# Separating Reflection Functions for Linear Radiosity

ALAIN FOURNIER

Department of Computer Science, University of British Columbia
Vancouver, BC, V6T 1Z4, Canada. fournier@cs.ubc.ca

**Abstract:** Classic radiosity assumes diffuse reflectors in order to consider only pair-wise exchanges of light between elements. It has been previously shown that one can use the same system of equations with separable bi-directional reflection distribution functions (BRDFs), that is BRDFs that can be put in the form of a product of two functions, one of the incident direction and one of the reflected direction.

We show here that this can be easily extended to BRDFs that can be approximated by sums of such terms. The classic technique of Singular Value Decomposition (SVD) can be used to compute those terms given an analytical or experimental BRDF. We use the example of the traditional Phong model for specular-like reflection to extract a separable model, and show the results in term of closeness to ordinary Phong shading. We also show an example with experimental BRDF data. Further work will indicate whether the quality of linear radiosity images will be improved by this modification.

**Keywords:** BRDF, separable BRDF, global illumination, form factors, singular value decomposition.

## 1. Introduction

There is no need to repeat here the principles involved in computing global illumination through the *radiosity* method (we will use this synecdoche, since it is now well accepted and understood). We will use here the notation and terminology of Cohen & Wallace [1] whenever applicable. We will include within the class *linear pair-wise radiosity*, or *linear radiosity* for short, the methods which use an equation of the form:

$$B_i = E_i + R_i \sum_{j=1}^{N} B_j F_{ij} \tag{1}$$

Where $R_i$ is only a function of the element $i$ (the reflectance $\rho_i$ in the classic case), and $F_{ij}$ are only geometric functions of the pairs $ij$, the *form factors*. Our goal in this paper is to show how one can extend considerably the class of reflective behaviours which still lead to a linear radiosity solution.

## 2. Separable Models and Form Factors

The starting point for the radiosity equations is the *rendering equation* as given for example in [1]. The corresponding geometry is shown in Figure (1).

$$L(\mathbf{x}', \omega') = L_e(\mathbf{x}', \omega') + \int_S f_r(\mathbf{x}', \omega, \omega') L(\mathbf{x}, \omega) G(\mathbf{x}, \mathbf{x}') V(\mathbf{x}, \mathbf{x}')\, dA \tag{2}$$

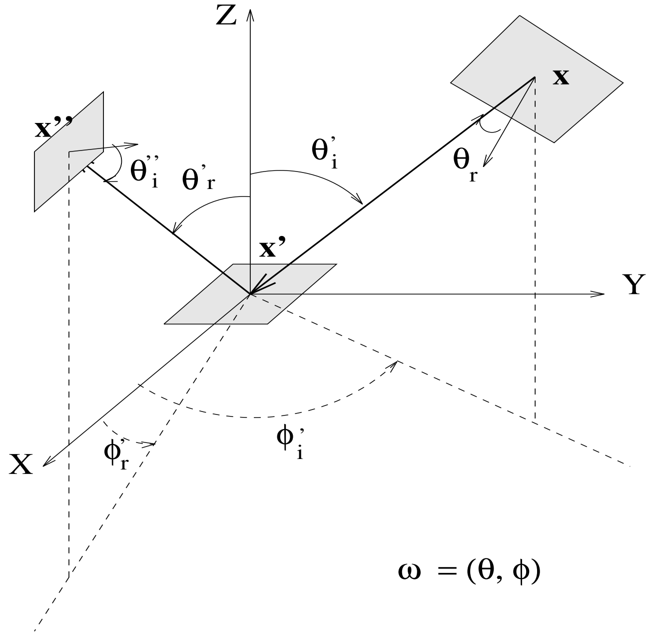


Figure (1). Geometry of 3-point transport

where $G()$ is a geometric factor:

$$G(\mathbf{x}, \mathbf{x}') = \frac{\cos\theta \ \ \cos\theta'}{|\mathbf{x} - \mathbf{x}'|^2} \tag{3}$$

and $V()$ is a visibility factor (1 if the points are mutually visible, 0 if not). Noting that for Lambertian reflectors we have:

$$f_r(\mathbf{x}', \omega, \omega') = \frac{\rho(\mathbf{x}')}{\pi}$$

and that the radiosity is $\pi$ times the radiance, one obtains:

$$B(\mathbf{x}') = E(\mathbf{x}') + \rho(\mathbf{x}') \int_S B(\mathbf{x}) \, \frac{G(\mathbf{x}, \mathbf{x}') V(\mathbf{x}, \mathbf{x}')}{\pi} \, dA \tag{4}$$

