Bringing Mathematical Research To Life in the Schools

by
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Bringing Mathematical Research To Life in the Schools[1]

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1. INTRODUCTION

This paper is concerned with giving school children an understanding and appreciation of doing research in mathematics. My basic thesis is that this is something that needs to be done, and something that can be done. The arguments for why it needs to be done include making mathematics more enjoyable and meaningful for students, encouraging students to pursue careers in the mathematical sciences, and building societal awareness, appreciation and support of the contributions of mathematical research. The techniques I propose for achieving this goal include giving children exposure to research mathematicians and their work, building connections between mathematical research and the activities children enjoy most, and having children do hands-on activities in which they explore accessible mathematical ideas and concepts. My ideas stem primarily from my experiences giving presentations in school classes on mathematics and being a mathematician. Much of the paper is devoted to concrete descriptions of presentations as well as the underlying philosophy behind them.

Since this is a paper motivated by my personal experiences, I will start by saying a few things about myself and my background to help place my ideas in context. Next, I discuss in more detail why I think it important to have school children (and their teachers and parents) understand and appreciate doing research in mathematics. I then describe the general framework I use to present mathematics and some specific examples of presentation modules I use in the schools. I also include a collection of miscellaneous tricks and tips I've accumulated while giving presentations. The final section discusses how these ideas and strategies can be extended to have a broader impact, as well as a proposal for a complementary approach based on video games.

1.1 About Me

I am a research mathematician working at the interface of discrete mathematics and theoretical computer science. I also spend a significant amount of time on administrative matters, having been the Head of my department for the last four years and a manager in the IBM Research Division for the four years before that. I am the product of Canadian universities, primarily the University of Alberta. At this point I have spent roughly half my working life in an industrial research lab in California, and the other half in universities, mostly in Canada. I am married to a mathematician/computer scientist, Nick Pippenger, and we have two children, a boy aged 10 and a girl 7. I have no formal training in mathematics education, though I have benefitted greatly from discussions with teachers and specialists in mathematics education. I have been visiting schools for

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over 15 years, first as a speaker about careers in mathematics for women, then as a parent volunteer in my children's mathematics and science classes, and most recently as a participant in the Province of British Columbia's Scientists in the Schools program . This program matches volunteer scientists with interested schools, and covers the expenses for scientists to give school presentations throughout the province[2]. Through this and similar activities I spend about a day per month working with school children.

1.2 Rationale And Objectives

In North America there is widespread agreement among politicians and educators on the importance of increasing interest in careers in science and engineering, especially among women and under-represented minorities (see [G], [R]. [E], [BH] etc.). In past decades many of the efforts were aimed at women and minorities, with fairness and equity as the primary reasons. Today, the primary justification is economic, with the realization that industrial competitiveness depends on the existence of a broad base of technical knowledge and skills within the population. There continues to be strong emphasis on women and minorities, partly because of the belief that there is a higher percentage of under-utilized talent in these populations.

In talking to high school students in both Canada and the United States I have found that many students view careers in science and engineering quite negatively. Students are frequently of the opinion that such careers are narrow, boring, demanding, difficult, not prestigious, not people-oriented, under-paid with respect to the degree of commitment and talent needed, and that these careers preclude having a satisfying family life. In addition they often blame science and engineering for creating the technology that led to problems with the environment. Student opinions of subject matter in school mathematics and science courses are not much better - boring, too difficult, and irrelevant are common criticisms. Mathematics is of particular importance for two reasons. First, the level of abstraction and the lack of knowledge about mathematicians and why mathematics is important makes mathematics more likely to be viewed as too difficult and/or irrelevant. Second, because of its integral role in most areas of science and engineering, fear, dislike and poor understanding of mathematics courses is a common barrier to pursuing a technical career. Obviously not all students dislike mathematics and science. Some high school students love these subjects and go on to pursue highly successful careers in these areas. The problem is that not enough students feel this way. Moreover, the peer culture is generally not supportive of students who work hard and/or do well in mathematics and science subjects. This discourages many from continuing their studies in these fields.

I spend time in the schools because having research mathematicians (and other scientists and engineers) communicate what they do, why they do it, and why they love doing it, can be an important part of the solution to the problem described above. I also do it because I enjoy the challenge of trying to make mathematics understandable to children, and I appreciate the new insights this effort brings me. Perhaps the biggest reason I do it is because it is such fun.

[2]For information contact SCIENCE WORLD BC, 1455 Quebec St. Vancouver BC Canada V6A 3Z7

My goal is to leave all the students (not just the mathematically talented ones) with the following set of beliefs:

1. Doing mathematics can be a lot of fun
2. Being a mathematician is GREAT in terms of interest, excitement, flexibility and salary
3. People that work hard become good at mathematics
4. Using and creating mathematics is important for the future of our society

Reaching **all** the students with my presentation is obviously important to achieving my objectives of making mathematics more enjoyable and meaningful for students, and the long-term building of societal appreciation for mathematical research. In addition I believe we will not see significant progress in attracting more good students to math, science and engineering until the peer support is there. Consider the difference in peer support for careers in mathematics and science versus professional sports careers. Both kinds of careers require a combination of many years of hard work, innate talent and luck. Both offer many rewards. The chances of achieving and sustaining a successful career are greater in a scientific field than in professional sports, and one could reasonably argue that the productive scientist does more for society than the successful athlete. Yet most North American children admire and support peers who succeed at sports but fail to appreciate (and even scorn) those who succeed in mathematics and science. Another interesting comparison is the strength of peer support for choosing careers in medicine and law versus those in mathematics, science or engineering, despite the similarities in training requirements and chances of success.

