Building Large Sets of Haptic Icons: rhythm as a design parameter, and between-subjects MDS for evaluation

by

David Richard Ternes

B.Sc., University of British Columbia, 2005

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF

Master of Science

in

The Faculty of Graduate Studies

(Computer Science)

The University of British Columbia

August 2007

Abstract:

Haptic icons (brief, tactile stimuli with associated meanings) are a useful new way to convey information through the modality of touch, but they are difficult to create because of our lack of understanding into what makes good haptic stimuli and how people will perceive them. This thesis aims to enlarge our capabilities to design and evaluate haptic icons, despite these problems. We seek to do this via two overlapping threads of research. In the first thread, we introduce the design parameter of rhythm as a means of extending the expressive capabilities of the simple tactile stimuli used in haptic icons. This allows us to create a set of expressive and perceptually distinguishable haptic stimuli larger by almost an order of magnitude than any previously created. In the second thread of research, we tackle the problem of how to evaluate the perceptual characteristics of such a large set of stimuli with real people. We develop a means of evaluation that allows us to collect perceived difference data by present each user with only a subset of the total stimulus collection, and then stitch together an aggregate picture of how the stimuli are perceived via data collected from overlapping subsets from different users.

To advance these two threads of research, two user studies are run in order to examine how our haptic stimulus set is perceived and to validate our new method of gathering perceptual difference data. One study uses an established but cumbersome technique to study our stimulus set, and finds that haptic rhythms are perceived according to several different aspects of rhythm, and that users can consistently differentiate between haptic stimuli along these aspects. The second study uses our newly developed data collection method to study the same stimulus set, and we find that the new technique produces results that show no significant difference from the established technique, but using a data collection task that is much quicker and less arduous for users to perform. We conclude by recommending the use of our new haptic stimulus set and evaluation technique as a powerful and viable means of extending the use of haptic icons to larger sets.

Table of Contents

List of Tables

List of Figures

Acknowledgments

Surprisingly enough, this has been quite the learning experience. I thank deeply everyone who has contributed along the way, helping me learn as much about myself as the topic of my research. First I must give thanks to my supervisor, Karon MacLean, whose guidance and instruction has made my research so rewarding. Your belief and support of my work drove me to always strive for my best. My second reader Tamara Munzner also aided me greatly in clarifying and strengthening my thesis, for which I am much indebted. I would also like to thank the people at Nokia, particularly Jukka Jalkanen and Patrick Wong, who were generous in providing prototype hardware for my research and were so accommodating to my questions and requests.

This thesis would also not have been possible without the help of numerous friends, colleagues and family members. Colleagues and friends Shawn, Steve, Ricardo, Rock, Nels and many others made my life easier and more enjoyable in countless ways. Mario Enriquez, whose work so much of my thesis relies upon, was instrumental in providing me with the tools and knowledge required to tackle all that is MDS and haptic. For her support, advice, and above all friendship, Nicole Arksey played no small role in keeping me sane and (relatively) productive during these last two years. I would also like to thank all my friends outside of school, whose insistence ensured I made it outside of the lab to enjoy myself once and while. Lastly I would like to thank my parents, Joyce and Willi, my brother Dan and his wife Meg, for providing a constant source of strength, encouragement, and positive thought.

Without you all it could not have been done.

Dave Ternes

University of British Columbia, August 2007

Chapter 1: Introduction

Can you touch abstract information? What does it feel like? These questions drive us in our research into haptic communication. We wish to convey information to people from computers enabled with haptic displays, and we wish to do so in the simplest and most transparent fashion possible. To accomplish this we build upon the concept of *haptic icons*: brief tactile stimuli that have been associated with a meaning. We believe that haptic icons present a new means of displaying information to people that can be discrete, convenient and informative while simultaneously decreasing the dependence on the visual and auditory channels of communication.

In our vision, we see haptic icons being integrated into almost any interface in which the visual and auditory channels are already used extensively, where haptic icons could provide information to users without requiring them to visually monitor the interface. We foresee haptic icons integrated into handheld devices, where they can support interaction without the user looking directly at the device, a considerable advantage in busy environments, or social situations in which discretion is required. We see haptic icons as a general-purpose design tool to help ease the flow of information from computer to human.

Researchers have worked hard to design haptic icons and test them with users. Across many different haptic devices and technologies, robust design parameters such as frequency, amplitude and waveform have been used to create haptic stimuli that users can easily discern and recognize [19, 6]. Multiple applications have been created using haptic icons, and have been found to be successful in conveying information in practical work contexts [10, 18]. These promising results are opening up a much larger area of research for work in haptic icons.

Yet many challenges still remain, not least of which is the gap in understanding between the design of haptic stimuli and how they will be perceived by users. Because our understanding of the sense of touch is quite primitive when compared to sight or hearing, we are constantly forced to test our stimulus sets with users to determine how their members are perceptually related. Another consequence of this is that we lack insight into what the important perceptual parameters of touch are, such that creating new design parameters is often simply done by guess-and-test. What is required is further sophistication in all aspects of our work on haptic icons: greater sophistication in how they are designed, and greater knowledge in how they will be perceived. This thesis works to fill this void, though much more work remains to be done.

1.1 Motive

It has been posited that a haptic equivalent of visual icons could be used to increase information flow in an environment where computer interfaces are rich in the visual but poor in the haptic. Following theory with testing, multiple researchers across the globe have shown promising results in the use of various tactile stimuli to present iconic information. Now it is time to push beyond current capabilities, expand benchmarks outwards and determine how far this concept can go. Presented in this thesis are both an expansion of the methodology for building and evaluating haptic stimuli as well as new, multifaceted design parameters with enough depth to support the creation of an expansive set of expressive, distinct haptic stimuli.

The motivation for the use of haptic icons is fairly straightforward. In modern interface design, the visual modality is heavily relied upon. Especially in interfaces such as the cockpit of a plane or the driver's seat of a car, the user's visual field is almost overwhelmed with information. But even in a simpler interface such as that of a cell phone, if it is placed in a busy environment where, either for social or practical reasons, the user cannot spend all his/her time looking at the device, the over-reliance on visual communication creates a bottleneck in information flow from device to user. The haptic modality opens up a new channel between device and user, one that can be constantly in contact with the user without him or her constantly attending to it. This is not to claim that simply moving something from the visual to the haptic domain will necessarily free the attentional resources formerly used to track the visual information: attention and multi-modal perception interact in complex ways that we are still only beginning to understand, and other, cognitive bottlenecks exist in aside from basic perception of stimuli.

Nevertheless, as an addition to interfaces already heavily dependant on other modalities, the advantages of using the haptic channel to display information have been consistently shown. For both critical control tasks and social communication, haptic icons can be used as a simple, straightforward means of conveying information to the user through the haptic channel. This simple, one-degree-of-freedom communication makes them, perhaps, the most basic building blocks of abstract haptic communication.

Research into haptic icons has generally focused on the use of short, simple vibrotactile stimuli to convey information to users [19, 6]. This focus should make them easy to create, easy to display on a device and easy to evaluate. Unfortunately, this claim has been the goal, but not the reality. In truth, research is still hamstrung by poor haptic displays on which even simple design of stimuli presents serious challenges. Poor displays lead to noisy results which make evaluation difficult, and evaluation of complex human interaction with technology is not a simple task to begin with, as the very existence of the field of Human Computer Interaction (HCI) attests.

Nevertheless, we are seeing more technology equipped with more advanced haptic displays every day, especially in handheld devices. If haptic display becomes more common, then the opportunity for new haptic-enabled applications increases sharply, in no small part due to increased user familiarity with the medium. Furthermore, as discussed above, handheld devices are often operated in busy, demanding environments where a different, discrete modality such as haptics can help make the difference between an easy–to-use, helpful application and one that simply causes the user more stress and frustration. Consequently we are applying our research to the development and application of haptic icons in the *real* world, in hope that they can solve real-world problems positively and efficiently.

We are not there yet. Research into haptic icons has generally been preliminary, involving relatively small numbers of icons used in laboratory environments. Developing stimuli has been done on a per-experiment basis, and has differed widely across researchers (see [27] and [18] as comparisons). What this thesis aims to do is to greatly increase the number of coordinated haptic stimuli that can be used in an experiment or in an application: increase via *gross number of stimuli in existence*, and increase via *easing development*. We hope to create the beginning of a general reference set of haptic stimuli that has been evaluated for consistency and distinctness. Moreover, we also aim to create a robust process for the creation and evaluation of large number of haptic stimuli, allowing other researchers to follow in our footsteps. Increasing by roughly an order of magnitude the number of haptic stimuli in a single, coordinated set that are available to researchers, we will be allowing for the creation of larger, more complex applications that can use haptic icons. These applications can then be applied to real-world situations, studied over longer periods of time, and evaluated for their usefulness and usability. Thus the contributions of this thesis are to be viewed as major steps towards a more wide-scale, ecologically valid evaluation of haptic icons. This thesis aims to break haptic icons out of the domain of "toy" research and into the domain of grounded, practical application work.

1.1.1 Icons vs. Stimuli

It should be made clear that, because our work deals strictly with the haptic stimuli themselves, and at no point attempts to attach meaning to them, we do not often refer to haptic *icons* throughout this document, instead haptic *stimuli*. Only when a stimulus has a meaning associated with it does it become an icon, and this semantic process is not the concern of this work as the stimuli must be designed first, before meaning can be attached to them. The process of assigning meaning is left to the designers of application who wish to use our stimuli to make haptic icons.

1.2 Overview and Approach

Two main problems stand in the way of our goal of creating a large, diverse set of haptic stimuli; solving them comprises the bulk of the contributions of this thesis. Both issues stem from the size of the set of stimuli that we wish to create. The first challenge is how to create so many different stimuli which are distinctive and expressive to potential users. This problem is approached here by the systematic and wide-scale use of tactile rhythms, a little-explored parameter for use in haptic stimuli that we find greatly extends the number of perceptually distinct tactile sensations that can be created, even with a duration as short as 2 seconds. The second challenge is to evaluate larger sets of stimuli, when traditional means of evaluation do not scale well due to time and fatigue. Our solution to this problem is a new method of gathering perceptual data that requires users to judge only a manageable subset of the complete stimulus set, with the total, overall perceptual picture stitched together using judgment data from multiple users.

Thus this thesis contains two intertwined, interdependent strands of research, one of method and one of design. The design is the creation of a set of 84 haptic stimuli (much larger than the previous standard of 36 set by MacLean and Enriquez [19]), using amplitude, frequency and rhythm. The method is an extension of the data-gathering techniques for multidimensional scaling (MDS) that are used to determine the perceptual characteristics of a set of stimuli. This method has expanded the size of stimulus sets that can be dealt with by a factor of three, enabling 150 stimuli, where 50 was the previous maximum. By running two separate but related studies we are able to both evaluate our stimulus set and validate our new data-gathering method. This duality informs the structure of this thesis: Figure 1.1 lays out the logical structure of the thesis, in terms of the two interlocking research strands.