Note that it has been implicitly assumed that the light sources are Lambertian emitters as well, which is not really necessary. To obtain equation (1), one discretizes the surfaces into elements. Integrating over $A'$ for a finite area, one obtains an average radiosity for $A'$:

$$\bar{B}' = \frac{1}{A'} \int_{A'} B(\mathbf{x}') dA'$$

The integration over $S$ is an integration over $A$ for each of the elements. Using the subscript $i$ for primed quantities, and $j$ for un-primed, and summing for all $j$ elements, one obtains equation (1), if $B_j$ can be taken to be constant over the element $j$, where $P_i$ is $\rho_i$ and where the form factors $F_{ij}$ are:

$$F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{G(\mathbf{x}, \mathbf{x}')V(\mathbf{x}, \mathbf{x}')}{\pi} \; dA_j \; dA_j \tag{5}$$

It has been shown, notably by Neumann & Neumann [5] that we can still use the same system of equations if the BRDF, instead of being constant as a function of the directions, is *separable*, that is can be written as the product of two functions, one a function of the incoming direction $I(\mathbf{x}, \omega_i)$ and the other a function of the outgoing direction $O(\mathbf{x}, \omega_r)$. So that with the notation used before:

$$f_r(\mathbf{x}', \omega, \omega') = I(\mathbf{x}', \omega) \times O(\mathbf{x}', \omega')$$

One should note that if the BRDF is to obey reciprocity, $O()$ and $I()$ should be the same functions for each point within a constant, but this is not affecting any of the formulas which follow.

We start again with equation (4), and substitute the separable BRDF. Since $O(\mathbf{x}', \omega')$ is not a function of the integration variable in the integral over $S$, we can get it out of the integral, and obtain:

$$L(\mathbf{x}', \omega') = L_e(\mathbf{x}', \omega') + O(\mathbf{x}', \omega') \int_S I(\mathbf{x}', \omega) \; L(\mathbf{x}, \omega)G(\mathbf{x}, \mathbf{x}')V(\mathbf{x}, \mathbf{x}') \; dA \tag{6}$$

To transform this relation between radiances into a relation between radiosities or irradiances, one need the basic relation:

$$B = \int_\Omega L(\omega) \cos\theta \; d\omega$$

where the integral is over the hemisphere in our case. So if we integrate both sides of equation (6), we get:

$$\int_\Omega L(\mathbf{x}', \omega') \cos\theta_r{}' \; d\omega' = \int_\Omega L_e(\mathbf{x}', \omega') \cos\theta_r{}' \; d\omega'$$

$$+ \int_\Omega \cos\theta_r{}' \; d\omega' \; [O(\mathbf{x}', \omega') \int_S I(\mathbf{x}', \omega) \; L(\mathbf{x}, \omega)G(\mathbf{x}, \mathbf{x}')V(\mathbf{x}, \mathbf{x}') \; dA \;]$$

One can call:

$$E(\mathbf{x}') = \int_\Omega L_e(\mathbf{x}', \omega') \cos\theta_r{}' \; d\omega'$$

In other word $E(\mathbf{x}')$ is the radiosity due to light emission (note that the distribution can be anything we want which is integrable). We can also note that only $O(\mathbf{x}', \omega')$ is affected by the integral over $\Omega$, so it all simplifies to:

$$B(\mathbf{x}') = E(\mathbf{x}') + [\int_\Omega O(\mathbf{x}', \omega') \cos\theta_r{}' \; d\omega'] \times [\int_S I(\mathbf{x}', \omega) \; L(\mathbf{x}, \omega)G(\mathbf{x}, \mathbf{x}')V(\mathbf{x}, \mathbf{x}') \; dA \;] \tag{7}$$

We can define $SO(\mathbf{x}') = \int_{\Omega} O(\mathbf{x}', \omega') \cos\theta_r{}' \, d\omega'$, in effect a normalization of $O(\mathbf{x}', \omega')$.