There are several factors that may account for these differences in attitudes. First, North American culture glorifies careers in sports, medicine and law. Consider, for example, the number of television shows and movies on sports heroes, doctors, and lawyers, compared to the number on mathematicians or other scientists. Second, children's opinions are influenced by their direct experiences. Children play sports (usually a pleasurable experience), go to doctors (whom they experience as being in a position of power and prestige), but rarely encounter mathematicians. Indeed, most children think of a mathematician as a person who does arithmetic all day. Third is the difference in earning levels of moderately successful professional athletes, doctors and lawyers compared to mathematicians and scientists (though at the moment, particularly in Canada, the pay gap between lawyers and academic mathematicians and scientists is probably more perception than reality). As individual mathematicians, it is difficult for most of us to affect the first and third factors, i.e. to change the media image of mathematicians, or to substantially raise our salaries. However, it is within our power to affect the second, namely to give children direct experiences of what mathematical research and researchers are like.

2. PRESENTATION FRAMEWORK

2.1 Underlying Philosophy

The design of all components of my presentations are based on a number of beliefs and objectives. I try to make all elements as interactive as possible to sustain student involvement and interest. My basic format is to ask the class questions whose answers lead to the discovery of the information I want to communicate. When necessary I elaborate on student answers to cover additional points. I present a handful of modules on loosely related topics rather than concentrating on a single theme. This makes it easier to ensure that some elements emphasize the importance of mathematical research to other aspects of our society, while some emphasize the enjoyment aspects of mathematical exploration. I stress the interdisciplinary and people oriented aspects of being a mathematician.

I relate as much material as possible to popular activities such as sports, video games, creative arts, adventure, and puzzles. For example, I often tell them that I have thought a lot about what kids of their age do that is most like being a researcher in mathematics, and that I have finally come up with the answer. I ask them to guess, and then I tell them my answer - playing video games. This is generally a surprise to them and their teacher. I ask them to think about what it's like to start playing a new video game. The answers I am looking for are that it's like exploring a new world full of puzzles and challenges; that the way one masters a new game is to figure out what combination of moves and tricks work in various situations and then to apply them to other situations; and that success depends not only on developing basic skills, but on thinking through problems, on experimenting, and on talking to other players and consulting books. Some of the basic skills one uses in being a mathematical researcher are different, but the rest of the process is very similar. In particular, the problem solving techniques, the exhilaration of success, and extended periods of intense concentration and absorption are amazingly alike.

There are many areas and aspects of mathematical research that cannot be presented in this manner. Fundamental and significant results often involve sufficiently sophisticated and abstract concepts that they are difficult to communicate to other mathematicians, let alone ten year-olds. Nevertheless, the image presented is accurate about a substantial fraction of mathematical research and researchers, and is analogous to the images school children have of other careers.

2.2 Format

Most of my presentations are from 75 - 90 minutes in length and are to classes of about 30 students aged 10 -12. Research in British Columbia has shown that this is the critical age range to reach students in terms of maintaining their interest in math and science [BH]. I also speak to audiences of other ages using the same material with minor adjustments. My audiences are usually very mixed with respect to mathematical interest and talent. My presentation consists of several parts: scene-setting questions, an auto-introduction, and two or three subject modules. A juggling lesson is often included for a brief change of pace at some point.

2.3 Scene-Setting Questions

I start my presentation by asking the class a series of questions of the form "How many people like x?" (and a few of the form "How many people absolutely hate x?"). Students express their opinions by raising their hands. As well as giving me valuable information about my audience, and setting the stage for my presentation, this has the advantage of getting everyone immediately involved in the presentation since the questions are chosen so that each student will respond positively to at least one question. A typical series of questions is shown in Figure 1(a). After this I point out that everyone liked at least one of the things I mentioned, and that I'm going to show them that there's something like each of these things in being a research mathematician.

(a) Scene-Setting Questions	(b) Who am I?
How many people love mathematics?	Maria Klawe
How many people absolutely hate mathematics?	Professor and Head, Dept. of Computer Science, UBC
How many people think mathematics is OK but not great?	Researcher discrete mathematics, theoretical computer science, computational geometry
How many people love science?	Teacher
How many people love doing art?	Mother
How many people love doing sports?	Slightly Crazy
How many people love playing video games?	Runner
How many people love solving puzzles?	Kayaker, Artist, ...
How many people love designing clothes?	

Figure 1.