The key to understanding the logic of this thesis is to understand the role played by the two major studies discussed herein, and how they pertain to the two strands of research described. Both studies seek to perform MDS on the same stimulus set; they both follow the basic steps of (a) determining how people group our set of rhythmic haptic stimuli, and then (b) feeding this data into the MDS algorithm, producing a n-dimensional plot of the stimuli where the placement of each stimuli relative to each other stimuli represents how perceptually similar the two were on average judged to be. How the two studies differ is in the method by which perceived similarity ratings were gathered. The first study uses the sorting technique developed by MacLean and Enriquez [19] to gather data. This is a proven method [20], but it is stretched to its limit by the sheer number of stimuli users are forced to deal with here. The second study uses a modification of the sorting technique that allows a larger number of users to each deal with a smaller subset of stimuli. This method, based on a form of between-subjects analysis, presents the user with a less taxing task that can be completed in a more reasonable timescale, but as an experimental technique the data integration must still be proven to produce valid results. The two studies thus provide two different looks in at the same stimuli set.

Since it utilizes a previously validated technique [20], we use the first study to investigate the perceptual characteristics of our haptic stimulus set. By completing an in-depth analysis of the MDS plot and examining precisely what characteristics of our stimulus set influences how they are perceived, we find strong and interesting effects of tactile rhythm—indicating its strength as a design parameter for use with haptic stimuli. Yet in performing this analysis, we also develop a "gold standard" against which our new data-gathering technique can be compared. Our second study uses the new method to examine the same set of stimuli, and produces results which are both quantitatively and qualitatively similar to the gold standard, thus allowing us to conclude that the technique itself is valid as well as more practical.

It might be noted that, if haptic icons are truly to be the touch-based equivalent of visual icons, the process that we are describing seems considerably more complex and involved than what would be expected of a visual icon design process. When needing a new visual icon, one might simply give a graphic designer some requirements as to what information the icon need convey, and then upon receiving the designers best guess at what the icon should look like, one would likely simply "eyeball" the result, ensuring its appropriateness. Even if more detailed user testing was performed, it would likely only be a part of a more wider usability analysis, and certainly would never reach the level of complexity and rigor exhibited in our MDS studies. Yet as a technology, as a psychological science, and as a symbolic medium, haptics and vision are by no means on the same playing field—thus comparing relative design processes leaves haptics at a considerable disadvantage. We have been studying and using vision, figuring out different ways of displaying visual information, for far longer, and in far greater detail, than ever has been done for haptics.

Relatively primitive displays and lack of knowledge make working with haptic icons considerably more challenging than visual icons. If we were to limit visual icons to abstract expressions only (as haptic stimuli themselves are limited); if we were to require only 6-pixel visual icons, with no more than a dozen different shades of grey to choose from; if creating each individual visual icon required an extended period of work writing code or mathematically modeling a waveform; if all these limitations were required of visual icon design, then perhaps a fair comparison of methodologies would be possible. As the field stands, our current thorough design process represents our best attempt at overcoming the constraints, both perceptual and technological, that the haptic modality places on icon design.

Thus, at the end of this research what we have accomplished is setting up a solid basis from which to extend the expressiveness and diversity of haptic icons. We developed a new design parameter that greatly increases the number of different haptic stimuli that can be created. We also developed a new technique for analyzing these stimuli to ensure their success as useful, informative signals. This creates a toolkit and a process that any new haptic icon developer could use to quickly and easily create a large set of discernable haptic stimuli tailored to his or her needs.

Chapter 2: Related Work

2.1 Abstract Tactile Communication

Though it could easily be considered a small niche of the research world, work on abstract tactile communication has flourished in the last few years, with groups from several different research labs across the world contributing to a great increase in knowledge about the design and application of haptic icons and informative tactile signals in general. Their results have all been generally positive, indicating a clear ability of people to discern, recognize and use haptic icons or similar types of stimuli in a variety of applications and contexts.

2.1.1 Haptic Icons

In the work of MacLean *et al.*, which our own work largely builds upon, the design and use of haptic icons has been study extensively across different platforms and applications. MacLean and Enriquez [19] created haptic icons using a force feedback knob, varying waveform, amplitude and frequency to create a set of 36 haptic stimuli. In order to determine how these icons were actually perceived, they were thoroughly analyzed using Multidimensional Scaling (MDS), an analysis technique that would prove invaluable for further work on haptic icons, and which is expanded upon in this thesis. They found that their three parameters, with some adjustment, could create an even spread of perceptually distinguishable stimuli. Their results showed both the utility of MDS as a tool for perceptual analysis of haptic sensations, and demonstrated that people can make consistent distinctions between well-designed haptic stimuli.

Following the work of MacLean and Enriquez were several more studies. Chan *et al.* [10, 11] developed a haptic icon-based protocol for turn-taking in a collaborative environment using a haptic mouse. It showed the efficacy of haptic icons used in cognitively loaded environments to communicate information unobtrusively to users. Luk *et al.* [18] used a novel piezo-driven skin-stretch display to present users with haptic icons in the context of a handheld device. Luk *et al.* showed that the haptic icon design paradigm could be applied equally well to new, handheld and non-vibratory platforms. Such robustness is an encouraging result if we wish to continue extending haptic icons in new and interesting directions.

2.1.2 Tactons

Coining the term "tactons" to describe their vibratory tactile icons, Brown *et al.* have also explored somewhat similar terrain, with similarly encouraging results. Like the work of MacLean *et al.* they have both developed design parameters for creating informative tactile stimuli and analyzed these stimuli to determine their perceptual nature. In [5], they introduce several different design factors to be used to create tactons, starting with frequency, amplitude and waveform as in [19], but adding to it duration and rhythm, as well the body location at which the stimuli is presented. These ideas were tested in-depth in several later papers. Firstly in [6] they tested icons with three different amplitude-modulated textures and three different rhythms (based on previous audio icon designs) and found strong recognition rates for each of their two parameters (80% and 93% for texture and rhythm respectively). In a still-developing work [7], Brown *et al.* added in a third parameter, location of stimulus presentation, and found that recognition rates dropped somewhat for three-parameter icons, but could be designed around if needed. In collaboration with a variety of other researchers, both Brown and Brewster continue to expand their research into tactons by examining crossmodal effects [14], various musical techniques such as crescendos [8], and applications in mobile phones [9].

2.1.3 Other Vibro-tactile Work

Research into tactual perception has been done at a more general level as well, not just in attempt to create some form of haptic icon. Tan *et al.* [23] have studied the display of tactual signals, comparing a variety of stimuli to determine the overall level of information that can be transmitted using artificial stimuli. Though their results considering the informational capabilities of tactile stimuli are a positive indication for us, they make no discussion of designing of stimuli for use in practical considerations, nor do they proceed beyond basic tactile waveforms in their stimulus set. Other researchers such as Klatzky and Lederman have performed research into related areas, for example texture perception using a stylus [16], and have gained similarly positive results. Representative of much of the work in this field, both of these works are directed at fairly low-level psychophysical findings, leaving considerable room still to be explored as we bring these tactile stimuli into more and more practical contexts.

Other important work on displaying tactile information has been done by van Erp [26], where he specified several design guidelines for communicating through vibro-tactile displays. He discusses several features present in the work of both MacLean and Brown, such as frequency, amplitude, temporal patterns (which are essentially rhythm), and display location. He also points out the dangers of masking and confusion that can occur using spatial and temporal effects. Van Erp has also worked on tactile melodies [27], which is, to date, the most significant analysis of tactile melodies and rhythms yet published. Van Erp and Spape take 59 real-world melodies and transfer them into the tactile domain. Using MDS along with other statistical methods, they determine two main perceptual characteristics: intrusiveness and tempo. However, these results are limited to a description of very complex and specific musical rhythms, such that direct application to the design of synthetic tactile stimuli from scratch would be difficult. Nevertheless, the results of their experiment show encouraging trends for tactile melodies and rhythms.

2.2 Multidimensional Scaling

Multidimensional Scaling (MDS) is a statistical technique that provides quantitative values describing the perceived dissimilarities between a given set of stimuli. The MDS algorithm takes an input of dissimilarity values and calculates a value describing the distance of each stimulus in relation to every other stimulus in a perceptual space. An appropriate number of dimensions are chosen based on the stress value, a measurement of model fit, where lower stress represents a better fit. Choice of the number of dimensions is determined by the benefits of an additional dimension in reducing the stress level against the loss of interpretability an additional dimension adds. Once an appropriate number of dimensions have been chosen, the data can be mapped visually and analyzed for clustering and trends [3].

The main reason that we use MDS in the study of haptic icons is its ability to pick out trends and grouping of stimuli, when no *a priori* knowledge exists about how the stimuli will be perceived. The application of MDS to the perceptual domain was pioneered by Roger Shepard at Bell Laboratories in the early '60s [21] as a general tool for analyzing interstimulus similarity amongst any grouping of stimuli for which the perceived differences were unknown. The technique was quickly picked up by many psychophysicists, being applied to such areas as, for example, musical timbre. Significant new understandings in how timbre was intuitively perceived were brought about by the use of MDS [13], an early positive indicator of its usefulness as an exploratory tool.

The haptic modality, compared to modalities such as vision or audition, is still largely lacking in a widely-accepted and intuitive description of its important perceptual characteristics, though the work of Tan [23] and other has taken great strides in gathering the basic perceptual knowledge to needed to gain such intuition. Lacking this intuition, MDS can provide clues to what these characteristic might be, without us knowing beforehand. MDS has thus been used extensively for the study of haptic stimuli, as well as other stimuli in different modalities for which there is a similar lack of intuitive understanding. In addition to the work of MacLean *et al.* described in section 2.1, other researchers have successfully used MDS to discover new information about novel stimuli. For example, [15] found a 3D percept map from a set of real tactile surfaces, finding dimensions such as hard/soft and slippery/sticky. Bonebright [2] used MDS to analyze everyday sounds, in hopes of furthering his work into designing informative sound icons.

Considerable effort has been applied to analyzing different types of MDS algorithms as well as different means of gathering dissimilarity data for analysis. However, for our purposes the standard SPSS MDS algorithm, ALSCAL, shall be used. Our main research interaction with MDS is in the means of gathering data; this shall be discussed in detail in Chapter 4.