In general for any $\mathbf{x}$ and $\omega$ we can write:

$$L(\mathbf{x}, \omega) = K \times O(\mathbf{x}, \omega)$$

for some positive constant $K$ (since $O()$ is the function giving the distribution of the reflected light at $\mathbf{x}$), and multiplying both sides by $\cos\theta \; d\omega$ and integrating:

$$B(\mathbf{x}) = K \times SO(\mathbf{x})$$

The ratio of these two equations gives the relationship between $L(\mathbf{x}, \omega)$ and $B(\mathbf{x})$:

$$L(\mathbf{x}, \omega) = \frac{B(\mathbf{x}) \times O(\mathbf{x}, \omega)}{SO(\mathbf{x})}$$

We can use this to replace $L(\mathbf{x}, \omega)$ in equation (7).

All told, we then obtain:

$$B(\mathbf{x}') = E(\mathbf{x}') + SO(\mathbf{x}') \int_{S} B(\mathbf{x}) I(\mathbf{x}', \omega) \, \frac{O(\mathbf{x}, \omega)}{SO(\mathbf{x})} \, G(\mathbf{x}, \mathbf{x}') V(\mathbf{x}, \mathbf{x}') \, dA \tag{8}$$

which is the new version of equation (4).

We can now discretize (8) as we did for (4). Integrating over $A$ for each element, averaging $B(\mathbf{x})$ over $A'$ and summing over all elements:

$$\frac{1}{A'} \int_{A'} B(\mathbf{x}') dA' = \frac{1}{A'} \int_{A'} E(\mathbf{x}') dA'$$

$$+ \sum_{\textit{all elements}} \frac{1}{A'} \int_{A'} SO(\mathbf{x}') dA' \int_{A} B(\mathbf{x}) \; I(\mathbf{x}', \omega) \, \frac{O(\mathbf{x}, \omega)}{SO(\mathbf{x})} \, G(\mathbf{x}, \mathbf{x}') V(\mathbf{x}, \mathbf{x}') \, dA$$

Using the same indices as before, and the same assumption about $B(\mathbf{x}) = B_j =$ constant over $A$, one gets:

$$B_i = E_i + SO_i \sum_{j=1}^{N} B_j \, \frac{1}{A_i} \int_{A_i} \int_{A_j} I_i(\omega) \, \frac{O_j(\omega)}{SO_j} \, G(\mathbf{x}, \mathbf{x}') V(\mathbf{x}, \mathbf{x}') \, dA_i dA_j = E_i + SO_i \sum_{j=1}^{N} B_j F_{ij}$$

where the new form factors are:

$$F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} I_i(\omega) \, \frac{O_j(\omega)}{SO_j} \, G(\mathbf{x}, \mathbf{x}') V(\mathbf{x}, \mathbf{x}') \, dA_i dA_j \tag{9}$$

Notice that the relation: $F_{ij} \times A_i = F_{ji} \times A_j$ always valid for diffuse reflectors, is here only true if $O_j(\omega) = I_i(\omega)$ and $SO_i = SO_j$. For different materials this is not going to be true, and even for the same material, first reciprocity has to apply, and since the same $\omega$ corresponds to different direction in $i$ and $j$ frames of references, this would be true only for special cases.

## 3. Sum of Separable Models

The basic problem with separable models is that very few real surfaces exhibit behaviour compatible with a straight separable model[1]. Neumann & Neumann presented an ingenious exception, the *lacquer* model, where a Lambertian reflector was covered by a layer of absorptant material (which obey Beer's law, but that is not necessary). This model, however, when implemented, gives a reflecting behaviour quite unsatisfactory compared to most real materials (see paper by Lewis [4] ).

