Probability and Time: Markov Models

Computer Science cpsc322, Lecture 31

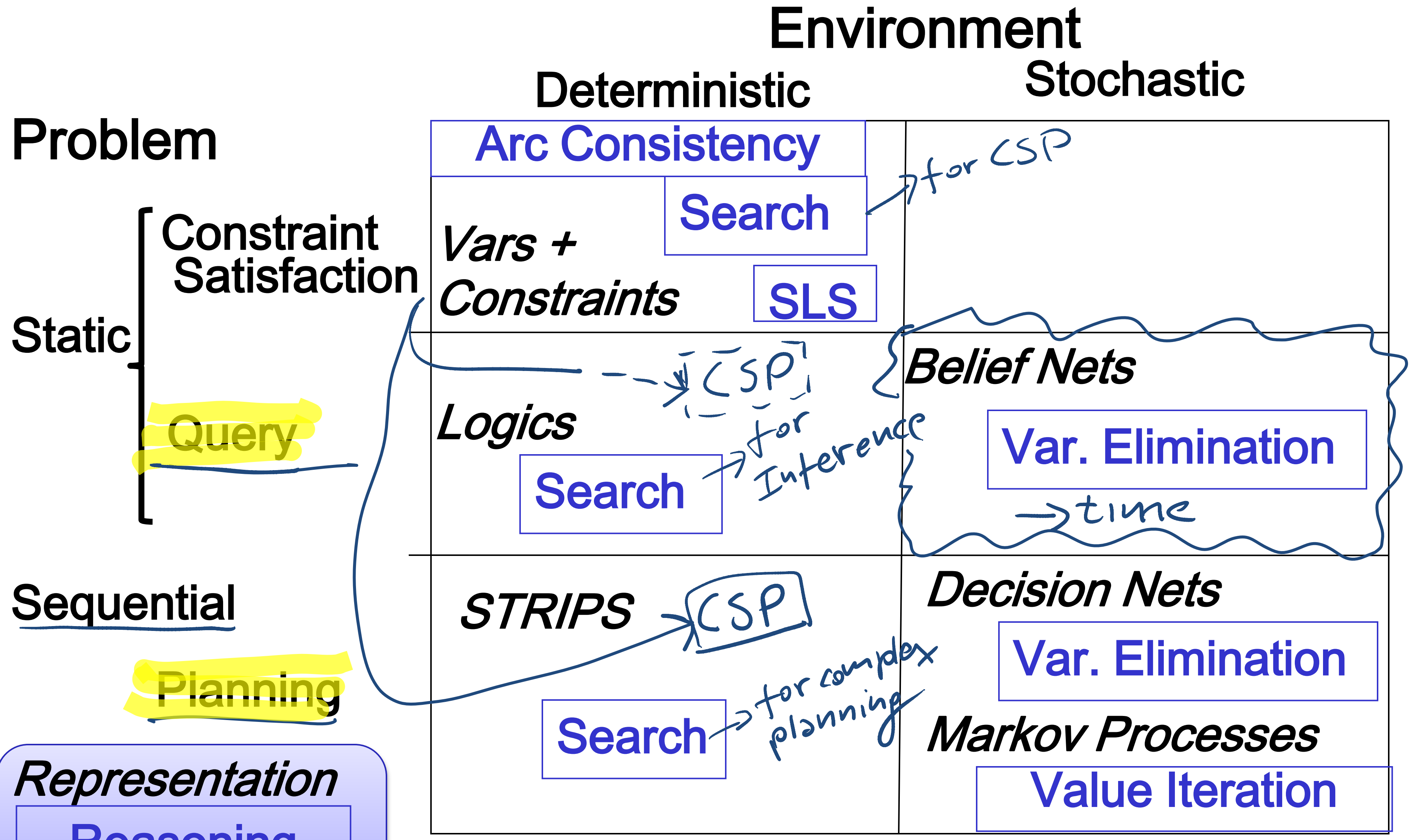
(Textbook Chpt 6.5.1)

Nov, 21, 2012

Lecture Overview

- Recap
- Temporal Probabilistic Models
- Start Markov Models
  - Markov Chain
  - Markov Chains in Natural Language Processing