2.2.1 Comparison of MDS Results

One aspect of MDS algorithmics that does bear mentioning is how best to compare the results of different MDS analyses. Two different MDS output maps based on subjective dissimilarity ratings from different users are almost guaranteed never to be the same, even if the stimulus set is the same. In canonical research into MDS from the psychology and psychophysics fields (such as [22]), it was originally considered that Pearson's *r*, the product-moment correlation coefficient, was sufficient to measure whether two n-dimensional MDS outputs were statistically correlated. For clarity, note that Pearson's *r* is unrelated to r^2 which is used elsewhere in this document to describe goodness of fit for MDS results. Borg and Leutner [4], with a simple example, proved that the product-moment correlation is in fact inappropriate for use with MDS outputs:

> Let A and B be two MDS configurations, consisting of $NP = 3$ points each, with distances $d(1,2) = 1$, $d(2,3) = 2$ and $d(1,3) = 3$ for A, and $d(1,2) = 2$, $d(2,3) = 3$, $d(1,3) = 4$ for B. The PM [Product-Moment] correlation of these distances is $r = 1$, indicating perfect similarity of A and B. This is false, of course, since the greatest distance in A is three times as great as the smallest, whereas in B it is only twice as long. Hence, A and B do not have the same shape: B forms a triangle, whereas A's points lie on a straight line, because they satisfy the equation $d(1,2) + d(2,3) = d(1,3)$.

Thus instead, Borg and Leutner proposed the use of the congruence coefficient defined as,

[1] $$c = \sum_i d_{Ai}\, d_{Bi} \,/\, (\sum_i d_{Ai}^2 \sum_i d_{Bi}^2)^{1/2}$$

where d_{Xi} is the *i*-th distance value between stimuli, in configuration X, and the sum is over all pairs of distances in the MDS output map. Due to the tendency of *c* to cluster close to its upper limit of 1, a transformation was performed, giving us the alienation coefficient *K*,

[2] $$K = (1 - c^2)^{1/2}$$

This formula was then empirically tested on randomly created dissimilarity matrices, at different dimensions and numbers of points, providing empirically derived constants describing similarity of MDS outputs at a 95% confidence interval. The alienation coefficient thus provides the only statistical measure specifically designed and tested to measure similarity of MDS outputs, and still stands as the state of the art for statistical comparison of MDS results—Borg's own book on MDS confirms this status [2]

In our own work we rely on the alienation coefficient to help validate our newly designed method of gathering data for MDS. By using two different data gathering methods on the same set of stimuli, we produce two MDS results that we hope to be similar. The work of Borg and Leutner gives us a statistical tool to compare the two results, to be used in conjunction with our standard visual interpretation of the MDS map.

Chapter 3: Creation of Haptic Stimuli

The simple haptic stimulus sets developed by MacLean and Enriquez [19] and built upon in several ensuing publications [11, 18, 12] are based upon varying three parameters: waveform, frequency and amplitude. However, using these three components has only produced a relatively small set of stimuli, when varied using today's tactile display hardware. While these parameter provide a solid basis for building tactile stimuli, we sought to find a means of creating new and interesting stimuli supplementing some of with more complex parameters that might yield better results..

Previous work on haptic stimuli has investigated the application of rhythm [6] and melody [27] to vibrotactile stimuli, with fairly positive results. The approaches taken were, however, quite different, with the work on rhythm using only a handful of very basic rhythms, while the research into melody used a broad range of actual musical melodies transposed into the tactile domain. So while their results point towards a positive use of rhythm and melody in developing expressive tactile signals, their goals were such that they do not cover a broad enough sample of the possible design space to reveal a consistent framework for dealing with these parameters in the tactile domain. Both studies lacked results that could be broadly generalized to all types of tactile rhythms. The initial studies by Brown *et al.* [6] used only three rhythms, which were too different and too few to establish any clear patterns. Van Erp and Spape [27] used far more stimuli, but because their stimuli were sampled non-systematically from real-world examples of music, they both lack a systematic description of their structure and suffer from the many possible learned musical associations that a participant in the study might have.

3.1 Description of Possible Stimulus Space

For our own purposes we felt that it was wrong to assume that the standards that inform normal auditory musical composition would apply to the sense of touch; the skin's sensory capabilities are attuned to different things than the ear, to say nothing of the effects that cognitive aspects of musical appreciation might have. This is not to say an approach attempting to utilize musical capabilities in the haptic domain might not be successful; rather, bringing the musical into the haptic is simply too large an issue to deal with within this thesis. Yet despite our wish to avoid borrowing too heavily from the musical domain, it was nonetheless considered most straightforward to represent rhythm and melody as a sequence of notes of varying length played at set intervals in a bar of "music."

For our purposes, we define rhythm as being the repeated, patterned recurrence of some set of variable length beats/notes, and we differentiate this from melody, which is concerned with the different tones that the notes in rhythm are played at. Changing the tones of the notes in a rhythm changes the melody, but not the rhythm; changing the length, number or placement of notes changes the rhythm and would also likely have an effect on the melody.

If we first simply look at the number of different combinations of notes that could be used in a rhythm or melody, we are immediately faced with an exponentially increasing set of possibilities. Limiting ourselves only to quarter notes in 4/4 time, we would have $2^4 = 16$ different way of arranging notes, with eighth notes $2^8 = 256$ variants, with sixteenth notes $2^{16} = 65536$. If we then consider playing melodies (*i.e.* the tone of each note is different, played at different vibratory frequency) or adding emphasis (*i.e.* playing different notes at different amplitudes) then the number of possibilities grows even larger.

Clearly we needed a means of reducing this huge number of variants down to a manageable handful, in a way that would produce tactile stimuli that were different enough to be perceptually distinguishable and while possessing shared features that would contribute to some natural perceptual groupings to increase learnability. We lacked a clear precedent into what would make a tactile rhythm or melody distinguishable yet also perceptually similar enough to the other stimuli used that some natural grouping would be evident. Thus we were forced to rely heavily on intuition and our own reasoning on how to move forward.

Very quickly it became clear to us that, as the above numbers indicate, some *a priori* design decisions were needed to limit the scope of this work. Our first observation was that all melodies have a rhythm, at least implicitly, and this suggested rhythm was a more fundamental parameter than melody and therefore should be focused on first. To eliminate melody as a confound and keep our search space manageable, we therefore utilized only monotone (non-melodic) rhythms: all notes in a particular rhythm were played at the same amplitude and frequency. The particular amplitude and frequency level at which a rhythm is played could still be varied, meaning that we have allowed rhythms that have the same number, type and placement of notes, but different overall frequency and amplitude levels. It is only variation of frequency and amplitude *within* a rhythm that we are choosing to disregard for the sake of simplicity. We felt that these initial design choices narrowed the field of possible stimuli down to an area that could be reasonably approached in a more rigorous manner. What follows is our analysis of the tactile rhythm space, and a description of how we narrowed down the field to our final selection of rhythms.

3.2 Sensory- and Hardware-Specific Limitations on Rhythm Space

Still without a set precedent on how we might partition the space of all possible tactile rhythms, we set out to study the space as best we could. By iteratively creating different haptic rhythms and observing how they felt, we were able to informally develop a set of rules that we felt tactile rhythms needed to obey in order to produce diverse yet associable stimuli. Some of these rules we believe, based on our own testing, to be necessary for creating any tactile rhythm, while others are more design heuristics that represent our own intuition on what makes good stimuli. In all cases though, these recommendations are based on our tests on a specific hardware platform (described in more detail in Chapter 5), and though it is likely that much of our work here is broadly generalizable, we cannot remove completely the confound of the specific hardware used. Nevertheless we feel that our extensive informal testing with a variety of different users has lead to consistent high-level recommendations.

3.2.1 High-level Limitations

Two facts were immediately obvious to us as soon as we started creating tactile rhythms. First, there needs to be a gap after each note played in a rhythm in order for the notes to be distinguished as separate. If the individual notes within a rhythm varied by frequency or amplitude, it would be possible to perceptually segment the different notes if the differences were large enough between adjacent notes; however, as we had already decided to limit ourselves to monotone rhythms, gaps in between notes were necessary.

The second fact was that unless a rhythm was repeated, it was not perceived as a rhythm, merely a set of isolated vibrations. Furthermore, the more times a rhythm was repeated, the stronger the sense of rhythm became. From our testing we found that four repetitions was a good compromise between having enough repetitions to create a strong sense of rhythm without requiring an overly long total duration for the stimuli. This observation led us to choose a total stimulus duration of 2 seconds, resulting in a 500 ms duration for each iteration of the rhythm. We felt two seconds to be about the longest a stimulus could last and still be useful in the context of a haptic icon, while the 500 ms duration was long enough to allow for a fair number of different notes to be packed into a rhythm (e.g. 4 bars of 4/4 time played at a brisk tempo, as elaborated below). Though from an auditory musical perspective 500 ms could be considered quite short for a bar of music, we were limited by our need to make the overall signal fairly short and yet still present enough repetitions. However, we did not find this to be too fast a pace to be playing our rhythms at, largely because we were not asking our users to pick out individual notes, just perceive an overall sense of the rhythm. Perception of the rhythms as a whole was still attainable, and benefit of the increased repetitions helped counteract the speed of the overall rhythm.

3.2.2 Shortest Note

With our single iteration time of 500 ms established, the next issue was to find the shortest length of note people could consistently perceive. From our informal testing we settled on a sixteenth of the total time (31.25 ms) followed by a break of a similar duration. We could have chosen to make the break shorter in time than the vibration, but in order to make it easier for us to line up where different notes (and breaks) fell, we chose to keep the on and off time the same. Thus the total time required for the smallest note was 62.5 ms, or exactly one eighth of the 500 ms single iteration time. Consequently the smallest note we use is called an "eighth" note because it takes up exactly one eighth of the time of a single iteration of the rhythm. Having now determined the smallest interval, we built all our rhythms along the basis of 16 consecutive time slots, which can either be on or off.

3.2.3 Selection of Different Note Types

We next sought to determine what other note lengths we should use to build up our rhythms. To do this, we made a series of rhythms containing only one note, with a note length varying from one to fifteen of the sixteen, 31.25 ms time slots (the 16th slot was required to introduce a break between rhythm iterations). Of these fifteen possible note sizes, the following observations were made.

- The difference between a 31.25 ms vibration and a 62.5 ms vibration is noticeable (*i.e.*, one vs. two consecutive time slots set as "on"), though not overpoweringly so.
- The difference between a 62.5 ms vibration and a 93.75 ms vibration (ie, two time slots vs. three time slots) is *not* consistently noticeable.
- This finding holds true for all longer notes: differences of one time slot (31.25 ms) are not noticeable.
- However, differences of 62.5 ms *are* noticeable for all longer notes.

These observations resulted in the following five types of notes, also described in Figure 3.1 in terms of the number of time slots they occupy.