Let us see what happens if we represent the BRDF as a *sum* of $M$ separable components. The equation is now at point $\mathbf{x}'$:

$$f_r(\mathbf{x}', \omega, \omega') = \sum_{m'=1}^{M'} I_{m'}(\mathbf{x}', \omega) \times O_{m'}(\mathbf{x}', \omega')$$

Similarly at point $\mathbf{x}$:

$$f_r(\mathbf{x}, \omega, \omega') = \sum_{m=1}^{M} I_m(\mathbf{x}, \omega) \times O_m(\mathbf{x}, \omega')$$

Substituting in equation (2), we have now:

$$L(\mathbf{x}', \omega') = L_e(\mathbf{x}', \omega') + \int_S [\sum_{m'=1}^{M'} I_{m'}(\mathbf{x}', \omega) \times O_{m'}(\mathbf{x}', \omega')] L(\mathbf{x}, \omega) G(\mathbf{x}, \mathbf{x}') V(\mathbf{x}, \mathbf{x}') \, dA \quad (10)$$

The integral over $S$ can now be separated into $M'$ integrals of the form:

$$\int_S I_{m'}(\mathbf{x}', \omega) \times O_{m'}(\mathbf{x}', \omega') L(\mathbf{x}, \omega) G(\mathbf{x}, \mathbf{x}') V(\mathbf{x}, \mathbf{x}') \, dA$$

In each of these integrals $O_{m'}(\mathbf{x}', \omega')$ is not dependent on $A$ and therefore can be removed from the integral:

$$O_m(\mathbf{x}', \omega') \int_S I_m(\mathbf{x}', \omega) \times L(\mathbf{x}, \omega) G(\mathbf{x}, \mathbf{x}') V(\mathbf{x}, \mathbf{x}') \, dA$$

We can now as before multiply by $\cos \theta_r' d\omega'$ and integrate:

$$B(\mathbf{x}') = E(\mathbf{x}') + \sum_{m'=1}^{M'} [SO_{m'}(\mathbf{x}') \int_S I_{m'}(\mathbf{x}', \omega) \; L(\mathbf{x}, \omega) G(\mathbf{x}, \mathbf{x}') V(\mathbf{x}, \mathbf{x}') \, dA]$$

We can also replace $L(\mathbf{x}, \omega)$ by:

$$L(\mathbf{x}, \omega) = B(\mathbf{x}) \frac{\sum_{m=1}^{M} O_m(\mathbf{x}, \omega)}{\sum_{m=1}^{M} SO_m(\mathbf{x})}$$

which gives;

$$B(\mathbf{x}') = E(\mathbf{x}') + \sum_{m'=1}^{M'} [SO_{m'}(\mathbf{x}') \int_S B(\mathbf{x}) \; I_{m'}(\mathbf{x}', \omega) \frac{\sum_{m=1}^{M} O_m(\mathbf{x}, \omega)}{\sum_{m=1}^{M} SO_m(\mathbf{x})} G(\mathbf{x}, \mathbf{x}') V(\mathbf{x}, \mathbf{x}') \, dA]$$

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1. Of course Lambertian reflectors are also rather rare, but you have to start somewhere.

Again using the indices $i$ and $j$, and replacing by the element averages (we will use $m$ and $n$ for the indices of the separable elements for element $i$ and $j$, respectively):

$$B_i = E_i + \sum_{m=1}^{M} SO_{im} \sum_j B_j \frac{1}{A_i} \int_{A_i} \int_{A_j} I_{im}(\omega) \frac{\sum_{n=1}^{N} O_{jn}(\omega)}{\sum_{n=1}^{N} SO_{jn}} G(\mathbf{x},\mathbf{x}')V(\mathbf{x},\mathbf{x}')\ dA_i dA_j$$

This is equivalent to having $M \times N$ pairs $ij$, with the pair equation:

$$B_{im} = E_{im} + SO_{im} \sum_j B_j \frac{1}{A_i} \int_{A_i} \int_{A_j} I_{im}(\omega) \frac{O_{jn}(\omega)}{\sum_{n=1}^{N} SO_{jn}} G(\mathbf{x},\mathbf{x}')V(\mathbf{x},\mathbf{x}')\ dA_i dA_j \tag{11}$$

or:

$$B_{im} = E_{im} + SO_{im} \sum_j B_j F_{im,jn}$$

The constraint on $E_{im}$ is $E_i = \sum_{m=1}^{M} E_{im}$. Of course putting for example $E_{i0} = E_i$ and all others to 0 will do. We have now $M \times N$ form factors:

$$F_{im,jn} = \frac{1}{A_i} \int_{A_i} \int_{A_j} I_{im}(\omega) \frac{O_{jn}(\omega)}{\sum_{n=1}^{N} SO_{jn}} G(\mathbf{x},\mathbf{x}')V(\mathbf{x},\mathbf{x}')\ dA_i dA_j$$

