MIDTERM - STATISTICS 302 (Section 201)

March 9, 2010

Student Name (Please print): ....

Student Number: ....

Notes:

- Total points equal 100.
- Show the work leading to your solutions in the space provided. Indicate clearly the part of the problem to which the work relates.
- This is a closed book midterm.

[**25**] Problem 1: Two coins fall "heads up" with probabilities p_1 and p_2, respectively. Both coins are tossed.
[**12**] **(a)** What is the probability that they show the same face?
[**13**] **(b)** If they do show the same face, what is the probability that the face they both show is "heads"?

Answer to Problem 1

Answer to Problem 1 (continued)

[**25**] Problem 2: Suppose you have a fair coin and you toss it 10 times.
[**8**] **(a)** What is the probability to get exactly 5 heads?
[**8**] **(b)** What is the probability to get exactly 3 heads in the first 5 tosses *and* 5 heads overall?
[**9**] **(c)** What is the conditional probability that you got 3 heads in the first 5 tosses given that you got 5 heads overall?

Answer to Problem 2

Answer to Problem 2 (continued)

[**25**] Problem 3: In a study of causes of power failures, these data have been gathered: 4% involve transformer damage, 60% involve line damage, 1% involve both problems. Based on these percentages, find the probability that a power failure involves:

[**6**] **(a)** line damage given that there is transformer damage;

[**6**] **(b)** transformer damage given that there is no line damage;

[**6**] **(c)** tranformer damage or line damage;

[**7**] **(d)** neither transformer damage nor line damage.

Answer to Problem 3 (continued)

[**25**] Problem 4: Consider a discrete random variable X which takes on non-negative integer values $0, 1, 2, 3, \ldots$ For some constant α, $0 < \alpha < 1$, we have

$$P(X = i) = \alpha P(X = i - 1) \text{ for any integer } i \geq 1.$$

[**10**] **(a)** Establish that

$$P(X = 0) = 1 - \alpha.$$

(Hint: use the fact that $\sum_{i=0}^{\infty} \alpha^i = \frac{1}{1-\alpha}$ for $0 < \alpha < 1$).

[**10**] **(b)** Compute the expected value $\mu_X = E(X)$ as a function of α.

(Hint: use the fact that $\sum_{i=0}^{\infty} i\alpha^i = \frac{\alpha}{(1-\alpha)^2}$ for $0 < \alpha < 1$).

[**5**] **(c)** Consider the random variable $Y = \frac{(1-\alpha)^3}{\alpha} X + 1$. Compute the expected value $\mu_Y = E(Y)$.

<u>Answer to Problem 4</u>

Answer to Problem 4 (continued)