# Propositional Definite Clause Logic: Syntax, Semantics and Bottom-up Proofs

Computer Science cpsc322, Lecture 20

*(Textbook Chpt 5.1.2 - 5.2.2 )*

March, 3, 2010

# Lecture Overview

- **Recap: Logic intro**
- Propositional Definite Clause Logic: Semantics
- PDCL: Bottom-up Proof

# Logics as a R&R system

Represent

- formalize a domain

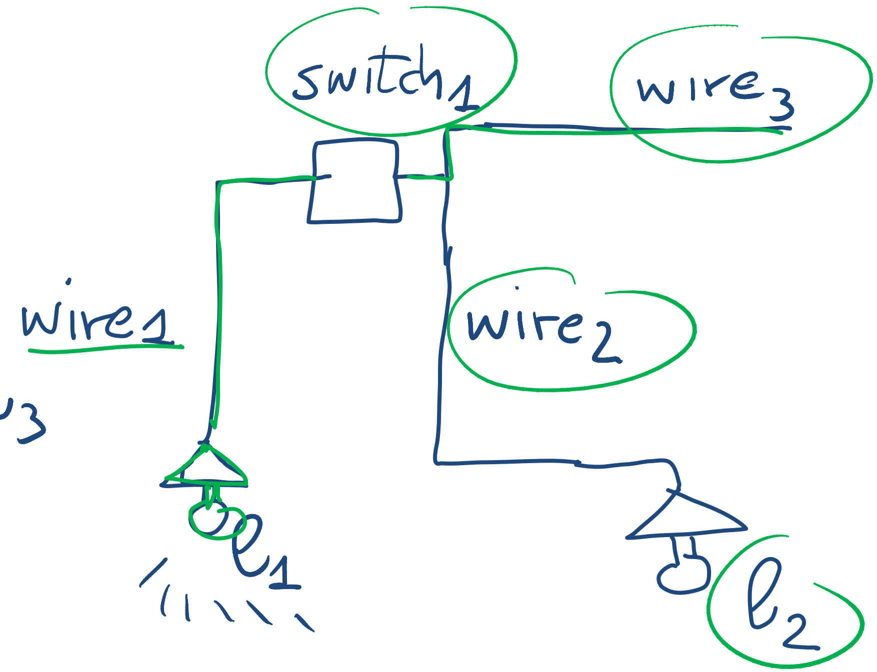
IF

$on\_l_1 \leftarrow live\_w_1$.

IF AND

$live\_w_1 \leftarrow on\_sw_1 \land live\_w_3$

. . . . .