2.4 Auto-Introduction

The goal of this section is to emphasize the 'mathematician as human-being' view. I tell the class that I'm going to start by telling them something about myself because one of the reasons I am there is so that they get to meet a real live mathematician, and put up my introductory transparency (Figure 1(b)). We then explore each of the items on the transparency. Using this approach has two advantages. It saves the teacher from having to introduce me, and it allows me to make the introduction interactive. Here's how a typical session goes.

Maria Klawe: I start with my name, explaining that everyone in the world, except my children, calls me Maria because it's hard to both spell and pronounce my last name correctly (Klawe rhymes with Ave as in Ave Maria). Among other things this lets the children know that I want them to call me Maria.

Professor and Head of the Department of Computer Science at UBC: At this point I ask "Who knows what UBC is?" Usually several children know that UBC stands for the University of British Columbia, and this leads us into a discussion of what a university is. Typical questions I ask include: "what is a university, why do people go to universities, when do people go to university, and who here thinks they will go to university". I draw analogies between professors and teachers, my department and their school, the role of the Head and their principal. I also talk about the difference between undergraduate and graduate students.

Researcher (discrete mathematics, theoretical computer science, computational geometry): I ask what it means to do research? Typically answers begin along the lines of "to study things" or "to look things up in the library to find out more information about a subject". I respond "Yes, that's one part of doing research, but can someone tell me others?" I am looking for answers that incorporate the ideas of discovery, exploration, creation. I explain that we are going to explore some of the areas in computational geometry and discrete mathematics that I work in. I ask what geometry means. Generally someone tells me it has to do with shapes. I ask them to guess what computational geometry means, and of course someone correctly guesses that it must have something to do with computers and geometry. This gives me a chance to say something about it, i.e. computational geometry concerns mathematical tools to help computers do tasks that involve shapes.

Teacher: This part doesn't need much explanation since they know what it means to be a teacher. At most I talk about the difference between teaching a class and teaching someone to be a researcher as one does in supervising a graduate student.

Mother: I ask the class why I put this on the transparency. Some of my favourite answers are: "You are proud of your children." "Your children drive you crazy (in reference to the next entry on the transparency)." "You want us to know that you have children so that we will think you will know how to talk to us." "You are looking for a husband and you think that someone would be more likely to marry you if you've already shown that you can have children." I answer "yes, but it's not the reason" to all except the last, to which I say that I already have the most wonderful husband in the universe (it's true, too). I then explain that the reason is that I want to destroy a myth. I say :

> "When I was your age (thirty years ago) there was a myth that girls (and women) couldn't do math and science. We now know that this myth was false, since there are many successful women mathematicians, scientists, and engineers. Now there is a new myth which is also false. The new myth is that women can't have both an ambitious career as a mathematician, scientist, or engineer, as well as enjoying and doing a good job as a mother. It's hard work to do both, but it's always hard work to be a good parent. In fact it's hard work to anything well, including being a child. Most of the women mathematicians and computer scientists that I know have children, and love being a mother as well as having their career. My own children are now 10 and 7 and I'm totally crazy about them, though they are, on occasion, unbelievably awful - but mostly they are unbelievably wonderful. The really important ingredients for success in combining motherhood and a career are having a husband who fully shares the responsibilities of being a parent, having a job that pays enough to afford good child-care, and having a job with enough flexibility to allow for spending more time with your children at critical times. Most careers in mathematics, science and engineering are pretty good with respect to the last two criteria."

Slightly crazy: I tell the class they have already probably figured this one out, but if they haven't, they're sure to do so during the rest of the presentation.

Runner: I reveal the fact that I run marathons (which explains part of the slightly crazy entry). Sometimes I ask how long a marathon is, or how many people have participated in a race. This year I have been announcing that I won my age-group division in the Vancouver marathon. The level of applause that greets this piece of information continues to amaze me. The first time I mentioned this was at a presentation a few days after the race. I mentioned it because I was still consumed with excitement about my performance. After I saw how much that class loved it, I decided to make it part of my regular routine. Sometimes I use this opportunity to tell the students how physically uncoordinated I am, how bad I was at all sports when I was in school, and how incredible it is to me that I have become a competitive athlete after all these years. I make the analogy that people who think they are bad at mathematics may well have hidden mathematical talents.

Kayaker, Artist: I say little about these except that ocean kayaking and landscape painting are also things I love and that are important to me.

3. SUBJECT MODULES

The first module is the one I use to stress the role that mathematics and mathematicians play in other fields. The other modules focus on getting the students to enjoy exploring some mathematical ideas, and on broadening their appreciation of mathematics. For the most part I have merely sketched the material covered in each module, but occasionally some of the questions I ask and typical answers are indicated.

3.1 Computer Animation

This module consists of showing a video of the Academy Award winning computer animation, Tin Toy [T], and exploring how the baby (the central character in the movie) was created. The underlying mathematical theme is how spline curves and surfaces are used to model complex objects in computer graphics. There are several other elements woven into the presentation: why computers are useful in animation; how other types of knowledge and skills (e.g. physiology, kinesiology, artistic, play writing, computer science) are needed to make a successful computer animated movie; the role of mathematics as a concise language for describing shapes and position; how complex objects are assembled from simple basic elements; the importance of modelling objects in the computer using small amounts of data; and analogies between two and three dimensions.