We can modify the equations to obtain a slightly better form, by multiplying $B_j$ by $\frac{SO_{jn}}{SO_{jn}}$ in equation (11). Denoting:

$$B_{jn} = \frac{B_j\ SO_{jn}}{\sum_{n=1}^{N} SO_{jn}}$$

we can write:

$$B_{im} = E_{im} + SO_{im} \sum_j B_{jn} F_{im,jn}$$

where the form factor is modified to be:

$$F_{im,jn} = \frac{1}{A_i} \int_{A_i} \int_{A_j} I_{im}(\omega) \frac{O_{jn}(\omega)}{SO_{jn}} G(\mathbf{x},\mathbf{x}')V(\mathbf{x},\mathbf{x}')\ dA_i dA_j$$

and therefore much more similar to the one given in equation (9).

Now the important practical questions are: for the BRDFs that one might actually want to use, how well can we separate them, and what is the size of $M$ and $N$? In effect we multiply the size of the problem by the average of $M \times N$ for all the pairs of elements. The additional cost of computing form factors is not that bad, since the visibility function is the same for all pairs $im$, $jn$. One must however, keep in mind that the alternatives can be much worse, from considering all triples of patches in the full solution, which multiplies the number of form factors by $n$ and the size of the solution by $n^2$ [3] to using various basis functions for input or output radiosity as for instance in Sillion *et al* (up to 200 coefficients) [7].

There are various ways to achieve an analytical sum of product decomposition of functions, from Taylor series expansion to spherical harmonics, but there is a numerical approach that has the advantage of being simple, almost universal, optimal under some conditions, and applicable to experimentally-measured BRDFs as well.

## 4. Singular Value Decomposition

Singular value decomposition (SVD) is a technique which given $\mathbf{A}$ a $P \times Q$ matrix[2], $P \geq Q$, decompose it into the product of three matrices:

$$\mathbf{A} = \mathbf{U} \;\; \mathbf{W} \;\; \mathbf{V}^T$$

where $\mathbf{U}$ is a $P \times Q$ column orthonormal matrix , $\mathbf{V}$ is a $Q \times Q$ column (and row) orthonormal matrix, and $\mathbf{W}$ is a $Q \times Q$ diagonal matrix. This decomposition is unique within row permutations. For more details see Golub [2] and for code see *Numerical Recipes in C* [6]. The result of SVD can be seen as writing each element of $\mathbf{A}$ as a weighted sum of the rows of $\mathbf{U}$ and the columns of $\mathbf{V}^T$, where the weights are the diagonal elements of $\mathbf{W}$:

$$A_{ij} = \sum_{k=1}^{Q} W_{kk} \times U_{ik} \times V_{jk} \tag{12}$$

If we take the $M$ largest elements (in magnitude) of $\mathbf{W}$, we then obtain an approximation of $\mathbf{A}$ with $M$ products per element. The SVD guarantees that the result will be optimal in the least-square sense for this number of terms.

The technique applies to functions in the following way. Given a function $f(x, y)$, we chose $P$ values $x_i$ of the abscissa and $Q$ values $y_j$ of the ordinate. The elements of the matrix $\mathbf{A}$ are:

$$A_{ij} = f(x_i, y_j)$$

Clearly each row of $\mathbf{A}$ corresponds to samples at constant $x$, and each column to samples at constant $y$. After applying SVD to $\mathbf{A}$, equation (12) means that all the samples can be approximated by the sum of the weighted products of $M$ elements of a row $\mathbf{U}$ and $M$ elements of a row of $\mathbf{V}$. These rows are only functions of $x$ and $y$ respectively. We can then fit a function to each relevant row of $\mathbf{U}$, $g_k(x)$ such that $g_k(x_i) = U_{ik}$ and fit a function to each relevant row of $\mathbf{V}$, $h_k(y)$ such that $h_k(y_j) = V_{jk}$ and we have:

$$f(x, y) = \sum_{k=1}^{M} W_k k \;\; g_k(x) \times h_k(y)$$

In other words we have expressed $f(x, y)$ as a sum of separated terms.