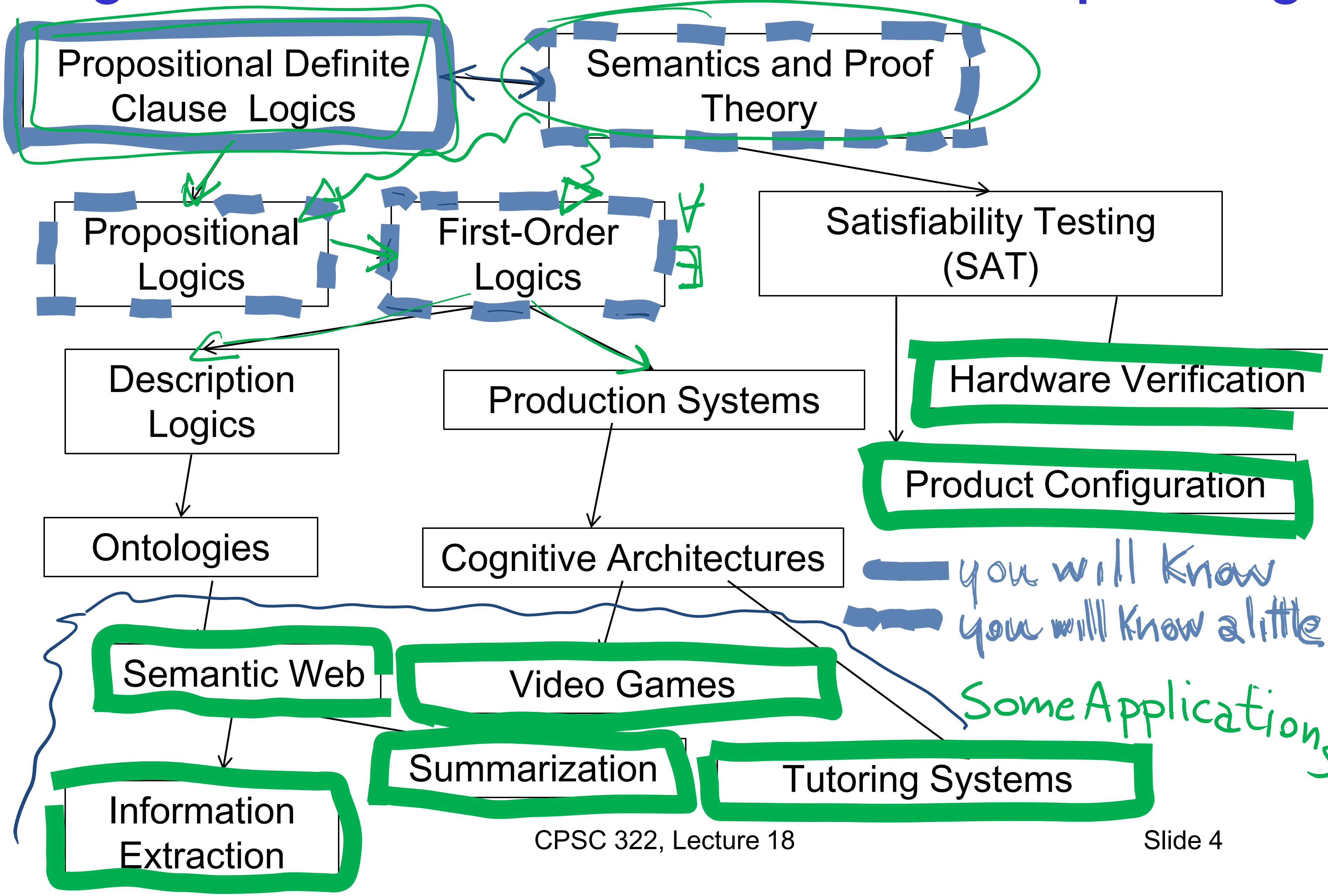



- reason about it

If the agent knows $on\_sw_1$ and $live\_w_3$

it should be able to infer $\boxed{on\_l_1}$

# Logics in AI: Similar slide to the one for planning



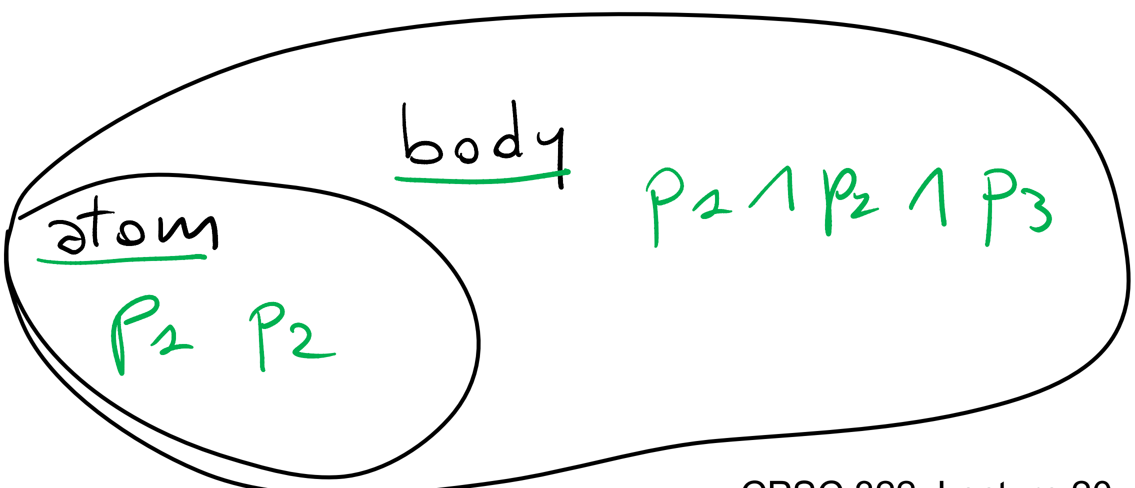
# Propositional (Definite Clauses) Logic: Syntax

