Painless Stochastic Gradient Descent: Interpolation, Line-Search, and Convergence Rates.

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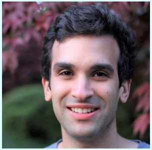
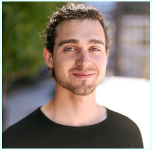



Stochastic Gradient Descent: Workhorse of ML?

"Stochastic gradient descent (SGD) is today one of the main workhorses for solving large-scale supervised learning and optimization problems."
—Drori and Shamir [8]

...and also Agarwal et al. [1], Assran and Rabbat [2], Assran et al. [3], Bernstein et al. [6], Damaskinos et al. [7], Geffner and Domke [9], Gower et al. [10], Grosse and Salakhudinov [11], Hofmann et al. [12], Kawaguchi and Lu [13], Li et al. [14], Patterson and Gibson [17], Pillaud-Vivien et al. [18], Xu et al. [21], Zhang et al. [22]

Motivation: Challenges in Optimization for ML

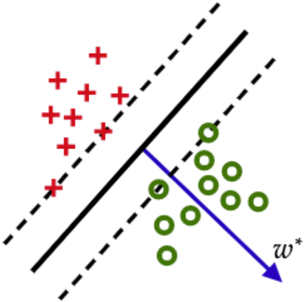
Stochastic gradient methods are the most popular algorithms for fitting ML models,

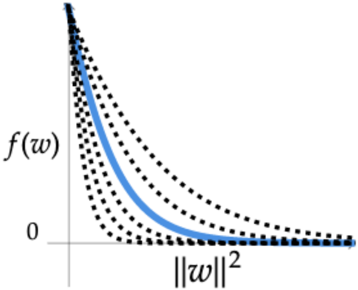
$$\textbf{SGD:} \quad w_{k+1} = w_k - \eta_k \nabla \tilde{f}(w_k).$$

But practitioners face major challenges with

- **Speed**: step-size/averaging controls convergence rate.
- **Stability**: hyper-parameters must be tuned carefully.
- **Generalization**: optimizers encode statistical tradeoffs.

Better Optimization via Better Models





Idea: exploit model properties for better optimization.

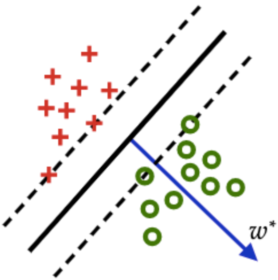
Interpolation

$$\textbf{Loss:} \quad f(w) = \frac{1}{n}\sum_{i=1}^{n} f_i(w).$$

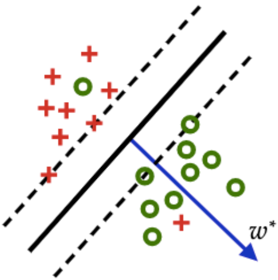
Interpolation is satisfied for f if $\forall w$,

$$f(w^*) \leq f(w) \implies f_i(w^*) \leq f_i(w).$$

Separable



Not Separable



Constant Step-size SGD

Interpolation and smoothness imply a **noise bound**,

$$\mathbb{E}\|\nabla f_i(w)\|^2 \leq \rho \left(f(w) - f(w^*) \right).$$

- SGD converges with a **constant step-size** [4, 19].
- SGD is (nearly) as **fast** as gradient descent.
- SGD converges to the
  - minimum L_2-norm solution for linear regression [20].
  - max-margin solution for logistic regression [16].
  - ??? for deep neural networks.

Takeaway: optimization speed and (some) statistical trade-offs.