Big Picture: R&R systems

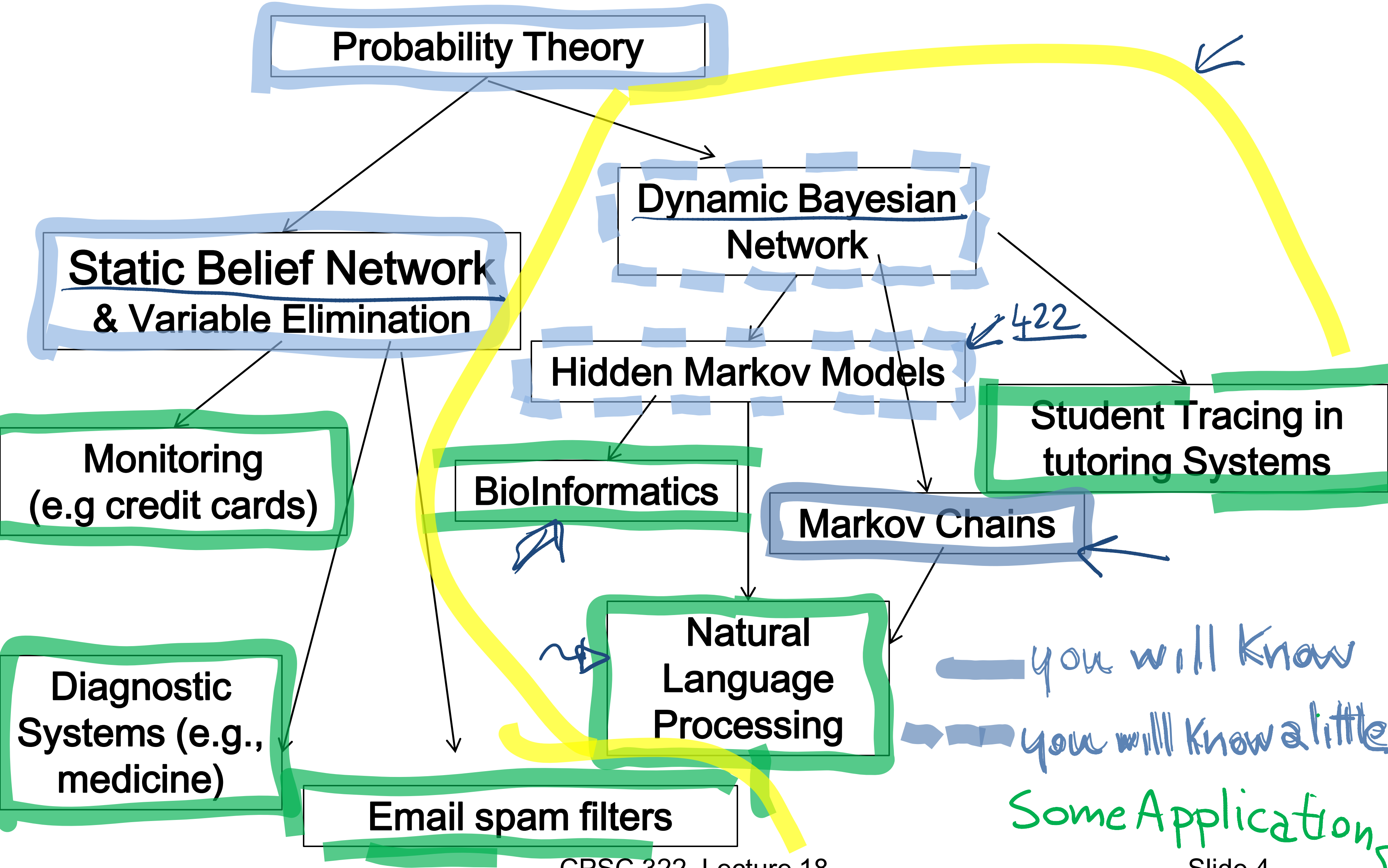


Problem		Environment: Deterministic	Environment: Stochastic
Static	Constraint Satisfaction	*Vars + Constraints* Arc Consistency Search → for CSP SLS	
	Query	*Logics* CSP Search → for Inference	*Belief Nets* Var. Elimination → time
Sequential	Planning	*STRIPS* → CSP Search → for complex planning	*Decision Nets* Var. Elimination *Markov Processes* Value Iteration

Representation

Reasoning Technique

Answering Query under Uncertainty



Probability Theory

Static Belief Network & Variable Elimination

Dynamic Bayesian Network

Hidden Markov Models

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Monitoring (e.g credit cards)