I start with a short introduction about the film, Tin Toy, which won the 1989 Academy Award in the Short Animated Feature category. Tin Toy was made by a team working at Pixar. *(Has anyone heard of Pixar? (no) Lucas Film? (maybe) Star Wars, Indiana Jones, etc.? (definitely - this leads to the story of how Lucas Film pioneered special effects and how Pixar and Industrial Light and Magic were spun-off as separate companies)).* I describe the key players on the Pixar team that created Tin Toy, and their education and training. John Lasseter is a traditional animator who began his studies with a four year fine arts program in animation and then went to work for Walt Disney (himself). Bill Reeves is one of the leading researchers in computer graphics. Bill did his undergraduate degree in Mathematics and his Ph.D. in Computer Science. Eben Ostby is a computer scientist and artist, and has studied both fields at university. I warn the class that Tin Toy is only five minutes long. I stress that they should keep in mind that Tin Toy is entirely computer generated and that while watching they should think about how it could have been done on a computer.

After the video we discuss how animations are done (a sequence of pictures with small changes between consecutive pictures), look at hand-drawn sequences for a stick figure waving its hand and a rotation of a cube, and discuss how I actually drew the sequences and how I could have saved time by using a computer drawing program. (The sequences are illustrated in Figure 2 but, like the rest of the figures in the paper, they were drawn on the computer because of the convenience with respect to word-processing.) I explain that in hand-drawing the stick-figure sequence I was able to use tracing to make the task easier. We note that using a copy-paste technique on the computer is easier still. For the cube rotation, I used a wooden block (I have the actual block there as a prop), and had to sketch each frame separately. We discuss why I couldn't use tracing for the cube sequence (all the shapes change). I explain that if we told the computer the position of each of the cube's corners, then the computer could easily calculate where the corners would move under a rotation and could thus draw a picture of what the cube would look like after each small rotation in the sequence.

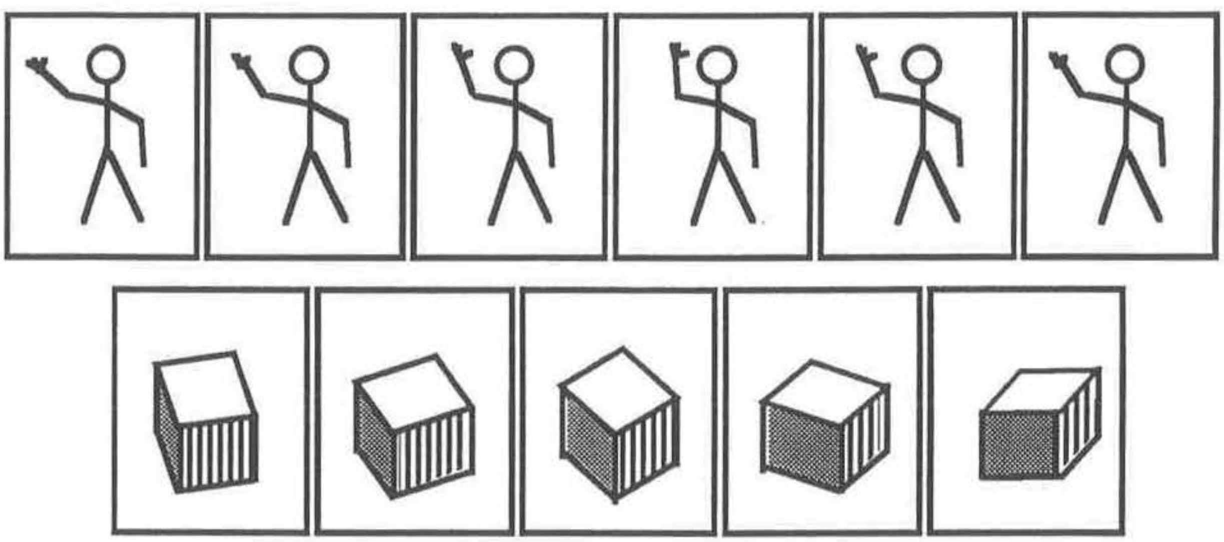


Figure 2.

We discuss how each object that we want the computer to draw must be represented by a model in the computer, how the position of a point is specified (coordinates), and how shapes such as line segments and circles are easy for the computer to work with since they can be uniquely specified using a small number of points. We talk about the basic shapes that are included in all computer drawing programs (e.g. rectangles, circles, polygons, etc.) and discuss why all programs have essentially the same set of shapes (they are commonly used, good building blocks, and fast for the computer to manipulate). We talk about why computers get slower when they manipulate complex objects (the calculations involve large numbers of points so they take more time). We talk about basic three dimensional shapes (spheres, pyramids, rectangular blocks, cones, rings, cones) and discuss where such shapes were used in Tin Toy. We talk about how the stick figure is composed of lines and a circle, and discuss whether the baby is similarly composed of basic 3D shapes.