- Eighth note (62.5 ms total play time: 31.25 ms on, 31.25 ms off)
- Quarter note (125 ms total play time: 62.5 ms on, 62.5 ms off)
- Half note (250 ms total play time: 187.5 ms on, 62.5 ms off)
- Three-quarter note (375 ms total play time: 312.5 ms on, 62.5 ms off)

- Whole note (500 ms total play time: 437.5 ms on, 62.5 ms off)

We used the larger break time of 62.5 ms (two time slots) rather than 31.25 ms (one time slot) because we felt it gave greater distinction between notes. The 31.25 ms break time is used only for eighth notes, in order to allow us to have two eighth notes played within the same amount of time as one quarter note, which was a feature we used to create several of the different groups of rhythms described in the next section.

3.3 Description of Stimulus Set

At this stage in our design process, we had pruned down our selection of possible rhythms considerably through *ad hoc* and informal testing. However, the rhythm space still remained relatively large. Having done all we could to develop rules describing which types of rhythms *not* to use, we now had to develop some positive heuristics as to which rhythms we *should* use. Again, using our intuition along with iterative informal testing, we developed four heuristics, which in turn created five groups of rhythms, each defined according to one or more of these rules.

These groupings were designed into the stimuli set from the start, and represent our best attempt at creating a diverse yet logically grouped set of tactile rhythms. We do not claim that this will end up to be the best grouping of rhythms that could be used. However, without a clear precedent into how tactile rhythms might be grouped and perceived, our own intuition, along with continual informal testing, was the best tool we could use. In our study results we discuss how our intuitive groupings were, in part, confirmed: some of our groupings were held out by the study, while other unanticipated perceptual groupings were also found. For clarity we believe it important to specify what groupings we built in to our stimuli beforehand and differentiate them from the *post hoc* groupings that our studies later revealed. The groupings below represent our initial best guess at how tactile rhythms might be grouped.

3.3.1 Heuristic One: Quarter Notes

Our first heuristic was to consider all possible rhythms that contain only quarter notes and pauses. This decision was based upon both our initial testing results which found quarter notes to be an easily recognizable duration, as well as the consideration that a straight 4/4 rhythm with notes on every downbeat would likely be considered one of the most simple and basic rhythms available. As noted above, there are 16 possible all-quarter note rhythms. However, many of these are perceptually indistinguishable from each other because of the repetition of the rhythms. As a specific example, consider all rhythms containing just one quarter note: the note could occur in any of the four slots, and thus we have four different rhythms. Yet an issue with 'monotone' rhythms (those without varying emphasis and thus a discernible downbeat) is that while looping, there are no indicators of its starting point. Thus all four of the single-note rhythms will feel the same once they have started, as the spaces between the 4 played notes (one in each iteration) will be the same in all cases. Similar situations occur for some rhythms of two and three quarter notes per iteration, and we therefore used just one instance of each of these cases. Thus considering all possible quarter note-only rhythms that are perceptually distinct from each other, we arrive at Group 1, the first five rhythms as indicated in Table 3.2.

3.3.2 Heuristic Two: Long Notes

Our second heuristic was to consider all rhythms containing only notes that are *longer* than quarter notes: *i.e.* half notes, three-quarter notes, and full notes. This decision was based upon the observed difference in sensation that longer vibrations gave as compared to shorter notes such as the quarter note, and because we felt that having a variety of different note lengths in our rhythms would be prudent if we wanted to obtain a good cross-section of different types of rhythms. Thus we have categorized quarter notes as being "short" and notes longer than a quarter as being "long." Similar issues of duplication due to repetition were present for this group, narrowing the number of possible rhythms down to four, creating Group 2 in Table 3.2.

3.3.3 Heuristic Three: Long and Quarter Notes

Our third heuristic was to consider rhythms which contained at least one quarter note, and least one of the longer notes used in Group 2. The goal here was to produce rhythms which had both quick and slow components. Again there were issues of duplication due to repetition, which pruned several rhythms, and the requirement that there be at least one quarter note meant that full notes could not be used. The final set of four possibilities is presented as Group 3 in table 3.2.

3.3.4 Heuristic Four: Substituting Quarter with Eighth Notes

Our fourth heuristic actually resulted in two groups of rhythms, as this heuristic was actually a means of modifying two of the groups already described. Because the number of different rhythms containing only eighth notes is 256 (to say nothing of rhythms containing combinations of eighth notes and other length notes), we felt daunted at the prospect of choosing some reasonable set of rhythms from this space. Consequently, we made the simple choice of taking the rhythms we created in Group 1 and Group 3, and in place of each quarter note (total playing time of 125 ms) we substituted in two eighth notes (playing time 62.5 ms a piece). This gave us a set of four eighth note-only rhythms (Group 4, analogous to Group 1) and four eighth note plus longer note rhythms (Group 5, analogous to Group 3). Group 4 has four rather than the five rhythms in Group 1 because the eighth note analog of rhythm 3 was felt to be very hard to distinguish from the eighth note analog of rhythm 2.

3.3.5 Complete Stimulus Set Used

With the 21 rhythms described in Table 3.2, we finally had set of diverse yet associable tactile rhythms. Thankfully we had chosen early on that, though each stimulus must be monotone in terms of the frequency and amplitude of all of the notes played within it, we can create different stimuli by simply playing the same rhythm at a different set level of frequency and amplitude. Keeping in mind our desire to have a large, but not overlarge set of stimuli, we thought it best to have two frequency levels (high and low) and two amplitude levels (high and low) that each of the 21 rhythms could be played at, creating 21 x 2 x 2 = 84 different stimuli. By having only two levels of frequency and amplitude, high and low, we hoped to ensure that the differences between each of the four frequency x amplitude levels would be quite strong. Table 3.3 gives the exact value, rhythm type by amplitude by frequency, that each of the 84 stimuli was given, and can thus be used as a lookup table for all further references to individual stimuli throughout this document.

3.4 The Space Untested

While our own selection of 21 different rhythms to use for our stimulus set was guided by a thorough loop of iterative testing, we by no means claim that our choices are the only ones that could have been made. Indeed there are many rhythms that we did not use, and that if studied, may well lead to further insights into tactile rhythms. As important as it is to understand the rhythms that we have chosen to use, it is also important to understand their relationship to the space of rhythms we did *not* choose.

Right away, the choices we made in Section 3.2 sharply decreased the number of possible rhythms that we were working with. First limiting ourselves to only monotone rhythms, and then specifying a time-span that could only allow notes no shorter than an eighth of a bar, we made somewhat arbitary, yet we feel reasonable steps towards a choosing a well defined rhythm space to work within. Many other choices could be made at this level, such as having varying (and slower) tempos, varying amplitude of notes within a rhythm, using melody, using crescendos and many other musical techniques—not even to mention multi-bar musical compositions. All these choices could likely lead to interesting new developments, but they are left to others to explore. We do believe, however, that many of these choices represent additions of considerable complexity to the rhythm space, such that we feel that in most cases we chose to *decrease complexity* of the parameters we were working with.

3.4.1 Unused Rhythms Possible Given Hardware and Sensory Limitations

Even narrowing down our rhythm space to a more manageable size, we still had more rhythms than we needed for our purposes. The choices that we made to select our final 21 rhythms are outlined above, but it is worth considering the rhythms that we did not choose. It is possible that some of these rhythms might be worth revisiting at a later date, especially if our chosen selection of rhythms does not truly represent an even cross-section of possible rhythms.

Given the limitation set out in Section 3.2 for the size and number of notes that can be fit into a rhythm, we can see that for rhythms containing only quarter or longer notes, we have exhausted all possible rhythms that could be used. Rhythm Groups 1, 2 and 3 specify all of the rhythms using only quarter notes, only notes longer than quarter notes, and both quarter and longer notes, respectively. Consequently for these note lengths we can be confident in the coverage of our different rhythms.

However, as specified in Section 3.3.4, we were not as thorough in our coverage of rhythms containing eighth notes. There are 256 rhythms containing only eighth notes, and though many of them are likely the same due to repetition, we still only use four. Though these four do have a good level of variance in terms of the number and placement of notes, we neglect many of the more complicated rhythms that could be created, as well as any rhythms with just one eight note separated by pauses on both sides (this because we were echoing the rhythms in Group 1, replacing one quarter note with two eighth notes). These more complicated rhythms could well have produced interesting, more nuanced results, yet for this initial exploration it was thought best to start with relatively simple patterns. Moreover, it was noted that the subtle differences in placement of a single eighth note were often very hard to notice perceptually, so we felt that fully examining this space would lead to diminishing returns.

Considering the combination of eighth notes with other, longer notes, there are yet more possible combinations that were not used. In rhythm Group 5, we combined eighth notes with longer notes, but many other possibly arrangements remain. Again, we felt that we had a fairly reasonable cross-section of different numbers and placements of eight notes, but by no means exhaustive. Taking, for example, rhythm 18 (containing a two-thirds note followed by two eighths), we could have also made this rhythm using just one eighth note, placing it either in the last or next-to last slot. While one of these rhythms might have produced slightly different results than rhythm 18, it is doubtful it would have been greatly different, given the shortness of the eighth notes. Furthermore, it is almost certain that users would not have been able to discern the difference between the single eight note being in the last or next-to-last slot. This sense of diminishing returns is strong when considering eighth notes, given that they lie almost on the threshold of perceptual distinguishability.

One last combination of notes that we did not use at all was combinations of quarter notes and eighth notes. This was largely because, as described in Section 3.2.3, we found the difference between these two note types to be quite perceptually weak. Beyond this, we deemed 21 rhythms to be a fairly large number of rhythms, and so did not want to over-extend our reach. As we wished to test these rhythms at different amplitude and frequency levels, given the importance of these parameters in prior research into tactile stimuli, the number of rhythms we developed would provide us with a large set of stimuli as it was. So we chose not to use all possible rhythms that we could have, mostly for practical reasons. Nevertheless we feel that the majority of the rhythms we did not use would have been perceptually quite difficult to distinguish between, and we feel confident that the rhythms we did select represent the strongest and widest selection we could have reasonably chosen.

Chapter 4: Subset Data Gathering Methodology for MDS

In order to be able to analyze the large sets of haptic stimuli that we are creating, we need to gather dissimilarity ratings from users that we can then feed into the MDS algorithm. However, collecting judgment data from people on that many stimuli at once is unwieldy and impractical. Our insight is to present users with less than the total stimulus set, and then create a total, aggregate view of the stimulus set by averaging overlapping data from multiple users. If we gather data simply by presenting users with pairs of stimuli and asking them to rate their similarity, it is easy to safely use a subset of possible stimulus pairs in a set, but pairwise comparisons also take far too long to perform and suffer from calibration and drift problems as subjects are being asked to make absolute judgments. Conversely, using a different data gathering method such as asking users to sort the stimuli into different groups based on perceived similarity is a far quicker task, but makes splitting apart the stimulus set much harder. Herein we address these challenges by developing a means of sorting stimulus sets that allows us to present users with only a subset of the total stimulus set, greatly shortening the total time and effort required of an individual asked to provide perceptual judgments. In practical terms, this brings about a three-fold increase in the number of stimuli that can be examined (from 50 to about 150). This is a significant result for tactile stimuli which are particularly difficult to gather perceptual data from; further, 150 may approach the limits of distinct stimuli that can be displayed and eventually learned given today's tactile display hardware.