BioInformatics

Markov Chains

Student Tracing in tutoring Systems

Natural Language Processing

Diagnostic Systems (e.g., medicine)

Email spam filters

🟦 you will know

▨ you will know a little

Some Applications

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Modelling static Environments

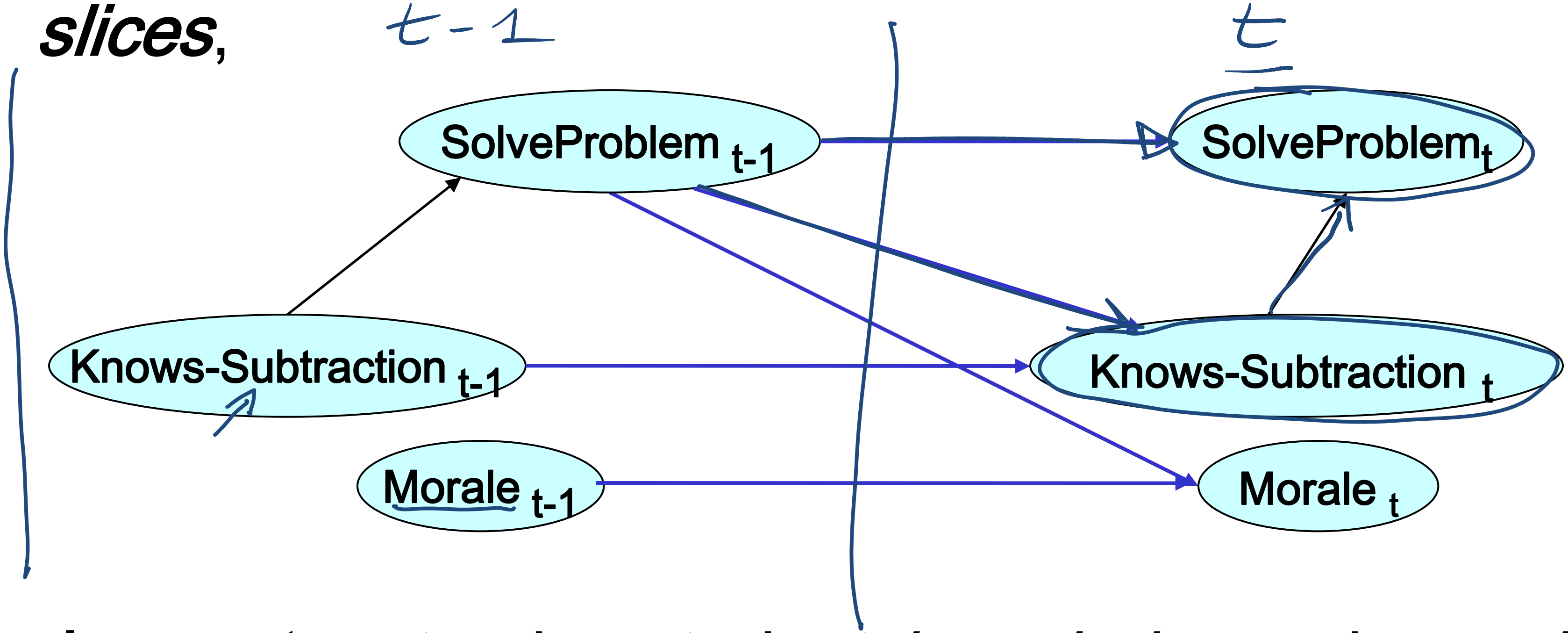
So far we have used Bnets to perform inference in **static environments**

- For instance, the system keeps collecting evidence to diagnose the cause of a fault in a system (e.g., a car).
- The environment (values of the evidence, the true cause) does not change as I gather new evidence
- What does change?

The system's beliefs over possible causes

Modeling Evolving Environments

- Often we need to make inferences about evolving environments.
- Represent the state of the world at each specific point in time via a series of snapshots, or ***time slices***,



