Example Heuristic Functions

(1)

- In the 8-puzzle, we can use the number of misplaced tiles



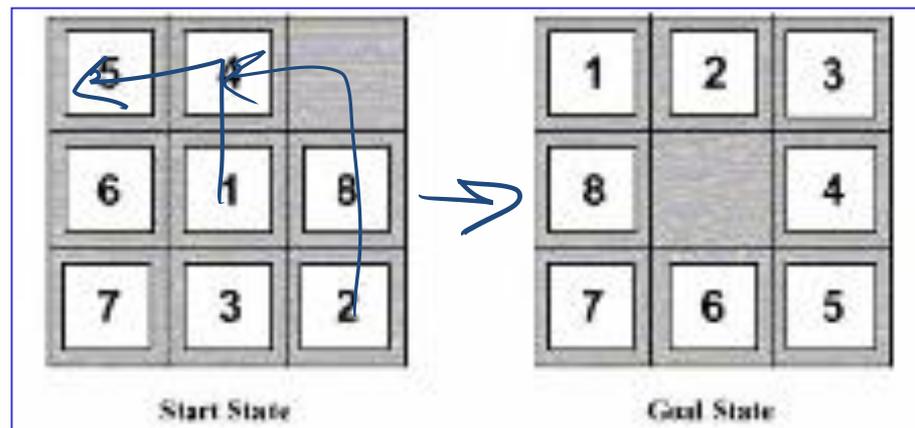
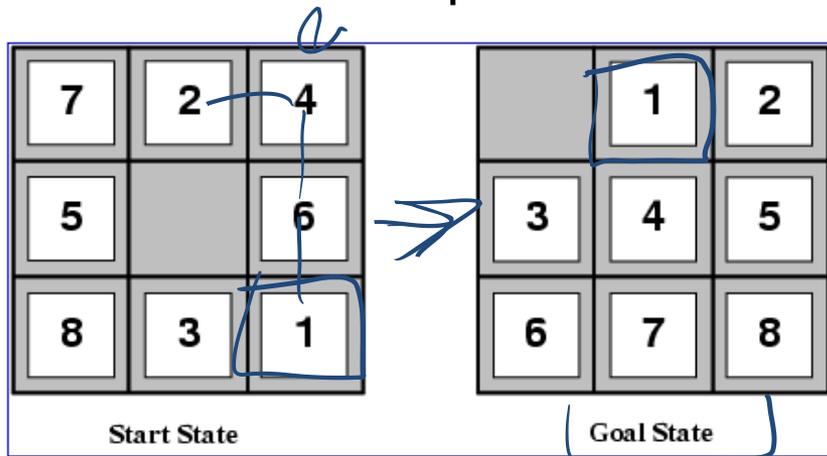
W 8



W 7

Example Admissible Heuristic Functions (2)

- Another one we can use the number of moves between each tile's current position and its position in the solution



tiles



$\{3, 1, 2, 2, 2, 3, 3, 2\}$ $\{2, 3, \dots, \dots\}$

$= 18$

How to Construct an Admissible Heuristic

You identify **relaxed version of the problem**:

- where one or more constraints have been dropped
- problem with fewer restrictions on the actions 

Robot: the agent **can move through walls** 

Driver: the agent **can move straight** 

8puzzle: (1) tiles **can move anywhere** 

(2) tiles can move to **any adjacent square** 

Result: The cost of an optimal solution in the relaxed problem is an admissible heuristic for the original problem (because it is always weakly less costly to solve a less constrained problem!)

How to Construct an admissible Heuristic (cont.)

You should identify constraints which, when dropped, make the problem extremely easy to solve

- this is important because heuristics are not useful if they're as hard to solve as the original problem!

This was the case in our examples

Robot: *allowing* the agent to move through walls. Optimal solution to this relaxed problem is Manhattan distance

Driver: *allowing* the agent to move straight. Optimal solution to this relaxed problem is straight-line distance

8puzzle: (1) tiles **can move anywhere** Optimal solution to this relaxed problem is number of misplaced tiles

(2) tiles can move to **any adjacent square**...

Another approach to construct heuristics

Solution cost for a subproblem

1 2 3 4

Simpler!

Original Problem

	1	3
8	2	5
7	6	4

Current node

1	2	3
8		4
7	6	5

Goal node

SubProblem

	1	3
@	2	@
@	@	4

1	2	3
@		4
@	@	@

Good

Heuristics: Dominance

If $h_2(n) \geq h_1(n)$ for every state n (both admissible)
then h_2 **dominates** h_1

Which one is better for search ?