We start from a restricted form of Prop. Logic:

$\neg (p_1 \vee p_2) \Leftrightarrow (p_3 \vee \neg p_5)$

Only two kinds of statements

- that a proposition is true
- that a proposition is true if one or more other propositions are true




definite clause is either an atom

or atom ← body

$p_3 \leftarrow p_1 \wedge p_2 \ldots$

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# Propositional Definite Clauses Semantics: Interpretation

Semantics allows you to relate the symbols in the logic to the domain you're trying to model. An **atom** can be….. T F

> **Definition (interpretation)**
> An interpretation $I$ assigns a truth value to each atom.

If your domain can be represented by four atoms (propositions):

q p s r

T T F F

$2^4$

So an interpretation is just a…possible world…

# PDC Semantics: Body

We can use the **interpretation** to determine the truth value of **clauses** and **knowledge bases**:

**Definition (**truth values of statements**):** A body $b_1 \wedge b_2$ is true in $I$ if and only if $b_1$ is true in $I$ and $b_2$ is true in $I$.

| | p | q | r | s | $p \wedge r$ | $p \wedge r \wedge s$ |
|---|---|---|---|---|---|---|
| $I_1$ | *true* | *true* | *true* | *true* | T | T |
| $I_2$ | *false* | *false* | *false* | *false* | F | F |
| $I_3$ | *true* | *true* | *false* | *false* | F | F |
| $I_4$ | *true* | *true* | *true* | *false* | T | F |
| $I_5$ | *true* | *true* | *false* | *true* | F | F |

# PDC Semantics: definite clause

**Definition (**truth values of statements cont'**):** A rule $h \leftarrow b$ is false in $I$ if and only if $b$ is true in $I$ and $h$ is false in $I$.

| | p | q | r | s | $p \leftarrow s$ | $s \leftarrow q \wedge r$ |
|---|---|---|---|---|---|---|
| $I_1$ | *true* | *true* | *true* | *true* | T | T |
| $I_2$ | *false* | *false* | *false* | *false* | T | T |
| $I_3$ | *true* | *true* | *false* | *false* | T | T |
| $I_4$ | *true* | *true* | *true* | *false* | T | F |
| | F | | | T | F | |
| ….. | …. | ….. | …. | …. | | |

In other words: *"if b is true I am claiming that h must be true, otherwise I am not making any claim"*

# PDC Semantics: Knowledge Base

**Definition** (truth values of statements cont'**):** A knowledge base *KB* is true in $I$ if and only if every clause in *KB* is true in $I$.

| p | q | r | s |
|---|---|---|---|
| T | T | F | F |

T

$KB_3$

$q \leftarrow r \wedge s$

$p$

F

$KB_1$

- $p$ ✓
- $r$ ✗
- $s \leftarrow q \wedge p$

F

$KB_2$

$p$ ✓

$q$ ✓

$s \leftarrow q$ ✗

# Models

**Definition (model)**
A model of a set of clauses (a KB) is an interpretation in which all the clauses are *true*.

# Example: Models

$$KB = \begin{cases} p \leftarrow q. \\ q. \\ r \leftarrow s. \end{cases}$$

| | p | q | r | s | |
|---|---|---|---|---|---|
| $I_1$ | *true* | *true* | *true* | *true* | M |
| $I_2$ | *false* | *false* | *false* | *false* | × |
| $I_3$ | *true* | *true* | *false* | *false* | M |
| $I_4$ | *true* | *true* | *true* | *false* | M |
| $I_5$ | *true* | *true* | *false* | *true* | × |

*Which interpretations are models?*

# Logical Consequence

**Definition (logical consequence)**
If *KB* is a set of clauses and *G* is a conjunction of atoms, *G* is a logical consequence of *KB*, written $KB \models G$, if *G* is *true* in every model of *KB*.

- we also say that *G* logically follows from *KB*, or that *KB* entails *G*.
- In other words, $KB \models G$ if there is no interpretation in which *KB* is *true* and *G* is *false*.

