Assume you have i.i.d. data $X_i \sim f(x|\,\theta^*)$ and you want to come up with an estimate θ_n of θ^*. You could obviously try to maximize the log-likelihood of the observation but alternatively you could consider the following estimate

$$\theta_n = \arg\max \ A_n(\theta)$$

where

$$A_n(\theta) = \sum_{i=1}^{n} g(X_i, \theta).$$

Here g is chosen 'appropriately'.

So how should we select g? You have to think of what is happening as $n \to \infty$. By the law of large numbers, we have

$$\lim_{n\to\infty} \frac{A_n(\theta)}{n} = \int g(x,\theta) f(x|\,\theta^*)\, dx := M(\theta).$$

The 'limiting' function $M(\theta)$ should admit as a (global) maximum θ^*. You also want θ^* to be the unique (global) maximum of $M(\theta)$ (otherwise it means you have identifiability problems). Assuming that this is the case, you can establish consistency of the estimate θ_n using a proof similar to the one given in your notes for the maximum likelihood estimate.

Once consistency is established, you can look at the asymptotic variance of the estimate. You use (in the scalar case)

$$0 = \left.\frac{\partial A_n(\theta)}{\partial \theta}\right|_{\theta_n} \approx \left.\frac{\partial A_n(\theta)}{\partial \theta}\right|_{\theta^*} + (\theta_n - \theta) \left.\frac{\partial^2 A_n(\theta)}{\partial \theta^2}\right|_{\theta^*}$$

so

$$\sqrt{n}\,(\theta_n - \theta) \approx \frac{-\frac{1}{\sqrt{n}} \left.\frac{\partial A_n(\theta)}{\partial \theta}\right|_{\theta^*}}{\frac{1}{n} \left.\frac{\partial^2 A_n(\theta)}{\partial \theta^2}\right|_{\theta^*}}.$$

We have by the law of large numbers

$$\lim_{n\to\infty} \frac{1}{n} \left.\frac{\partial^2 A_n(\theta)}{\partial \theta^2}\right|_{\theta^*} = \int \left.\frac{\partial^2 g(x,\theta)}{\partial \theta^2}\right|_{\theta^*} f(x|\,\theta^*)\, dx$$

and by the central limit theorem

$$-\frac{1}{\sqrt{n}} \left.\frac{\partial A_n(\theta)}{\partial \theta}\right|_{\theta^*} \xrightarrow{\text{D}} \mathcal{N}\left(0, \int \left(\left.\frac{\partial g(x,\theta)}{\partial \theta}\right|_{\theta^*}\right)^2 f(x|\,\theta^*)\, dx\right)$$

as by definition of θ^*

$$\left.\frac{\partial M(\theta)}{\partial \theta}\right|_{\theta^*} = 0 = \int \left.\frac{\partial g(x,\theta)}{\partial \theta}\right|_{\theta^*} f(x|\,\theta^*)\, dx$$

so by Slutzky's theorem we have

$$\sqrt{n}\,(\theta_n - \theta) \xrightarrow{\text{D}} \mathcal{N}\left(0, \left(\int \left.\frac{\partial^2 g(x,\theta)}{\partial \theta^2}\right|_{\theta^*} f(x|\,\theta^*)\, dx\right)^{-2} \left(\int \left(\left.\frac{\partial g(x,\theta)}{\partial \theta}\right|_{\theta^*}\right)^2 f(x|\,\theta^*)\, dx\right)\right)$$