Tutoring system tracing student *knowledge* and *morale*

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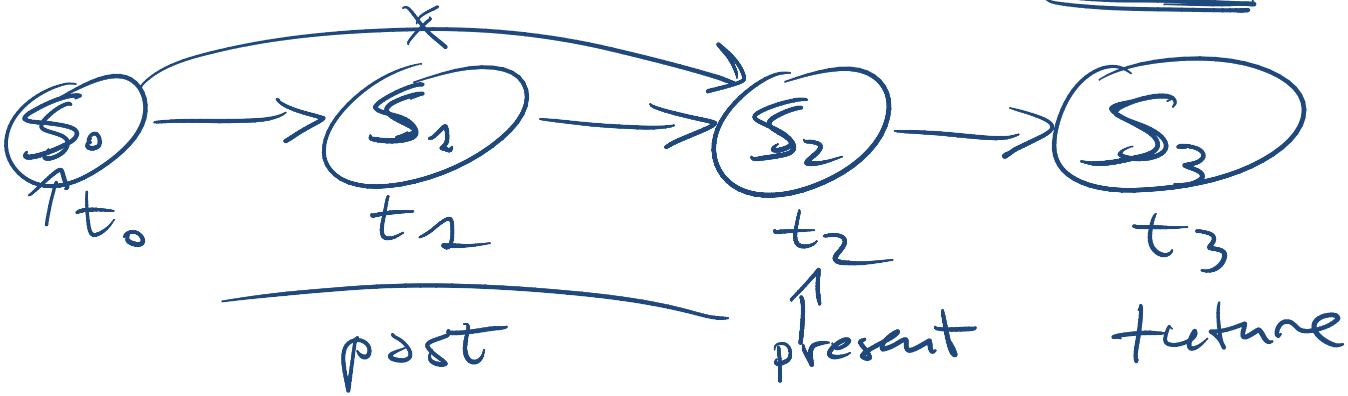
Simplest Possible DBN

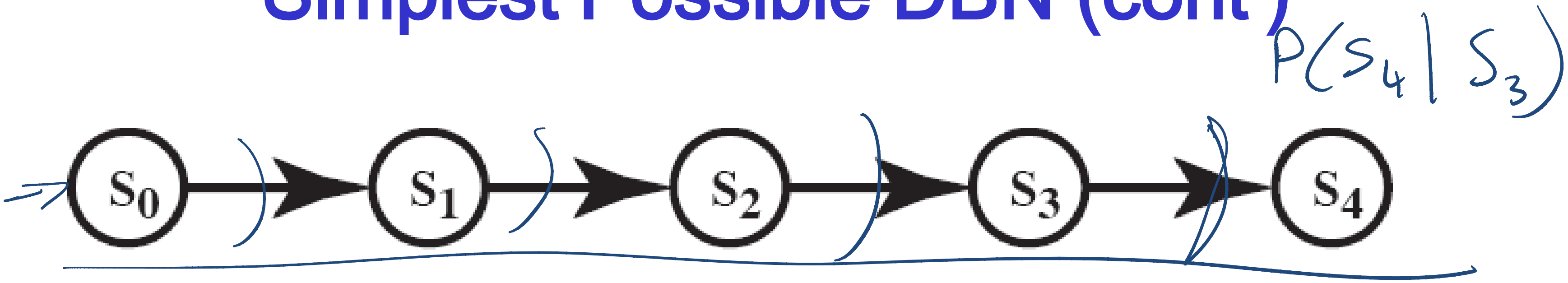
- **One random variable** for each time slice: let's assume S_t represents the state at time *t*. with domain $\{\boxed{s_1 \ldots s_n}\}$



- **Each random variable depends only on the previous one**
- Thus $P(S_{t+1} \mid S_0 \cdots S_t) = P(\underline{S_{t+1}} \mid \underline{S_t})$

- Intuitively S_t conveys all of the information about the history that can affect the future states.
- "The future is independent of the past given the present."

Simplest Possible DBN (cont')



- How many CPTs do we need to specify?

4 $P(S_1 | S_0)$ $P(S_2 | S_1)$ etc.

- *Stationary process assumption:* the mechanism that regulates how state variables change overtime is **stationary**, that is it can be described by a single transition model

- $P(S_t|S_{t-1})$ is the same for all t

Stationary Markov Chain (SMC)

A stationary Markov Chain : for all t >0

- $\underline{P(S_{t+1}| S_0,\ldots,S_t) = P(S_{t+1}|S_t)}$ and Markov assumption
- $\underline{P(S_{t+1}|S_t)}$ is the same stationary

We only need to specify $P(S_0)$ and $P(S_{t+1}|S_t)$

- Simple Model, easy to specify ←
- Often the natural model ←
- The network can extend indefinitely ←
- **Variations of SMC are at the core of most Natural Language Processing (NLP) applications!** also used in the PageRank algo (used by Google to rank web pages)