The challenges imparted by this new data gathering method are of two types. The first is in development of an algorithm for splitting up a stimulus set into subsets that can be sorted individually by users and then successfully stitched back together again to form an aggregate picture. Secondly this method faces several challenges in its experimental validity, there being some potentially confounding effects of judgments gathered from only part of the total stimulus set. As we are proposing a novel method for gathering dissimilarity data for MDS, and several potential problems are clearly extant, a means of validating this method must be developed and applied in a real-world experiment. This validation process is described in Chapter 7, but first in this chapter we outline the new data gathering method and discuss its strengths and weakness.

4.1 Other Methods for Dealing with Large Set Sizes

One of the limitations of the basic MDS procedure is that it requires a dissimilarity rating for every pair of stimuli involved. A dissimilarity matrix for a stimuli set of size n contains $n(n-1)/2$ dissimilarity values (since the dissimilarity ratings are symmetric it is only a half matrix, hence the division by two). Consequently the number of dissimilarity values required increases quadratically with the number of stimuli. As the number of stimuli being compared becomes large, it is an increasingly laborious task to gather all these dissimilarity values. Subject fatigue and loss of calibration quickly become a problem. If we are to study a set of 84 (or more) different haptic stimuli using MDS, then we will need a method of gathering data that overcomes this problem of size.

Tsogo *et al.* performed a review of established data gathering techniques for dealing with oversized sets of stimuli [25], i.e. set sizes which are too large for the acquisition methods available for those data. According to their review, there are two main simplifying approaches available to mitigate this problem: using incomplete dissimilarity matrices, and gathering comparisons via sorting tasks, rather than individual pair-wise comparisons. Both of these methods reduce the length of time and amount of work required from users to gather perceptual data, but both eventually come up against hard limitations as to the total number of stimuli they can handle. In this context "pair-wise comparisons" is the method whereby each possible pairing (disregarding order) of two different stimuli in a set are presented to a user, who is then asked to provide a rating of similarity; a "sorting task" is the method whereby users are presented with the entire stimulus set and ask to "sort" or categorize them into groups according to perceived similarity. A third data gathering method that we would also add to Tsogo's accounting is the use of a pre-determined scale that can be applied individually to each stimuli. In this case, the user is presented with each stimulus individually, and asked to rate it on a pre-determined Likert-type scale, as used by Van Erp, for instance, to study tactile melodies

[27]. Since this method requires only one judgment per stimuli, it is only $O(n)$, while pair-wise comparisons require the full $O(n^2)$ comparisons. Sorting tasks require a number of comparisons between those two extremes, though the exact number is not fixed due to its dependence on the individual sorting strategies of each of the users. Nonetheless, these three techniques span the range of data gathering methods for MDS, each with their own strengths and weaknesses, to be discussed below.

4.1.1 Incomplete Dissimilarity Matrices

The insight behind incomplete dissimilarity matrices is that it is not always necessary to have difference ratings for all pairs of stimuli from all subjects. Spence and Domoney [22] investigated how incomplete dissimilarity matrices can be dealt with in perceptual MDS methods, when the data comes from a standard pair-wise comparison task. For every dissimilarity value $d(i,j)$, comparing the ith to jth stimulus, a pair-wise comparison task requires that those two stimuli are presented to a user, and that the user provides a rating of how perceptually similar the two stimuli are. In this case $d(i,j) = d(j,i)$ because judgments are symmetric and the order of stimuli does not matter, requiring $n(n-1)/2$ total dissimilarity values.

Spence and Domoney make two important claims. First, that for an individual, it is not necessary to have a complete dissimilarity matrix in order to get an accurate result from MDS, though each stimulus must have at least one dissimilarity value (connecting it to at least one other stimulus in the matrix), and it should be possible to move from any one stimuli to any other, by chaining along dissimilarity values (*i.e.* there are no unconnected islands of stimuli). Second, that since each judgment in a pair-wise comparison task is independent, it is possible to combine multiple incomplete dissimilarity matrices from different users to create an average, complete dissimilarity matrix. The first claim frees us from having to always guarantee that each individual compares every stimulus in the set, while the second claim means that we can combine dissimilarity values from different individuals to make a total, averaged picture—though unfortunately Spence and Domoney's work on incomplete matrices was performed on data from individuals, rather than averaged values, so the combinations of these two claims cannot be directly made..

Most encouraging of Spence and Domoney's results was their finding that even with one-third of the entries in an individual's dissimilarity matrix removed (either at random or in a cyclical pattern), the resulting difference map (MDS output) varied less than 10% from the map derived from the complete matrix. This result is promising, indicating that we need not be overly concerned with getting *every* difference rating for a set of stimuli. That there is a certain amount of looseness to how many difference ratings must be gathered, and from whom they must be gathered from, gives us much greater room to devise new data gathering methods of our own.

However, if we wish to gather only particular dissimilarity values in either a cyclical or random fashion as Spence and Domoney recommend, we are required to use pair-wise comparison between each stimulus, because it is the only technique that allows you to individually pick the exact dissimilarity values you want without getting any other values, unlike other data gathering methods such as sorting. The number of pair-wise comparisons needed for a set increases exponentially with the number of stimuli. Removing one-third of all entries may be a fairly large amount, but it is still one-third of a quadratically increasing amount, meaning that the incomplete dissimilarity matrix method will always eventually run into issues of having too many stimuli to judge in one experimental task. Furthermore, pair-wise comparisons have been found to have almost twice the level subjective fatigue as compared to sorting tasks [1]. So this method is limited both in number of stimuli that can be compared as well as accuracy of ratings given.

4.1.2 Sorting Tasks

Whereas Spence and Domoney show that it is not necessary for each participant to compare every stimulus to every other stimulus in the set, the sorting task method seeks to make the act of comparing stimuli much more efficient by having subjects compare all stimuli at once and sort (or categorize) them into discrete groups based on perceived similarity. Dissimilarity matrices can then be created using the number of times that two stimuli occurred in the same group as a measure of their similarity (and inversely, their dissimilarity). Often participants are required to sort the stimuli multiple times into different numbers of groups, in order to give varying levels of resolution.

A variant of this method was used by MacLean and Enriquez [19] and analyzed in [20], to determine the perceptual characteristics of a series of haptic stimuli. Their results show that allowing users to sort a large set of stimuli into different numbers of groups provides a strong and robust measurement of the perceived differences between a fairly large set of stimuli, while greatly shortening the overall time of the experiment. Unfortunately, even this method is still limited in the number of stimuli that can be judged. Maclean and Enriquez tested 30 stimuli in their study, a number significantly lower than our own goal, and informally guessed that a maximal reasonable set size by this method and using stimuli of this sort is 40 or 50.

4.1.3 Per-stimulus Judgment Tasks

The per-stimulus judgment task is a particular type of data gathering method, that differs largely from the previous two methods in that it provide a confirmatory rather than explanatory description of the stimulus set. Van Erp and Spape [27], in a study particularly relevant to our own, analyzed 59 different tactile melodies by asking participants to judge the melodies according to 16 different pre-determined criteria such as "cheerful" or "polished," each on its own 5-point Likert scale. This allowed them to get judgments on all 59 melodies within a reasonable time-span and gave them data to which they were able to apply MDS. However, it is clear that they approached the data with a fixed belief about what aspects of the stimuli would be important to peoples' perceptions—specifically the 16 criteria on which they asked participants to judge the stimuli. Though one could, by finding unexpected correlations between parameters, perhaps indirectly discover new features of the stimulus set, it would be difficult to directly discover any completely new and unforeseen perceptual parameters. While this technique may be acceptable when some idea about the nature of the stimuli already exists, in our own case we know so little about the stimuli that we wish to make no assumptions about how people will perceive them going into our experiment, so as to minimize any bias we might have on the results.

4.2 Design of Proposed Subset Data Gathering Method

Though the methods described in Section 4.1 have been widely used to handle large sets of stimuli, they still fail to sufficiently reduce the time and effort needed to gather data for the goals defined here. Though the per-stimuli judgment task would be fast enough, it contains too many assumptions about the stimuli for our purposes. Both sorting tasks and incomplete matrices are valid attempts at reducing the number of comparisons needed to get useful MDS data, but again, take too long to be practical. Sorting 84 haptic stimuli takes approximately two hours to perform.

Yet it might seem strange that we so quickly came to the limits of the existing methods for dealing with large stimulus sets. The reason this is so is because of the nature of haptic stimuli, especially in our own case. With a two second duration, the comparison of any two stimuli will take at a bare minimum four seconds, and likely much more if an individual wishes to feel the stimuli multiple times. Compared to visual stimuli, which can be viewed simultaneously and in the manner of a few milliseconds, it quickly becomes clear why data gathering for our own stimuli is so much more difficult. Add to this the general lack of experience people have with haptic stimuli (compared to aural or visual stimuli) and we are confronted with a situation where the cost of each single comparison is considerably higher for haptics compared to other modalities that MDS is regularly used for.

Given this difficulty, we asked whether the sorting task could be combined with incomplete matrices to further cut down on the number of comparisons need to gather perceptual data. This idea forms the basis for our novel MDS data gathering method: using a sorting task on a subset of the total stimuli, and building up an aggregate result by piecing together dissimilarity data from multiple differing subsets. By using less than the total number of stimuli in a sorting task, we can ensure that a participant will be able to complete their experimental task within a reasonable timescale. However, splitting up the simulus set into subsets creates several difficulties which are discussed throughout the remainder of the chapter. As soon as each participant no longer experiences every stimulus in the set, many potential issues arise of study design (which and how many stimuli should they get) and study validity (can judgments given from only a subset of the stimuli apply to the whole).

Through informal testing it was determined that a subset of size 50, with 3 complete sortings into 3 different numbers of groups, would take a participant roughly an hour to complete, and so this became our target subset size. By giving participants different subsets that cover different portions of the set of stimuli, we can gather dissimilarity data about all of the stimuli in the total set. Averaging together the results from the different participants can then give us a total picture of the perceptual space for a given set of stimuli. What this subset method of gathering MDS data does is essentially forego unreasonably long individual experiment session durations by using a larger number of participants to gather the same amount of data.