# Example: Logical Consequences

| | p | q | r | s |
|---|---|---|---|---|
| $I_1$ | true | true | true | true |
| $I_2$ | true | true | true | false |
| $I_3$ | true | true | false | false |
| $I_4$ | true | true | false | true |
| $I_5$ | false | true | true | true |
| $I_6$ | false | true | true | false |
| $I_7$ | false | true | false | false |
| $I_8$ | false | true | false | true |
| .... | .... | ... F | ... | ... |

Models

$$KB = \begin{cases} p \leftarrow q. \\ q. \\ r \leftarrow s. \end{cases}$$

$2^4 = 16$ interpretations in total, only 3 are models

remaining 8 cannot be models because q is false

*Which of the following is true?*

- $KB \models q$, $KB \models p$, $KB \not\models s$, $KB \not\models r$

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# One simple way to prove that G logically follows from a KB

- Collect all the models of the KB
- Verify that G is true in all those models

a set of atoms $P_1, P_2, \ldots$

*Any problem with this approach?*

you have to check all the $2^n$ interpretations

intractable time complexity

- The goal of proof theory is to find proof procedures that allow us to prove that a logical formula follows form a KB avoiding the above

is logically entailed by

# Soundness and Completeness

- If I tell you I have a **proof procedure for PDCL**
- What do I need to show you in order for you to trust my procedure?

- $KB \vdash G$ means *G* can be derived by my proof procedure from *KB*.
- Recall $KB \models G$ means *G* is true in all models of *KB*.

**Definition (soundness)**

A proof procedure is sound if $KB \vdash G$ implies $KB \models G$.

**Definition (completeness)**

A proof procedure is complete if $KB \models G$ implies $KB \vdash G$.

# Bottom-up Ground Proof Procedure

One rule of derivation, a generalized form of ***modus ponens***:

$q \leftarrow p$, $p$ we can derive $\Rightarrow q$

If "$h \leftarrow b_1 \land \ldots \land b_m$" is a clause in the knowledge base, and each $b_i$ has been derived, then $h$ can be derived.

You are forward chaining on this clause.

(This rule also covers the case when $m=0$. )

# Bottom-up proof procedure

$KB \vdash G$ if $G \subseteq C$ at the end of this procedure:

$C$ :={};

**repeat**

**select** clause "$h \leftarrow b_1 \wedge \ldots \wedge b_m$" in $KB$ such that $b_i \in C$ for all $i$, and $h \notin C$;

$C := C \cup \{ h \}$

**until** no more clauses can be selected.

# Bottom-up proof procedure: Example

BU

$C = \{f, r, b, a, e, z\}$

$z \leftarrow f \wedge e$

$q \leftarrow f \wedge g \wedge z$

$e \leftarrow a \wedge b$

$a$

$b$

$r$

$f$

$C$ :={};
**repeat**
  **select** clause "$h \leftarrow b_1 \wedge \ldots \wedge b_m$" in $KB$ such that $b_i \in C$ for all $i$, and $h \notin C$;
  $C := C \cup \{h\}$
**until** no more clauses can be selected.

BU can derive $r \wedge z$

$KB \vdash_{BU}$ $r$? $q$? $z$?

$r, z$

BU cannot derive

$KB \nvdash_{BU} q$

# Learning Goals for today's class

**You can:**

- Verify whether an **interpretation** is a **model** of a PDCL KB.
- Verify when a conjunction of atoms is a **logical consequence** of a knowledge base.
- Define/read/write/trace/debug the **bottom-up proof procedure**.

# Next class

(still section 5.2)

- Soundness and Completeness of Bottom-up Proof Procedure
- Using PDC Logic to model the electrical domain
- Reasoning in the electrical domain

# Study for midterm (Wed March 10 )

Midterm: ~6 short questions ( *10pts each*) + 2 problems (*20pts each)*

- Study: textbook and **inked** slides
- Work on **all** practice exercises and **revise assignments**!
- While you revise the **learning goals**, work on **review questions** **(posted)** I may even reuse some verbatim ☺
- Will post a **couple of problems** from previous offering (maybe slightly more difficult /inappropriate for you because they were not informed by the learning goals) … but I'll give you the solutions ☺