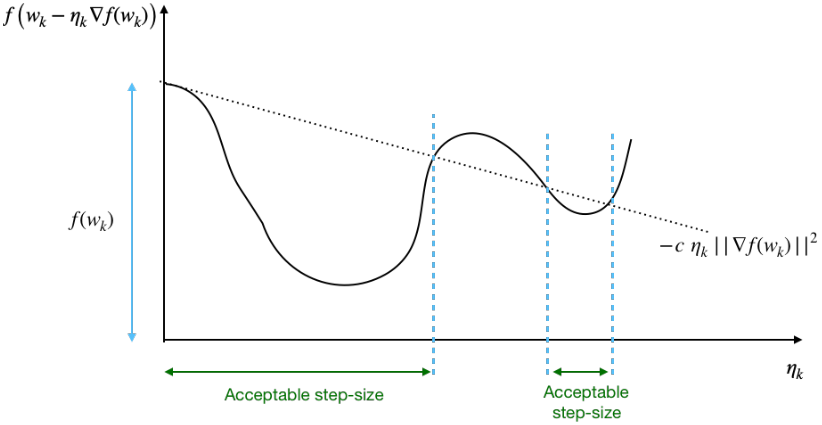
What about **stability** and **hyper-parameter** tuning?

Is grid-search the best we can do?

```
for i, step_size in enumerate(np.logspace(-4,1,12)):
    opt_params["step_size"] = step_size
    results[i] = run_experiment(opt_params, exp_params, data_params, model_fn,
                                objective, error_fn, training_set, test_set)
```

Painless SGD: Tuning-free SGD via Line-Searches

Stochastic Armijo Condition : $f_i(w_{k+1}) \leq f_i(w_k) - c\,\eta_k \|\nabla f_i(w_k)\|^2$.



Painless SGD: Stochastic Armijo in Theory

Theorem 1 (Strongly-Convex). *Assuming (a) interpolation, (b) L_i-smoothness, (c) convexity of f_i's, and (d) μ strong-convexity of f, SGD with Armijo line-search with $c = 1/2$ in Eq. 1 achieves the rate:*

$$\mathbb{E}\left[\|w_T - w^*\|^2\right] \leq \max\left\{\left(1 - \frac{\bar{\mu}}{L_{max}}\right), (1 - \bar{\mu}\,\eta_{max})\right\}^T \|w_0 - w^*\|^2.$$

Theorem 2 (Convex). *Assuming (a) interpolation, (b) L_i-smoothness and (c) convexity of f_i's, SGD with Armijo line-search for all $c > 1/2$ in Equation 1 and iterate averaging achieves the rate:*

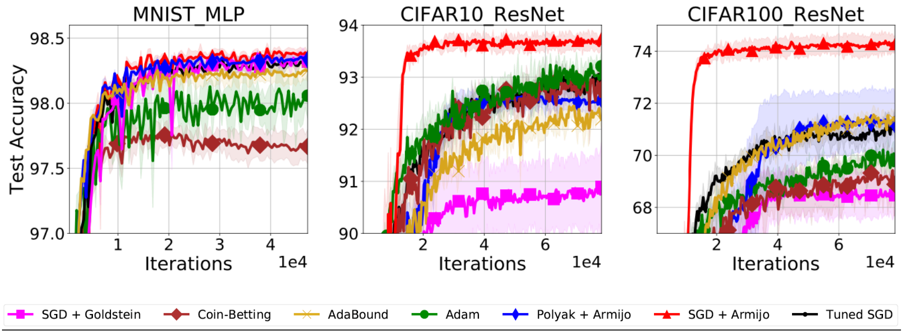
$$\mathbb{E}\left[f(\bar{w}_T) - f(w^*)\right] \leq \frac{c \cdot \max\left\{\frac{L_{max}}{2\,(1-c)}, \frac{1}{\eta_{max}}\right\}}{(2c-1)\,T} \|w_0 - w^*\|^2.$$

Theorem 3 (Non-convex). *Assuming (a) the SGC with constant ρ and (b) L_i-smoothness of f_i's, SGD with Armijo line-search in Equation 1 with $c = 1 - \frac{L_{max}}{4\rho L}$ and setting $\eta_{max} = \frac{2}{\sqrt{5}\rho L}$ achieves the rate:*

$$\min_{k=0,\ldots,T-1} \mathbb{E}\,\|\nabla f(w_k)\|^2 \leq \frac{10\rho L}{T}\,(f(w_0) - f(w^*)).$$