4.2.1 Creation of Subsets

The primary challenge to this new method is in determining how large to make each subset, and how to distribute stimuli amongst subsets in order to ensure each individual's results can be aggregated into the whole to produce accurate overall judgment ratings. Because we wished to avoid biasing results, we chose to create random subsets, giving each random subset to just one participant to judge. We hoped this would minimize data bias due to the way a particular subset was perceived. However; the creation of random subsets is actually a somewhat more complicated matter if we are concerned with gaining an even coverage of judgments across the entire dissimilarity matrix. Uniform coverage is desirable because it minimizes the number of participants needed in order to achieve a required number of observations for each point in the matrix.

To this end, we developed a program that attempts to minimize the number of randomized subsets required to ensure that each value in the dissimilarity matrix has at least the specified number of observations. The algorithm is given the total size of the stimulus set that is to be used, the size of the subsets desired, as well as the minimum number of observations that each point in the dissimilarity matrix needs to have, and produces as many randomized subsets as is required by the given parameters.

Unfortunately it is non-trivial to produce a group of subsets which provide *only* the requisite number of observations to each point, due to the fact that every stimulus that is added into a subset will be compared with all other stimuli in the subset. For example, if a subset contains stimulus 2, and we are still lacking comparisons of stimulus 2 with 3, and 2 with 4, it is impossible to get those comparisons without also getting a comparison between 3 and 4. If another subset already exists with stimuli 3 and 4, then an overlap between the two subsets is unavoidable. This problem becomes progressively more complicated as the number of subsets increases, requiring more and more comparisons in order to achieve a minimally overlapping set. We have dealt with this problem with the following algorithm.

Description of Subset Algorithm

Our algorithm contains a two-dimensional array which keeps track of the number of observations (*NO_cur*) for each value in the dissimilarity matrix, and tries to make sure each value reaches the minimum number of observations (*NO*) without going over. It does this by continually adding in stimuli to new subsets, trying to bring *NO_cur* up to *NO* for each value. Thus at all times a list is kept for each stimulus detailing how many observations are needed against which other stimuli. This list is called a stimulus' free-set (as in, there exists free space to add in new observations), and it is essentially a list of stimuli that this stimulus still needs to be compared with (which means they must appear in the same subset).

The algorithm begins by selecting the first stimulus to be placed in a new subset, choosing the stimulus with the largest free-set (*i.e.* the largest number of stimuli it still needs to be compared against). Thus the most "greedy" stimuli are always dealt with first. Then the following loop begins:

- A stimulus is selected that is in the free-sets of all the simuli already in the subset.
- If there is not one single stimulus that all the stimuli already in the subset have in their free-set (as is often the case), then the new stimulus is selected according to the following criteria (in decreasing priority):

1. The new stimulus should occur in the largest number of free-sets of stimuli already in the subset. If the stimulus is *not* in an already selected stimulus' free-set, the previously selected stimulus will end up with more than *NO* observations for that point. By minimizing the number of values in the matrix that receive more than *NO* observations, we ensure that we get as even coverage of observations as possible.
2. Provided the new stimulus is in as many free-sets as possible, the next check is how much of its own free-set overlaps with the other stimuli's free-sets. The stimuli with the largest amount of overlap is chosen. This will increase likelihood of meeting criteria 1 when more stimuli are selected in the future.
3. The new stimulus should have the largest free-set possible (ie, it should be the most "greedy"). Though ultimately, the greedy stimuli need to be dealt with most urgently, if we only ever grabbed the greediest stimuli without regard to anything else, we might quickly reach a point where it is impossible to add in new stimuli to the subset without creating many values in the matrix with greater than *NO* observations.

- Once the new stimulus is chosen, it is noted which required observations have now been accounted for (and which non-required observations have now been added as well).
- The process then repeats itself until the subset has been filled, and then starts again on a new subset until all required observations have been filled.

Pseudocode

```
User specified constants:

NO - minimum number of observations needed for each value in
        dissimilarity matrix
Stimulus_set_size - size of the total stimulus set to be used
Subset_size - size of subset to be used

Main Loop:

Variables:
MATRIX - 2D array, of Stimulus_set_size, used to keep track of how many
```

```
        observation each value in the dissimilarity matrix will have,
        given the subsets thus specified
NO_temp - number of observations that we wish to obtain in this
iteration of the loop

For NO_temp = 1 to NO
    While MATRIX still has values < NO_temp, do

        Call CreateSubset

        Add observation to MATRIX caused by new subset

        Save new subset to file

    End loop
End For
```

CreateSubset:

```
Variables:
SUBSET - array returned containing all stimuli in this subset
STIMULI - array of all available stimuli that could still be added
    to SUBSET

While SUBSET < Subset_size Do

    Populate STIMULI with all stimuli not in SUBSET

    //
    // First Criterion
    //
    For each stimulius in STIMULI
        number_of_conflicts = how may values in MATRIX would
                        be > NO_temp, if stimulus was added to SUBSET

        If number_of_conflicts < min_conflicts,
           min_conflicts = number_of_conflicts
    End For

    Remove all stimuli from STIMULI with
                        number_of_conflicts > min_conflicts

    If size of STIMULI is one,
```

```
    add remaining stimulus to SUBSET, iterate loop


//
// Second Criterion
//
For each stimulus in SUBSET
     free_list = list of stimuli that still need to be compared
                    to stimulus in order to reach NO_temp
End For

subset_free_list = intersection of all stimuli's free_list

If subset_free_list is empty,
     skip to Third Criterion

For each stimulus in STIMULI
     free_list = list of stimuli that still need to be compared
                    to stimulus in order to reach NO_temp

     overlap_size = size of intersection of free_list
                    and subset_free_list

     If overlap_size < min_overlap,
        min_overlap = overlap_size

End For

Remove all stimuli from STIMULI with overlap_size > min_overlap

If size of STIMULI is one,
     add remaining stimulus to SUBSET, iterate loop


//
// Third Criterion
//
For each stimulus in STIMULI
     free_set_size = number of values in MATRIX along stimulus'
                    row or column that are < NO_temp

     If free_set_size < min_free_set_size,
        min_free_set_size = free_set_size
```

```
    End For

    Remove all stimuli from STIMULI with
                        free_set_size > min_free_set_size

    If size of STIMULI is one
        add remaining stimulus to SUBSET, iterate loop
    Else
        randomly choose stimulus from STIMULI,
        add to SUBSET, iterate loop

End Loop
```

Problems with algorithm

Because this algorithm was not the focus of this thesis, the actual end program written takes one major shortcut for the sake of efficiency and simplicity. In order to find a truly minimal number of subsets, the calculation performed for Criterion 2 should not just check whether the next stimuli will have a maximum amount of overlap with the free-sets of the stimuli in the subset, but should also check whether stimuli that are in that overlapping free-set, if chosen, would produce good results. That is to say, it is possible that choosing a stimulus that has a smaller amount of overlap compared to some other stimulus might actually work out better in the long run, because the stimuli that are in that overlap might in fact be better choices than the stimuli in the larger overlapping free-set. That is, choosing these stimuli might not lead as quickly to a point where the only stimuli that can be added in will create overlap points where the number of observations is greater than *NO*. We realized this error, but had to cut short our development time in order to proceed with the rest of our research.

Consequently the subsets created are not necessarily the most mathematically optimal non-overlapping subsets, though for our purposes they do provide reasonable coverage and randomization (see Appendix C for examples of actual subsets used in our studies). One exception to this is the tendency for the distribution of overlapped and non-overlapped points to clump together, as stimuli chosen in earlier sets tend to be used less in the later sets, which are more constrained in which stimuli they can select. A more thorough and mathematically complete algorithm could be developed, but is outside the scope of this work. This effect can be seen clearly in Chapter 7, Sections 7.2 and 7.3, where this subset method is used in a full study and its results are discussed.

4.2.2 Robustness and Scalability

As this new data gathering method is designed to accommodate larger sets of stimuli, it is reasonable to ask how large a set this method can handle. The tradeoff that our method offers is that instead of increasing the length of time that a particular individual must spend judging stimuli, an experimenter may simply increase the number of individuals judging stimuli, with the amount of time per individual staying constant. Once an experimenter has figured out how large a stimuli set a person can be reasonably expected to sort within a target time frame (usually an hour), and decided what the minimum number of observations each point in the aggregate dissimilarity matrix should have, then our subset algorithm will be able to provide as many subsets as needed to acquire the requisite number of observations. At this point it is simply an issue of finding enough participants to run through each of the subsets, and then the data will be collected. Thus, theoretically, whatever the set size, it is only an issue of using enough participants in order to gain the necessary data.

However, in reality there are several concerns in regard to the scalability and robustness of this technique in the face of increasingly large stimuli sets. The first is the size of the subsets used to gather judgments: the smaller the subset, the more participants required to gather data, as well as the greater the potential for disagreement in judgments from different subsets, especially if the superset is large or perceptually complex. Another concern is the number of subsets (and thus participants) that will be required to satisfy the total number of observations specified. Last is the number of overlapping observations required in order to overcome any variability brought about by the large number of different participants contributing to the overall average, as well as any noise brought about by subsets whose small size might create idiosyncratic judgments from participants.

Size of Subset Required For Data Collection

One of the first issues an experimenter must deal with when using the subset method is determining the size of the subsets to be used. In order to gather the necessary judgment data as quickly as possible, as large a subset size as possible is desired (the exact benefit, in terms of decreased number of participants, is discussed later below). Consequently, it is suggested that through trial runs with sample participants, the maximum size of stimulus subset that can be sorted within a reasonable timeframe (usually about an hour) be determined for a given hardware and stimulus set combination. However, the maximum size is not the only concern with regard to subset size: there is also the issue of whether there is a *minimum* size which a subset must be larger than, in order to gain judgments that, when averaged together, will accurately reflect the entire stimulus set.

Though finding a time-driven maximum subset size is not overly difficult to determine, the issue of minimum subset size is slightly less clear-cut. Small subset sizes would limit the number of different stimuli a participant was exposed to, giving them a smaller "world view" from which to make their judgments. While their judgments within this world view would be valid, averaging them with other judgments that came from different world views would likely cause noise in the data. It is always possible to gather more observations in an attempt to counter the noise, but if the subsets were different enough it could be that no amount of observations could cause the values to converge in agreement.

What would likely determine if a given subset size was too small to produce converging results would be the actual number of underlying perceptual dimensions of the total stimulus set. The more perceptually complex the stimulus set (*i.e.* the more dimensions it has), the more variability there might be in judgments from different subsets. If the stimuli were only ever perceived as being "A" or "B" then even with very small subset sizes, there would likely be very little disagreement about how each of the stimuli were grouped. It is when there is a wide variety of stimuli that subsets can end up with far more of one type of stimuli than another, and perhaps another type of stimuli not present at all. It is this type of uneven distribution that would cause greater variability between subsets and thus would require greater numbers of observations to counteract. This creates a somewhat paradoxical situation: how can we know how complex the stimuli set is (and thus the size of subset needed), when that is the very thing that MDS is supposed to discover?