Stationary Markov-Chain: Example

Domain of variable S_i is {t , q, p, a, h, e}

six possible values

We only need to specify…

$P(S_0)$

Probability of initial state

t	.6
q	.4
p	0
a	0
h	0
e	0

Stochastic Transition Matrix

$P(S_{t+1}|S_t)$

S_{t+1}

S_t

	t	q	p	a	h	e
t	0	.3	0	.3	.4	0
q	.4	0	.6	0	0	0
p	0	0	1	0	0	0
a	0	0	.4	.6	0	0
h	0	0	0	0	0	1
e	1	0	0	0	0	0

← $P(S_{t+1}|S_t=q)$

← $P(S_{t+1}|S_t=p)$

Markov-Chain: Inference

Probability of a sequence of states $S_0 \ldots S_T$

$$P(S_0,\ldots,S_T) = P(S_0)\,P(S_1|S_0)\,P(S_2|S_1) \ldots\ldots$$

P(u,e,e) →

$P(S_0)$

t	.6
q	.4
p	0
a	0
h	0
e	0

$P(S_{t+1}|S_t)$

	t	q	p	a	h	e
t	0	.3	0	.3	.4	0
q	.4	0	.6	0	0	0
p	0	0	1	0	0	0
a	0	0	.4	.6	0	0
h	0	0	0	0	0	1
e	1	0	0	0	0	0

Example:

$$P(t, q, p) =$$

$$P(t) * P(q|t) * P(p|q)$$

$$.6 \times .3 \times .6 = .108$$

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Key problems in NLP

Noun Verb

"Book me a room near UBC"

w_1 w_2 w_3 w_4 w_5 w_6

Assign a probability to a sentence (a sequence of words)

$$\boxed{P(w_1,..,w_n)?}$$

- Part-of-speech tagging
- Word-sense disambiguation,
- Probabilistic Parsing

Summarization, Machine Translation........

Predict the next word

$$P(w_n \mid w_1 \ldots\ldots w_{n-1}) = = P(w_1 \ldots w_n) / P(w_1 \ldots w_{n-1})$$

- Speech recognition
- Hand-writing recognition
- Augmentative communication for the disabled

$$\boxed{P(w_1,..,w_n)?}$$

Impossible to estimate ☹

$\boxed{P(w_1,..,w_n)?}$

Impossible to estimate!

Assuming 10^5 words and average sentence contains 10 words ....

$(10^5)^{10} = 10^{50}$

would contain $\uparrow$ probabilities

Google language repository (22 Sept. 2006)

collected from the whole web

contained "only": 95,119,665,584 sentences

$\sim 10^{11}$

Most sentences will not appear or appear only once ☹

What can we do?

Make a strong simplifying assumption!

Sentences are generated by a Markov Chain

$$\underline{P(w_1,..,w_n)} = \overbrace{P(w_1|<S>)}^{w_1 \text{ at the beginning of a sentence}}\underline{\prod_{k=2}^{n} P(w_k | w_{k-1})}$$

$$= P(w_1|<S>) P(w_2|w_1) P(w_3|w_2) .. P(w_K|w_{K-1})$$

P(The big red dog barks)=

**P(The|<S>) *** P(big | the) * P(red | big) * ...

* P(dog | red) * P(barks | dog)

These probs can be assessed in practice!

Estimates for Bigrams

$P(w_i | w_{i-1})$

Silly language repositories with **only two sentences**:

"<S> *The big red dog barks against the big pink dog*"

"<S> *The big pink dog is much smaller*"

Count how many times in your documents you have "big red" and "big"

$$P(red \mid big) = \frac{P(big,red)}{P(big)} = \frac{\overset{\text{count}}{\dfrac{C(big,red)}{\cancel{N}_{pairs}}}}{\dfrac{C(big)}{\cancel{N}_{words}}} = \frac{C(big,red)}{C(big)} = \frac{1}{3}$$