This leads into a discussion of how smooth shapes could be modelled in a computer. We start by exploring how a two dimensional smooth curve drawing of a ghost might be modelled. We discuss how using a freehand drawing tool would involve too many points (a point for each pixel on the computer screen). We look at the advantages (small number of points) and disadvantages (corners) of a straight-line approximation (I use an analogy with connect-the-dots drawing). This leads to the idea of using circular arcs instead of straight lines to connect points on the ghost line-drawing. I explain that computer models actually use another type of curve called a spline. A spline is like a circular arc in that it can be described with a small number of points, but it gives slightly smoother approximations.

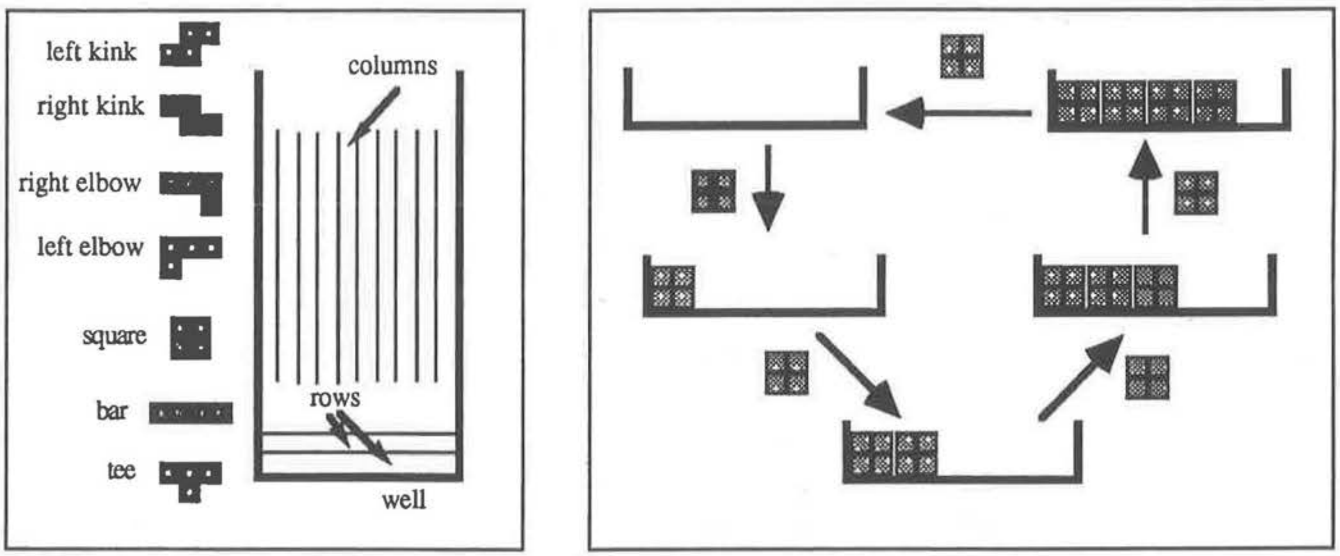
Now we move to the actual technique that was used to model the baby. I pull out my well-worn doll (courtesy of my daughter's doll collection), and explain that what the Pixar team did was to buy a doll, and draw several thousand dots on the doll, putting more dots where more detail was needed. The Pixar team then used a three-dimensional analogue of a mouse to input the coordinates of each dot. The dots were then connected with (flat) triangles to form a faceted surface (I illustrate this with a surface constructed from Zaks), and finally each flat triangle was replaced by a curved triangular patch so as to get a smooth surface. At this point we observe that this only gives a stationary model of the baby, so we discuss some of the techniques used in the animation of the baby (e.g. observing and analyzing the motions of real babies, and modelling of some facial muscles to get the facial expressions).

We next discuss what makes Tin Toy successful as a movie (characterization, plot, humour, graphics), and what skills were needed to make it. A good hands-on activity associated with this module is to pass out some simple smooth curve drawings (ghosts, etc.) and ask students to explore where to place 20 dots in order to give the best straight-line (connect-the-dots) approximation.

3.2 Tetris

In this module I present two topics based on the video game Tetris. The first explores whether there is a winning strategy when there is only a single type of piece, and is based on the M.Sc. thesis of J. Brzustowski [B]. The second concerns packing a set of pieces into a rectangle. It is based on a trick played by R. L. Graham in his invited lecture at the first ACM-SIAM Symposium on Discrete Algorithms in 1990. The primary objectives of this module are to explore how creating a mathematical formalism (finite state machine) helps to solve real-world problems (finding winning strategies for Tetris), and to examine easily understandable proofs that combine geometric reasoning with counting. In addition, students are exposed to the technique of forming generalizations and variants of a familiar object, and to how knowledge is gained by studying simplified versions.

I use a transparency to briefly define the game of Tetris and the names of the objects involved (see Figure 3(a)). My experience is that almost all students have had some experience playing Tetris, but the transparency is needed to establish common names for the elements of the game. We then consider the problem of finding a winning strategy under the assumption that there is no time limit for choosing where a piece should be placed. The definition of a winning strategy is a strategy that allows one to play indefinitely without the well ever overflowing. We discuss how the game can be varied by limiting the pieces to a subset and by changing the number of columns and depth of the well.



(a) (b)

Figure 3.