Thankfully, practical considerations ensure that we rarely come truly face-to-face with this issue. To begin with, as alluded to above, and discussed more thoroughly below, very small subset sizes are quite impractical since they require a huge number of subsets (and thus participants) even to gain a bare minimum number of observations. Secondly, when deciding upon a stimulus set to study, experimenters are rarely without any intuition as to how many perceptual dimensions there might be—it is usually clear roughly how perceptually complex a set of stimuli is. Furthermore, in analysis of perceptual MDS results, it is very rare to deal with a result of dimensionality greater than 4 due to problems of visualization; a quick review of papers involving perceptual MDS finds very few analyses larger than even 3 dimensions. Consequently the level of perceptual complexity that a stimulus set has is often specifically designed in order to ensure its interpretability. This does not mean experimenters could not unwittingly produce a stimulus set too complex for the size of subset they specified, but careful selection of stimulus set, as would be done for any MDS study, will likely minimize this danger.

As a general rule, an experimenter should ensure that the subset size is large enough such that several stimuli that exhibit any given type of parameter (and any of the particular levels that that parameter might have) be present in any random subset. Though, as discussed above, there is no guarantee that there might be unforeseen characteristics in the stimulus set, as long as each of the various known (or assumed) major parameters has some representation in each subset, then it is likely that most important perceptual characteristics will be gathered. The simplest way to achieve this is to have a subset size as close to the superset size as possible without the subset become too large to sort with a reasonable effort. Of course this is generally not possible (since it was too-large supersets that this technique was designed to deal with in the first place), and so, as the subset size decreases and the chances of particular dimensions of the stimulus set being left out of a given subset increase, more randomized subsets (judged by more participants) are needed to counteract the increased level of noise and disagreement.

Number of Subsets Required For Data Collection

When each participant is tested on a unique subset, the number of subsets required is equal to the number of participants required. We will discuss here a version of our subset creation algorithm in which each participant is tested on a unique subset, which provides the specified number of observations in the fewest subsets possible. Repeating subsets with multiple participants can also be done if desired (this is, in fact, done in Chapter 7 to help validate the subset method), but is less efficient in terms of number of participants run.

In our subset creation algorithm, there are three factors that affect the number of subsets required for an experiment: the size of the total stimuli set (*NT*), the size of the subset (*NS*), and the minimum number of observations required (*NO*). In our own practical experience, a bare minimum of five observations per value in the dissimilarity matrix is required for reasonable results, though there are exceptions, as discussed below. Assuming that the size of the subsets is large enough to capture significant characteristics of the stimulus superset, as discussed above, and holding the minimum number of observations constant, it is the ratio between *NS* and *NT* that affects the total number of subsets required. The total size of the stimulus set does not have an effect, as the coverage of both subset and total set grows as a quadratic function of their size, thus ensuring that our subset algorithm will produce the same number of subsets for a pairing of $NS = 50$, $NT = 100$ as $NS = 5$, $NT = 10$; there will simply be a factor of ten fewer stimuli in the subsets produced for the latter as opposed to the former.

In Figure 4.1, we show a curve of the number of sets required to obtain at least five observations in each point in a dissimilarity matrix plotted against the *NS*/*NT* ratio. As can be seen, the smaller the ratio, the greater the number of subsets required, to the point that any ratio lower than approximately one third will likely require far more participants than any experimenter would be willing to run. What this means is that though our new technique is theoretically unbounded, in reality there is a cap on how large a stimulus set can be tested with it. We cannot state exactly what the upper limit on total set size is though, because it is dependant on how many stimuli could be put in a subset. For our purposes, with our target subset size of 50, we would probably not want to have a total stimulus set size of much greater than 150. However, our target size of 50 is at least partly limited by our means of sorting and the nature of the stimuli themselves.

Number of Observations Required for Data Collection

The curve in Figure 4.1 represents, in truth, a lower bound for the number of subsets needed for a given *NS/NT* ratio, as *NO* is fixed at five, a value we have found sufficient for our own purposes, but may under other circumstances be insufficient. More observations are needed to deal with noisy data, which can result from two main causes: greater variance within the pool of participants, or too-small subsets. Individual differences will always be a factor, but the effect of subset size on the noisiness of the judgment data will vary from stimulus set to stimulus set.

As discussed above, the likely determining factor in the size of subset (and thus the number of observations needed) is actually the true underlying dimensionality of the stimulus set. Obviously if *NS* was equal to *NT*, then all subsets would be the same and the only cause of variance would be from individual differences. But as *NS/NT* gets smaller, the difference between individual subsets increases, as they have less chance of overlap. This means that the view each participant has on the stimulus set differs by more and more. We advocate completely randomizing subset selection in order to cover over differences between subsets, so that each aggregate dissimilarity value is built up of values from enough different subsets so as to cover over any large, subset-specific variances. Given this, it would seem clear that smaller subset sizes would require a larger number of observations per dissimilarity value, in order to deal with the higher level of variance. Perceptual judgments from an *NS* of 2, for example, would likely differ hugely, and it may even be that no number of observations would ever create a complete picture of the stimulus set with such a small *NS*.

Thus in each case, the particular combination of *NS*, *NT* and *NO* that will meet an experimenter's needs will have to be decided individually. Nevertheless, given a not too diverse set of stimuli, our method can cut the number of stimuli that need to be presented to a user by a third, while still only requiring a very feasible number of participants to run the experiment. Such a decrease is of great use for our own goals, but also of general use to anyone who wishes to gather perceptual dissimilarity data about a large set of stimuli.

4.3 Potential Threats to Validity of Method

The subset method combines two previously validated experimental methods—sorting tasks and incomplete dissimilarity matrices. Joining the two together, however, by no means implies the validity of the combination. Certain characteristics of the sorting method throw into question the validity of results produced by using comparisons collected using anything other than the entire stimulus set.

4.3.1 Incomplete Individual Results

Firstly, compared with the straight pair-wise comparison task where it is easy to remove just a single comparison (because each comparison is independent of all others), in the sorting task it is impossible to remove one comparison without removing all of the comparisons of a given stimulus. This is because in a sorting task all stimuli present are compared against all others, so removing one stimulus removes all the comparisons of that stimulus against all the other stimuli present. Thus there is no way to create an MDS plot from just one individual containing all of the stimuli in the set (as in the incomplete but completely connected set advocated by Spence & Domoney [21]), and if each individual is given a different subset, it also means that each individual's MDS plot will involve (at least some) different stimuli. Consequently it will be very hard to compare individual MDS plots directly with each other, as a means of determining how consistent different people were in judging the stimuli set. Comparison of individual results is a useful tool in proving the quality of the averaged results, and the subset method is hurt by not having a direct means of performing this comparison.

4.3.2 Subset-relative Judgments

Unfortunately, creating average results from a series of incomplete dissimilarity matrices is also problematic due to the inter-dependant nature of all perceptual judgments performed in a sorting task. In a pair-wise comparison task, each comparison is dependant only on the two stimuli being compared (subject learning effects over time are assumed to be negligible). However, in a sorting task, each stimulus is being compared, either explicitly or implicitly, relative to all other stimuli present, and thus a change of one stimulus could completely change the groupings of all other stimuli. What this means is that an incomplete dissimilarity matrix produced by a sorting task comparison of one subset of stimuli versus a different but overlapping subset of stimuli could produce radically different dissimilarity values even for the stimuli in the intersection of the two subsets.

As an example, consider a total stimulus set in which one stimulus was played at ten times the amplitude of any of the other stimuli. If a participant was presented with a subset without the very loud stimulus, the relative amplitude differences of the remaining stimuli would seem more salient. However, if the very loud stimulus was present in the subset, the participant might now judge all the remaining stimuli to be at the same amplitude level, because the differences between the rest of the stimuli is so small compared to the difference between the very loud stimulus and all the rest. In this way the presence or absence of one stimulus could potential produce very different sorting strategies leading to very different results. Thus any attempt at averaging over all dissimilarity values could be potentially covering over a very noisy set of data, producing averages that essentially reflect no real-world population. However, it is not clear how strong an effect the relative nature of these judgments would have on the resulting dissimilarity matrix; or stated another way, how many repetitions, assembled from many participants, would be required in order to diminish the effect of this noise (as discussed in Section 4.2.2). This potential threat to validity is why we emphasize the randomization of subsets. We believe that cases of the above happening will likely be fairly rare, if the stimulus set is well designed. Thus if all participants are presented with a unique, randomized subset, we believe that instances of this problem occurring should, on average, be covered over by the far greater number of reasonable, well-formed subsets.

4.3.3 Ability to Discover Overall Perceptual Trends

An additional problem that arises from averaging together different subsets is that due to overlap points, some values in the dissimilarity matrix will be averaged over observations from a larger number of participants than other values. This might, in essence, make certain dissimilarity values more "trustworthy" than others—that is, less likely to contain aberrant, fluke results. One option for dealing with this problem is to use some sort of weighted MDS, with each dissimilarity value weighted according to the number of observations or the standard deviation. However, by choosing to use a method that specifically aims to make the number of observations per value in the dissimilarity matrix as even as possible, we can deal with this issue without resorting to more complicated MDS models. Thus it will be important in experimentation to check the number of observations and/or consistency of the standard deviation of the various values within the dissimilarity matrix to ensure that they are not having an adverse effect on the MDS results, though we will not have a strict mathematical means of analysis for these features.

4.4 Pilot Study: Initial Study on Voicecoil Vibrators

An exploratory pilot study was run to determine the issues involved with both the rhythmic haptic stimuli discussed in Chapter 3, as well as the subset method of data gathering for MDS discussed above. The results of this study were used to inform the more thorough and detailed studies discussed throughout the remainder of this thesis.

4.4.1 Apparatus

For this study, vibrotactile stimuli were emitted from the transducers VBW32 Skin Stimulators from Audiological Engineering Corp. MA. The peak frequency transmitted by the device is 250Hz with a usable output range from 100Hz to 800Hz. The transient response of the device is 5ms. The experimental software responsible for presenting the vibratory stimuli was written in VB6, which logged results in a .csv format. The experiment was run on a Dell laptop running Windows XP.

4.4.2. Participants

Thirteen university undergraduate and graduate students (8 females) with ages ranging from 19 to 36 years were recruited for this study.