$P(w_i | w_{i-1})$

$10^5 * 10^5$ matrix

$\cancel{10^5}$

$$\boxed{P(w_i \mid w_{i-1}, w_{i-2})}$$

some models use two preceeding words

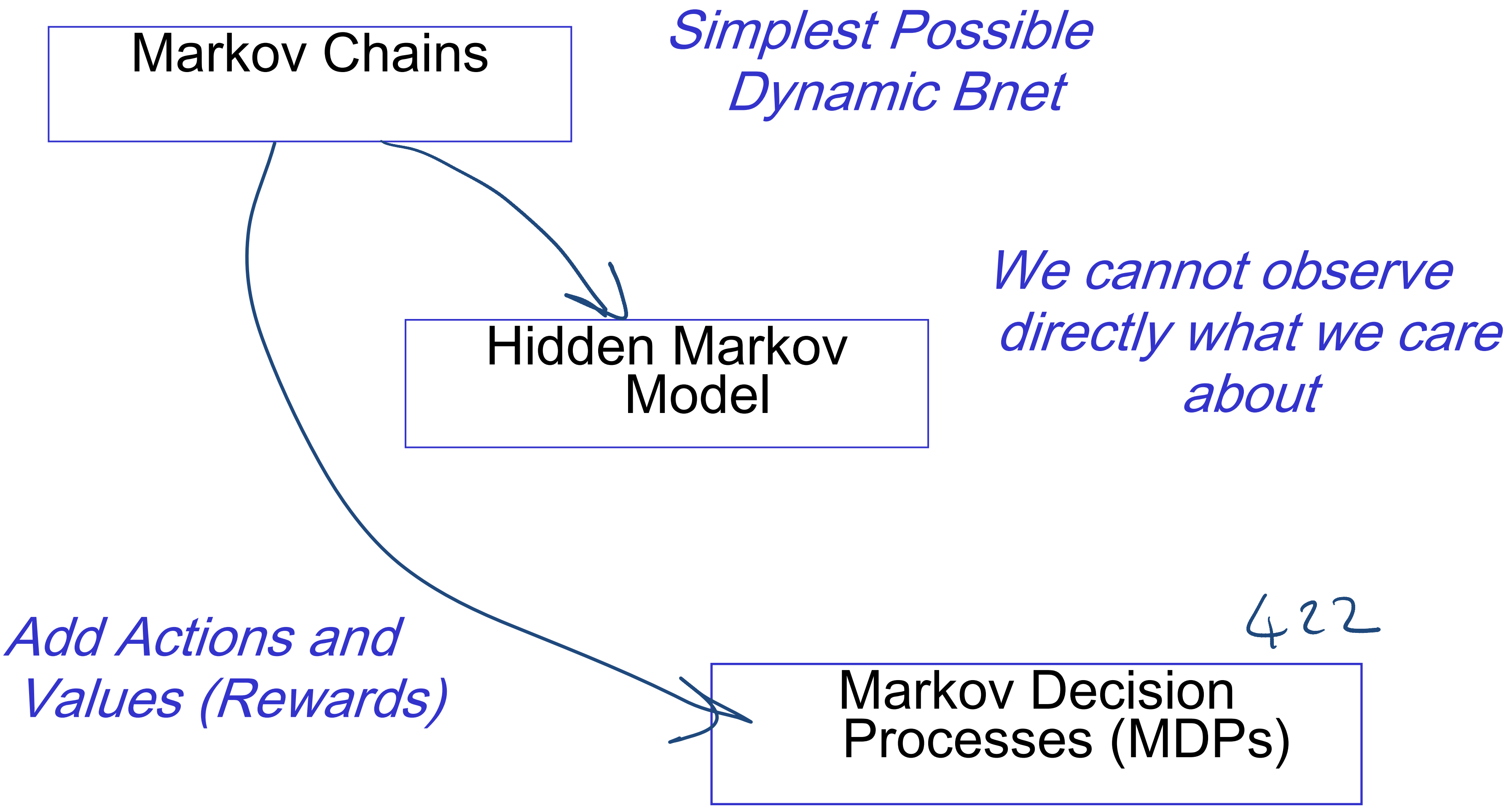
Learning Goals for today's class

You can:

- Specify a Markov Chain and compute the ✓ probability of a sequence of states ✓

- Justify and apply Markov Chains to compute the probability of a Natural Language sentence (NOT to compute the conditional probabilities - slide 18)

Markov Models



Markov Chains

Simplest Possible Dynamic Bnet

Hidden Markov Model

We cannot observe directly what we care about

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Add Actions and Values (Rewards)

Markov Decision Processes (MDPs)

Next Class

- **Finish Probability and Time:** Hidden Markov Models (HMM) *(TextBook 6.5.2)*
- **Start Decision networks** *(TextBook chpt 9)*

Course Elements

- **Assignment 4 is available on Connect this afternoon .** Due on Nov the 28th (last class).

TA evaluations