The samples have to be chosen so that more weight is given to ranges that are important for the function at hand. The error made comes from both the SVD and the fit with the basis functions used. In both cases these errors are known from their

2. Usually the dimensions are noted $M \times N$, but we do not want to promote confusion with the number of separate factors in the BRDF.

respective methods. In our case we want to take a function of four variables and separate it into sum of products of two two-variable functions. Fortunately the method is very much the same. We chose $P$ pairs $(\theta_i \phi_i)$ for the incoming direction and $Q$ pairs $(\theta_j \phi_j)$ for the outgoing direction. direction. For each tuple of pairs, we compute:

$$A_{ij} = f_r(\theta_i, \phi_i, \theta_j, \phi_j)$$

Again the pair should be chosen according to the function at hand. In practice for functions known analytically, we picked random direction whose projections on the unit circle are uniformly distributed. The justification is that according to the Nusselt analog they represent a uniform distribution in the diffuse form factor.

For experimental data, one should obtain or extract by interpolation a $P \times Q$ array of values.

## 5. Decomposition of Phong Model

To test the practicality of this solution, we made a preliminary study for an analytical formulation, namely *Phong* shading. We used a reciprocal version of Phong shading [4] for which the BRDF has the simple form:

$$f_r(\omega, \omega') = k\ (\cos\alpha)^n$$

where $\alpha$ is the angle between the bisector of $(\omega, \omega')$ and the normal to the surface (the $Z$ axis in its local frame). Of course $\cos\alpha$ is known as $\mathbf{N}.\mathbf{H}$ in the trade; this is the "Blinn" modified version of Phong. We chose this because it is popular, used by most renderers, adjustable to high glossiness through the value of $n$, and a difficult case. The only redeeming value here is that it is isotropic. The process was to take a user-defined number of samples in incoming and outgoing directions (the size of the matrix), an exponent $n$, and the number of terms $t$ to be used in the approximation. Once the SVD was done, the first $t$ rows of $\mathbf{U}$ and the first $t$ rows of $\mathbf{V}$ were fitted to a bicubic equation (16 coefficients) by least square (actually using SVD again). The choice of bicubic is quite arbitrary, and we have not yet explored better basis functions. As mentioned above, each direction is chosen randomly, with projections uniformly distributed over the unit circle.

| $n$ | Size $P = Q$ | Number of Terms | RMS after SVD | Max $\Delta$ | RMS after fit | Max $\Delta$ |
|---|---|---|---|---|---|---|
| 1 | 40 | 5 | 0.011 | 0.087 | 0.019 | 0.17 |
| 1 | 40 | 10 | 0.00088 | 0.00095 | 0.016 | 0.14 |
| 4 | 40 | 5 | 0.029 | 0.27 | 0.032 | 0.27 |
| 4 | 40 | 10 | 0.0046 | 0.041 | 0.016 | 0.077 |
| 8 | 40 | 5 | 0.050 | 0.55 | 0.052 | 0.55 |
| 8 | 40 | 10 | 0.0091 | 0.096 | 0.019 | 0.12 |
| 32 | 40 | 5 | 0.088 | 0.91 | 0.10 | 0.92 |
| 32 | 40 | 10 | 0.039 | 0.40 | 0.073 | 0.34 |
| 32 | 40 | 15 | 0.013 | 0.080 | 0.069 | 0.33 |

The table shows the results for a range of exponents and number of terms (separate components $M$) kept. As expected, for small exponents the BRDF is easy to separate, and is approximated within a few percent with 5 terms. Even when $n = 8$, 10 terms give a 2% accuracy. For an exponent of 32, 15 terms are necessary to do a good job. Notice that the fit of the separated functions is sometimes responsible for most of the RMS error. As mentioned before we have not yet spent much time on this aspect of the fit. Note that the maximum difference between computed and fitted values can be alarmly high. We suspect that more weight has to be put on the direction pairs causing a specular highlight. Also the fit used here did not use isotropy of the input or force isotropy of the result.