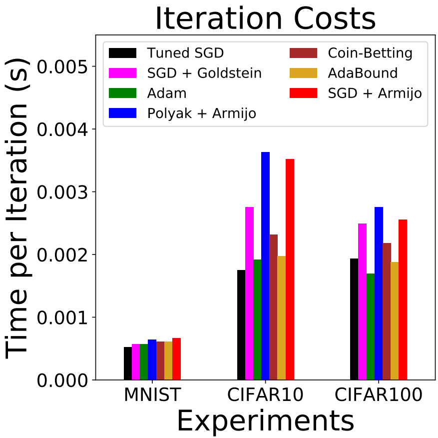
Painless SGD: Stochastic Armijo in Practice

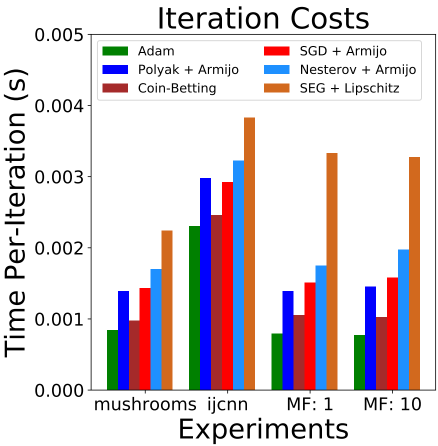
Classification accuracy for ResNet-34 models trained on MNIST, CIFAR-10, and CIFAR-100.



Painless SGD: Added Cost

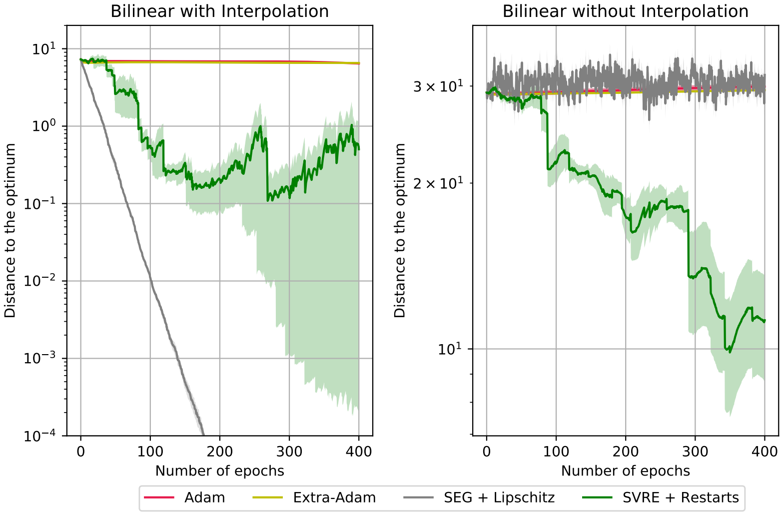
Backtracking is low-cost and averages once per-iteration.





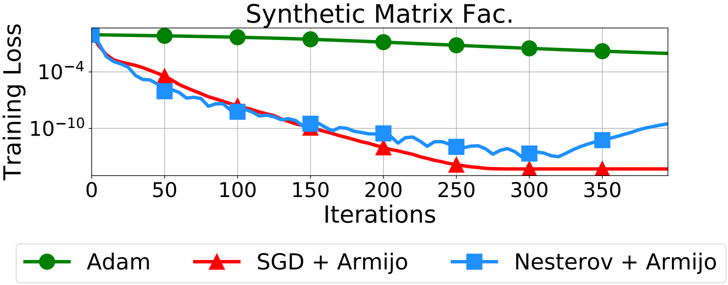
Painless SGD: Sensitivity to Assumptions

SGD with line-search is **robust**, but can still fail catastrophically.



Questions.

Bonus: Robust Acceleration for SGD



Stochastic acceleration is possible [15, 19], but

- it's **unstable** with the backtracking Armijo line-search; and
- the "momentum" parameter must be **fine-tuned**.

Potential Solutions:

- more sophisticated line-search (e.g. FISTA [5]).
- stochastic restarts for oscillations.

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