We start with the case where the only type of piece is the square and the number of columns is even. Everyone agrees that there are winning strategies for this game. I put up a transparency illustrating such a strategy (Figure 3(b)). This introduces the notation (a graphical version of a finite state machine) we use to describe strategies. We discuss the case where the number of columns is odd. Everyone agrees that there is no winning strategy in this case. I ask them to explain why. Next we look at the case where the only piece is the left kink and the number of columns is 8. After some discussion we find a winning strategy. Now we explore what happens if the number of columns is 7. Trial and error convinces the class that there cannot be a winning strategy but they find it more difficult to explain why. I lead them to a proof from [B] based on alternately colouring the columns of the well red and blue. Each left kink covers 2 red squares and 2 blue squares no matter where it is placed. Whenever a row is cleared the number of red squares removed is one more than the number of blue squares removed since there are 4 red columns but only 3 blue columns. Thus the number of occupied blue squares must accumulate, and the well eventually will overflow.

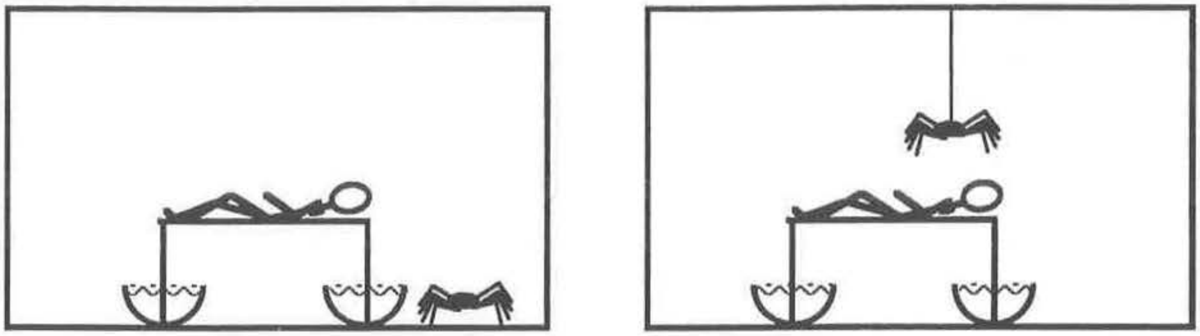
For the second topic we look at trying to pack all seven Tetris pieces into a 4 x 7 rectangle. After the students have tried long enough to be convinced that it might be impossible, we attempt to prove that it cannot be done. We start by trying to apply the colouring idea from the preceding paragraph. The students discover that alternately colouring rows or columns does not work, and eventually (with hints if necessary) arrive at the idea of colouring the rectangle like a checkerboard. Now, since all pieces except the Tee cover two squares of each colour, any placement of the 7 pieces must cover 15 squares of one colour and 13 of the other. As the rectangle has 14 squares of each colour, the packing is impossible. I leave the class with the problems of trying to pack all the pieces except the Tee into a 4 x 6 rectangle (fairly easy), and trying to pack all the pieces with an extra Tee into a 4 x 8 rectangle (harder). This module works best if each student has a set of pieces to manipulate.

3.3 The Spider Puzzle

In this module I present the class with a simply stated geometric puzzle involving a student who wishes to avoid being crawled on by a spider while asleep. We then, as a group, gradually discover the solution by sketching proposed solutions (and demonstrating their failures) on an overhead projector. The main objective of this module is to explore a mathematical problem requiring different problem-solving skills from those present in traditional school mathematics.

The spider puzzle is defined as follows. Jenny (I begin by asking for a volunteer who is afraid of spiders so I can give the student in the puzzle a name) is terrified of spiders. She knows there is a spider somewhere on the walls, floor or ceiling of her bedroom, but does not know where. The spider is too small to be seen but can be felt and Jenny wants to avoid having the spider crawl on her during the night. The spider can crawl on any surface except water, and can also drop down on a vertical thread. It is so small that it can land on an arbitrarily small surface (except water). Jenny places each leg of her bed in a bowl of water (Figure 4(a) shows the two-dimensional version of this problem) which prevents the spider from climbing up the legs of her bed. However the spider can still drop down onto Jenny from the ceiling (Figure 4(b)). The problem is to figure out what Jenny should construct in her bedroom to protect her from the spider. The construction must obey the constraint that Jenny and the spider remain in the same connected component (I express this by saying that Jenny and the spider must still breathe the same air)[3].