A. h_1

B. h_2

C. *It depends*

Heuristics: Dominance



8puzzle: (1) tiles can move anywhere

(2) tiles can move to any adjacent square

(Original problem: tiles can move to an adjacent square if it is empty)

Iterative deepening (not using any heuristic)

search costs for the 8-puzzle (average number of paths expanded):

↳ depth of solution

<u>$d=12$</u>	IDS = 3,644,035 paths
	$A^*(h_1) = 227$ paths
	$A^*(h_2) = 73$ paths
<u>$d=24$</u>	IDS = too many paths
	$A^*(h_1) = 39,135$ paths
	$A^*(h_2) = 1,641$ paths

	h_1	h_2
If tile in correct position	0	0
If tile 1 move from correct position	1	1
otherwise	1	>1

why

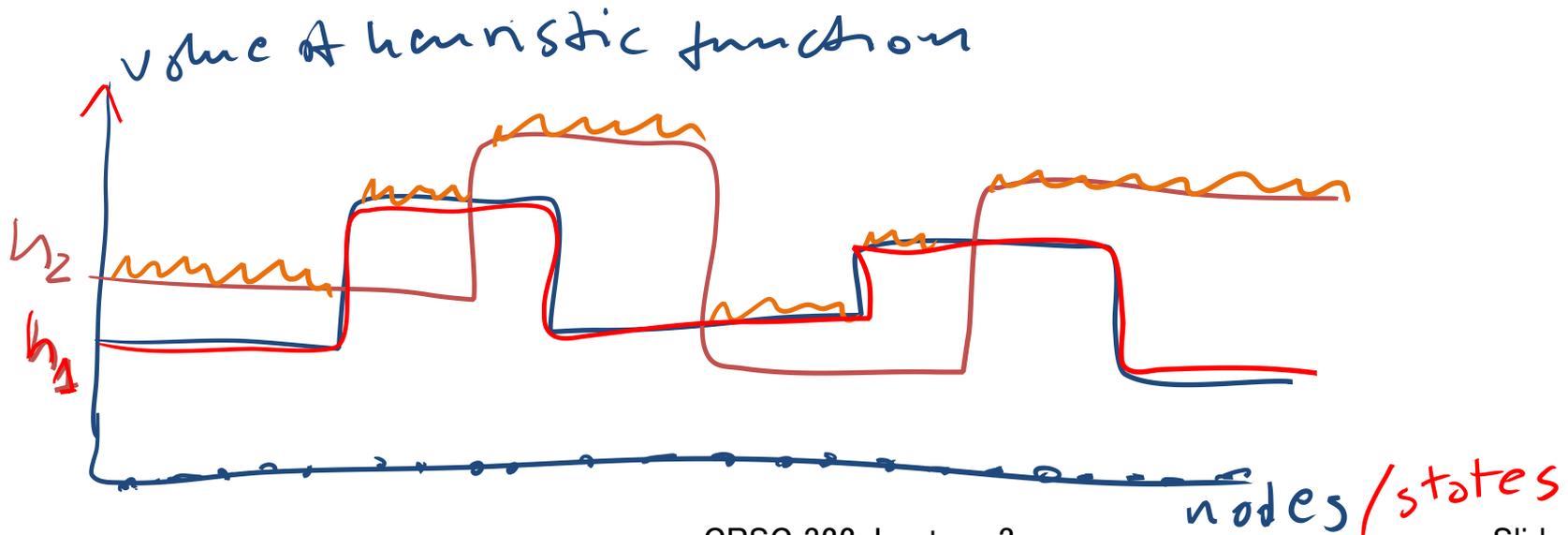
Combining Admissible Heuristics

How to combine heuristics when there is no dominance?

If $h_1(n)$ is admissible and $h_2(n)$ is also admissible then

$h(n) = \max(h_1, h_2)$ is also admissible

... and dominates all its components



Combining Admissible Heuristics: Example

In 8-puzzle, solution cost for the 1,2,3,4 subproblem is substantially more accurate than Manhattan distance in some cases

So....

1 of each tile from its position in goal

sum of

max

better heuristic

Learning Goals for today's class

- Construct admissible heuristics for a given problem.
- Verify Heuristic Dominance.
- Combine admissible heuristics 
- From previous classes Define/read/write/trace/debug different search algorithms
 - With / Without cost
 - Uninformed

Next Class

- Best-First Search
- Combining LCFS and BFS: A* (finish 3.6)
- A* Optimality