Of course the numbers here are not telling the whole story. The symmetry, or lack thereof, of the recovered functions can be important in many cases. What we have to offer for now is some images computed with the separated BRDFs. Figure (2) shows spheres shaded by a built-in Phong shader (non-reciprocal), a reciprocal interpreted one (for sanity check), and a diffuse surface. The specular surfaces have a small ambient term added (0.1). The eye is at (0, -20, 0), the balls all around the origin, and there are two directional light sources, one at (0.4, -0.9, 1) and the other at (-0.87, 0.5, 1). One can see that the separated functions (in this case with $n = 8$ and 5 terms selected) do a decent specular surface. Remember that you can put this BRDF into your favourite radiosity-based renderer at very little added cost.

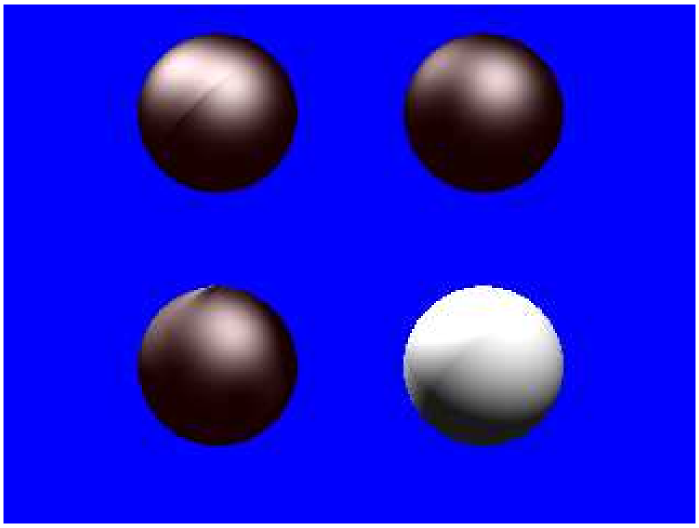

Figure (2): Phong model, original and reconstructed, with two lights. Upper left: non-reciprocal Phong, upper right: reciprocal Phong, lower left: separated reciprocal Phong, lower right: diffuse.

### 6. Decomposition of Experimental Data

As a quick test we used Greg Ward's experimental data [8]. It came in two flavours, a raw data file with 27330 experimental pairs of directions (file `g50bw.brdf`), and a "filtered" one with 2647 data pairs (file `g50bw.rdu`). To use this data we inserted the values into a quad tree where the node are $(\theta, \phi)$ bins, and split the nodes when they had too many data points. At the end the nodes with enough data points (typically 40 to 50) were used. The centre pair value of each node was the sample values used for building the matrix. The reflectance values were determined by interpolation for these samples. Usually a $50 \times 50$ matrix is output. The following table show some results.

| Data Points $P = Q$ | Size of Terms | Number after SVD | RMS $\Delta$ | Max after fit | RMS $\Delta$ | Max |
|---|---|---|---|---|---|---|
| 2647 | 50 | 5 | 0.021 | 0.29 | 0.031 | 0.30 |
| 2647 | 50 | 10 | 0.011 | 0.083 | 0.029 | 0.25 |
| 27330 | 50 | 5 | 0.033 | 0.35 | 0.18 | 5.2 |
| 27330 | 50 | 10 | 0.019 | 0.35 | 0.18 | 5.2 |

It can be noted that the filtered data works well, when the raw data fails. In fact in the latter case the situation does not improve with more terms. Better use of these experimental data is tied in with reconstruction from irregularly sampled data, a topic we are currently investigating.

## 7. Conclusions

We have shown that expressing BRDFs as sums of separable functions leads to a simple implementation within linear radiosity systems. We have also shown how singular value decomposition helps us produce these sums from analytical or experimental data. The experiments with Phong BRDF and Greg Ward's experimental data are promising, but there is still a lot to explore in order to fine tune the approach. The next step, obviously, is to incorporate this into a linear radiosity system, and we have done so starting with *rad*, the program from S.N. Pattanaik (images to follow).

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