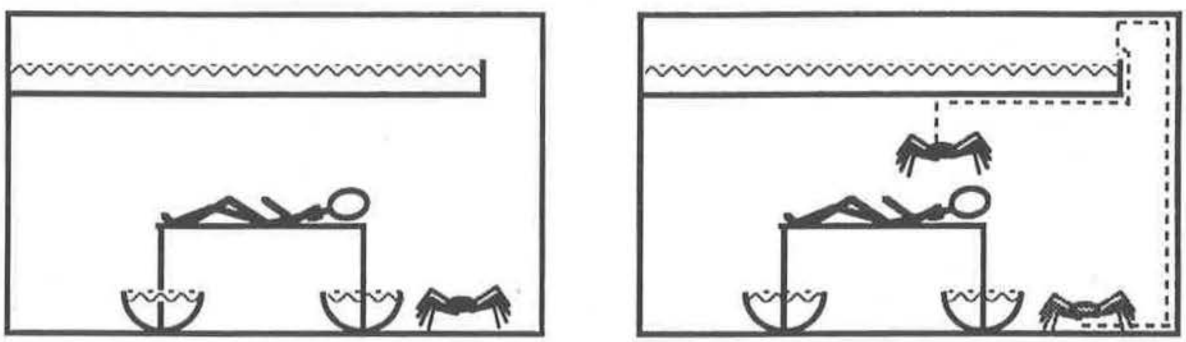
[3]I do not know the origin of this puzzle - I heard it from a computer scientist, Xioanan Tan, who was intrigued by the fact that her 11 year old son was able to solve it almost instantaneously while many professional mathematicians took several hours to find the solution.



(a) (b)

Figure 4.

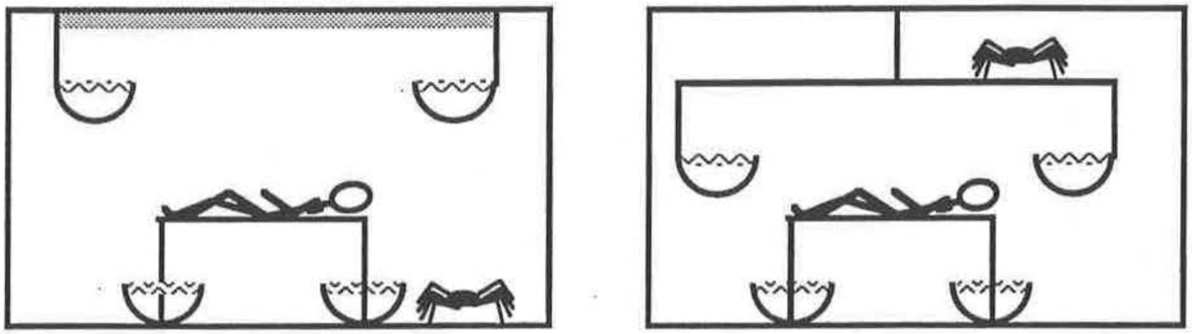
I put a transparency of Figure 4(a) (drawn in permanent ink) on an overhead projector and ask the class for ideas (it helps to hand out a paper copy to each student so that they can try drawing solutions). As soon as someone has an idea for a solution I ask him or her to draw the solution (in washable ink) on the transparency. I then ask the class whether it is a valid solution, and if not, why not. Initial attempts tend to be along the lines shown in Figure 5(a). Students enjoy drawing routes the spider (Figure 5(b)) could use to get to Jenny on the transparency.



(a) (b)

Figure 5.

After the first few attempts fail, several silly suggestions are made (shoot the spider with a water gun, buy Jenny a scuba diving outfit and fill the room up with water, etc.). Eventually (sometimes after some hints) someone suggests an idea along the lines shown in Figure 6(a), and the class realizes that the structure shown prevents the spider from reaching the shaded portion of the ceiling if the spider was not already there. At this point students usually start to have so many ideas and suggestions simultaneously that the best idea is to let students continue to work on the problem individually. Most students arrive at some variant of the solution shown in Figure 6(b) in a few minutes.



(a) (b)

Figure 6.

4. MISCELLANEOUS TIPS

Over the years I have assembled a number of minor tricks and strategies to help my presentations. Here are the ones I've found most useful.

Wear the right clothes: It really helps to wear clothes that the students identify with. For the last few years I have dressed in very bright colours and worn my fanciest running shoes.

Be prepared to cope with unavoidable delays in starting your presentation: Often it is necessary to wait for the arrival of part of the audience or a crucial piece of equipment. I use such periods to learn the names of the students in the audience. They like the interaction and knowing many of the names helps in soliciting answers to questions during the presentation.

Switch gears from time to time - a juggling lesson: I usually include a juggling lesson that purports to demonstrate the principle of mathematical induction, but primarily provides a break so that students who have lost the thread can tune in again. The demonstration also allows me to stress the fact that in mathematics as in athletics, practice and hard work can compensate for lack of innate talent.

Use props: the more outlandish the better.

Include hands-on activities: Like props, the more the better is the rule. Even for the juggling lesson, I get volunteers to come up and try out the basic steps.

Stick to simple audio-visual equipment: Canadian schools all have overhead projectors and VCRs, but the existence of other equipment (computers, etc.) is much less uniform.

Give your presentation in the classroom rather than an auditorium: The atmosphere is more intimate, the students expect to be involved and to interact, and are more comfortable.

Allow for enough time: 75 minutes is the absolute minimum if I want the class to participate effectively, especially in hands-on activities.

5. BROADENING THE IMPACT THROUGH KITS AND VIDEO GAMES

5.1 Extensions

The impact of the approach presented in this paper could be increased in several ways. First, more materials for teachers to use in preparatory and follow-on activities surrounding presentations are needed. I would like to see mathematicians and teachers collaborate in creating such materials and in exploring how these types of presentations can be used to support the curriculum. The second is to get substantially more mathematicians (and other scientists) to regularly give presentations in the schools. Programs like B.C. Scientists in Schools are helpful in this regard since they make scientists and schools aware of the possibilities. They also facilitate the process by matching schools with scientists, by providing funds for travel and supplies, and by providing scientists with helpful information on preparing and giving school presentations. Even with such support, preparing a successful presentation involves a significant investment of time and thought, as well as a good deal of trial and error. We should develop kits containing videos, hands-on activities, props, etc. on a number of different mathematical subjects. Such kits could then be made available to prospective volunteers through workshops on building complete presentations around the kits.

5.2 Video Games

Video games are an excellent vehicle to use to increase the exploration of mathematical concepts by children. Children love playing video games for extended periods of time on a daily basis. The exploratory and interactive nature of video games is ideally suited to exploring mathematical concepts. There are already a number of games that are mathematical in nature, e.g. Tetris and its variants (Welltris, Hatris, Columns, Dr. Mario), Pipedream, Boxxle, etc.; adventure games, such as Mario Brothers III, seem to be more popular, perhaps because they are less abstract and present a wider variety of environments and challenges. A promising approach is the integration of mathematical exploration into adventure games. I suggest we develop adventure games in which the problems, puzzles and reward systems are mathematical, though the goal, characters and environment are not. In most adventure games the protagonist must accumulate various resources (money, food, special tools and powers, etc.) to progress through the game. Consider the examples below in which the player explores the relationships among perimeter, shape and area while attempting to optimize the amount of resource obtained.

Upon discovering a treasure chest, the player is asked to position a slider on a scale to select the width and length of the base of the chest. The length of the portion of the scale to the left of the slider determines the width of the base, and the length on the right determines the length of the base. The chest is then redrawn with the appropriate dimensions, and its lid springs open to reveal a single layer of coins arranged in a regular array. The player thus receives an amount of money proportional to the area of the base of the chest. Since the sum of the base's width and length is fixed, the player quickly learns that the slider should be set at the middle of the scale. This example can be improved (in terms of video game effectiveness) by introducing elements of skill and hand-eye coordination. For example, the player positions the slider by throwing a dart at the scale. As the player reaches higher levels in the game, the shooting of the dart is hampered by random obstacles that appear and disappear, obstructing a clear shot at the middle of the scale unless the player achieves the correct timing. Another example that focuses on area-exploration is to allow the player to draw a curve with fixed arc-length on a background consisting of a regular array of coins. The fixed length curve is represented by a rope that the player drags through the array of coins. The player is rewarded with the number of coins lying in the interior of the figure formed by adding the line joining the two endpoints of the rope. The idea that the reward is the number of coins in the interior can be graphically communicated to the player by an animation showing the coins roll out of the interior one at a time and stack themselves while a voice enumerates them. At higher levels in the game the underlying lattice of the coin array is altered causing the player to explore how the optimal shape changes. Another variant allows the player to throw two darts at a target. The lattice of the coin array is given by the two vectors from the centre of the target to the points hit by the darts.

It is easy to think of other mathematical concepts that can be explored in similar ways. Fractions lend themselves to puzzles based on filling shapes with pieces of other shapes. Addition and subtraction of positive and negative numbers can be handled in many ways. Consider balancing weights where anti-gravity weights are included, or swimming in streams with currents. It is more challenging to find games to illustrate the multiplication of positive and negative numbers, particularly the multiplication of two numbers that are both negative, but even this can be done (as suggested by Frank Tompa from the University of Waterloo) with games involving lenses that invert as well as magnify.

If done in isolation the creation of video games that explore mathematical concepts is unlikely to be enough to convince children of the joys and importance of learning mathematics. If the games are designed well, the children will be largely unaware of the educational component involved. Thus it is important to provide complementary materials that build a bridge to the standard curriculum. Such materials would include clue books (e.g., "How to win at ...") that explain the mathematics needed to solve the puzzles and optimize the rewards. Practice modules in which the students can hone their skills at each set of concepts and puzzles are also needed. The practice module would provide students with a broader range of examples to explore on the given topic. Finally, sets of practice module experiments and game examples for sections of standard textbooks must be developed so that the students see the connection between the abstract techniques and concepts and their applications in the games.

Implementing this proposal will not be easy. Talented and experienced video game designers must be engaged in the project to ensure that the video games are successful as video games. Otherwise children will not play them. Initially it may be hard to convince game designers that the market is large enough to justify their time commitment. Another problem is the negative image of video games, particularly among parents and teachers. Contributing factors to this image include the frequent emphasis on violence, the lower appeal to girls, and the belief that playing video games wastes time and encourages aggressive and competitive behaviour. These issues can be addressed by ensuring that the video games are nonviolent and gender-neutral, and by doing a good job of communicating the educational benefits. In addition, advances in distributed computing enable the development of multi-person games in which players must cooperate to achieve their goals. Despite the inherent difficulties, the video game approach appears to be one of the most promising avenues for getting large numbers of children to enjoy math and science, and deserves serious investigation in the immediate future.

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