4.4.3 Stimulus Set

Via brief preliminary psychophysical testing, two frequencies and two amplitude levels were determined for use with the 20 of the 21 rhythms discussed in Chapter 3. We were still in development of our stimulus set at the time, and so did not use rhythm 4 in our set (see Table 3.2 for explanation of rhythm numbers). The two frequency levels were 150Hz and 300Hz, and the amplitudes were defined as the maximum volume output of the vibrators, along with the threshold amplitude level, as determined for each of the frequency levels. This resulted in a set of 80 stimuli, comprised of 20 rhythms x 2 amplitudes x 2 frequencies.

4.4.4 Procedure

We generally followed the method of [19]. Participants sorted the entire 80-stimulus rhythmic haptic stimulus set using the apparatus described above. Each participant completed 3 sorting tasks on the same stimulus set. At the beginning of the study, participants were instructed to feel each stimulus by clicking on to each numbered tile organized at the bottom of the screen, and to group stimuli that felt the same in the same boxes. Participants were also told that they could feel the stimuli as many times as they needed by clicking on the tile again, and were allowed to change their mind about the groupings by clicking and dropping the tile in the desired box. In the first sort, participants were told to group stimuli into whatever number of discrete, non-overlapping groups they felt was appropriate to describe the perceived dissimilarity between stimuli. For the remaining two sorting tasks, participants were required to sort the stimuli into a specified number of groups, either 3, 9 or 15. Of these three group numbers, the one closest to the number of groups chosen in the first sorting task was not used, with the remaining two numbers randomly assigned to the second and third sorting tasks. Having three repetitions of the sorting task performed on the same set of stimuli and varying the number of groups that the stimuli are sorted into as we have done has been shown previously in haptic MDS studies to yield good resolution for perceived differences [20].

4.4.5 Results and Discussion

An average dissimilarity matrix was constructed from the participants' data, and then run through the SPSS ALSCAL algorithm for 1 to 5 dimensions. Graphing the resulting stress values shows no clear elbow indicating a point of diminishing returns in terms of goodness of fit (Figure 4.2); instead both 2 and 3 dimensional results provide reasonable stress levels, while higher dimensions are somewhat decreased in improvement. For the sake of parsimony as well as ease of interpretation, the 2D solution was chosen as the primary solution for analysis, though the 3D solution is given some consideration as a secondary tool for analysis.

Several features are immediately evident from visual inspection of the 2-D perceptual map (see Figure 4.3). First is the clear circular arrangement of the stimuli around the center of the graph. According to MacLean and Enriquez [19] this circumplex arrangement is a common result in perceptual MDS studies, including those involving haptic stimuli, resulting from judgments of stimuli as either very similar or very dissimilar according to the frequency and amplitude of the vibrations used, regardless of rhythm. This trend is further emphasized by the projection of the design parameters of amplitude and rhythm onto the perceptual map, as they both neatly bisect the map in nearly orthogonal directions. This projection is done by averaging the location of all the stimuli that have one value of the parameter, plotting the points, and drawing an axis between these points. The length and placement of these axes on the map indicate their overall importance in the perception of the stimuli. This result is consistent with those of MacLean and Enriquez who found frequency and amplitude were both extremely important perceptual features, as well as being highly correlated in terms of perception.

The second salient point is the even spread of stimuli around the circumplex distribution and the general lack of clustering amongst the stimuli. A cluster of stimuli around one position indicates that people perceptually group them together as being related in the visible dimensions. A lack of any such clustering indicates an overall level of distinctiveness and discernability of our stimulus set, such that even stimuli that are perceived as being perceptually similar to each other are nonetheless perceived as distinct and separate sensations. MacLean and Enriquez [19] with a similar circumplex arrangement, nonetheless also showed clear clustering in mid-process results, a result distinctly different from our own.

It is noted that in that previous work, iterations were performed for the distinct purpose of designing non-clustered stimuli set with maximum perceptual 'spread', with the MDS result guiding adjustments. In other cases, clustering might be desirable in order to promote 'family' associations of meanings (e.g. Enriquez, Chita & MacLean [12]).

Thus we are left with the question of why the MDS plot shows no clear clustering, while other similar studies have. Specifically we wonder what effects rhythm has had on the perception of the stimuli, and whether it has played any role in this lack of clustering. The only clear rhythm effect is the one outlier stimulus, situated far outside the circumplex of stimuli, which is the full 4 quarter note rhythm, stimulus 1, played at the highest frequency and amplitude. That this is the most distinctive of all rhythms is to be expected, as it was the simplest according to our interpretation of rhythm, and it was played at the most clearly discernable frequency and amplitude combination used. Yet the remaining stimuli's marked descent into a cloud of opaque rhythm effects is made all the more frustrating because of the tantalizing promise of this one outlier.

We are thus left with ambiguous, noisy results and several possible explanations for this ambiguity. Given that the experimental method used was untested, one obvious possible explanation is that the new data gathering method introduced too much noise into the data. However, since the stimulus set is also unique and untested, it could also be suggested that the results of this study accurately reflect the difficulties people had in perceiving similarities amongst rhythmic haptic stimuli. Yet another issue is that vibrators used to display the stimuli may have lacked sufficient dynamic range and responsiveness to effectively display the more complicated haptic stimuli used here. The difficulty in resolving these possible explanations led us to design and implement the studies described in Chapters 6 and 7, hoping to determine the validity of the experimental method used and the true perceptual nature of the rhythmic haptic stimuli created.

Chapter 5: Methods

The apparatus used in the pilot study – voicecoil vibrators attached to the sound-card output of a PC – has been used before successfully to create and test haptic stimuli. However, we felt that we may have been approaching the limits of this setup. The voicecoil vibrators were not as precise in their output as might be desired, leading to worries that our more complicated haptic stimuli might not be being displayed in complete detail. Furthermore, the voicecoils were separate from the PC, making them somewhat awkward to naturally introduce into regular application use. Thus when the opportunity arose to use prototype hardware from Nokia based on piezo technology, we gladly took advantage of it. It offered more precise timing and control over feedback, and an embedded platform that could support new haptic applications without any additional peripheral devices. In 5.1, we describe the hardware, a relatively new type of handheld haptic display on which very little haptic icon work has been done previously. In 5.2, we discuss the sorting program that we were required to write, in order to perform data gathering on the new handheld platform.

5.1 Discussion of Hardware Platform

The Nokia 770 (Figure 5.1) is a handheld internet tablet, with a large (90x54 mm) high-resolution (800x480) screen, ARM-based processor, and runs a modified version of the Debian Linux distribution. While the 770 is already commercially available, Nokia has added haptic feedback to a prototype model, identified as the 770T (see [16] for details). Though visually identical to the 770, the 770T has a piezo-mounted touchscreen, which allows the screen to be pulsed with small displacements in the axis orthogonal to the screen, giving the sensation of a single "click" when done once, and of a continuous vibration when done repeatedly at tightly spaced intervals. This technique can give quite convincing and satisfying haptic feedback, all within the context of a handheld device.

We are much indebted to Nokia for supplying us with several of these devices along with their technical support. What follows is a discussion of the new hardware platform's suitability to our own ends, with regard to creating and analyzing a large set of rhythm-based haptic stimuli.

5.1.1 Control of Haptic Feedback

Haptic feedback in the 770T is controlled through the use of feedback scripts, which are compiled into byte-code and sent to the hardware that controls the piezos. The feedback scripts consist of a series of commands for driving the piezos. There are five main commands: *charge*, *discharge*, *delay*, *loop* and *voltage set*. The *charge* command tells the device to begin charging the piezos and can specify the speed (by specifying the resistance of a current limiting resistor) that the piezos will be charged at. This creates the leading edge of a single "click" motion. The *discharge* command causes the piezos to discharge, thus creating the trailing edge of a single "click" motion. The *delay* command is used to specify timing between clicks and between charges and discharges. The *loop* command is used, as would be expected, to simply specify the number of times a set of commands should be repeated. Lastly the *set voltage* command sets the overall voltage level to which the piezos will be charged. No more than 255 total commands can be used in any one feedback file.

These feedback files, once compiled and loaded into the hardware, can be associated with a given type of GUI widget (for example, a button or a scroll bar) or specific individual widgets, and the feedback will then be played whenever the click event for the specified widget is fired. The 770T hardware only has space for 16 user-defined feedback files to be loaded into the hardware at one time, though multiple widgets can be mapped to the same feedback file.

In order to create sustained vibrations which can be used to make up a rhythm, consecutive series of closely spaced clicks had to be placed together to build up what is essentially a square wave playing at a given frequency. These vibrations give us the notes that can be used to make rhythms, while the delay command gives us the off-notes. Thus a single haptic feedback file could be used to make an single haptic stimulus from our rhythm set.

5.1.2 Baseline Perceptual Data

According to [22] the most perceptually salient parameters to be varied with the piezo touchscreen are the duration of the voltage curve and the speed at which the leading edge of the curve rises. These parameters corresponded roughly to the perceived amplitude or "strength" of the feedback. This claim was confirmed via our own informal user testing; in further agreement with [22], our testing also showed that the height of the voltage curve had an insignificant effect on the perceived strength of sensation, thus it was decided that the default voltage level of 173 V would be used for all feedback.

In order to create rhythms, we first needed to determine how to make continuous vibrations that were distinct. This necessitated a small, informal experiment in which users were presented with different combinations of feedback strength and vibration frequency, and asked to order them from strongest to weakest. For amplitude, we used wave durations of 0.5 ms (low) 1 ms, 2 ms, 4 ms and 10 ms (high amplitude) and resistance levels starting at 13.2 (low) and moving up to 1.0 kOhm (high amplitude) in 10 even intervals, thus controlling the sharpness of the leading edge of the voltage curve (curve rise time) as well as the length of the curve. High resistance levels produced low perceptual amplitudes because they decrease current thus slowing curve rise time Frequencies ranged from 150 Hz to 300 Hz, at 50 Hz intervals.

From this it was found that there were generally four levels of perceived intensity of signal. Frequencies of 150, 200 and 250 Hz were all perceived essentially the same; they felt very strong and distinct to the touch. Vibrations played at 300 Hz felt much softer. Only the two extremes of voltage curve rise time were distinctively different, but they did tend to dominate the perception of curve duration. Voltage curve durations of greater than 1 ms were found to cause no perceptual differences when occurring within a vibration, while the difference between 1 ms and 0.5 ms was evident, but perceptually it was generally overwhelmed by frequency and curve rise time. Thus for the purposes of creating rhythms, we selected one *wave duration* (1 ms), two *curve rise levels* (1.0 and 13.2 kOhm), and two *frequencies* (200 Hz and 300 Hz). Thus we have a high and a low amplitude and a high and a low frequency vibration, providing 4 different vibrations that we could use for our rhythms.

5.1.3 Advantages and Disadvantages of Hardware

From our initial experiences using the 770T, we observed that compared to the voicecoil vibrators as used in the pilot study, the piezo-driven 770T provided much more crisp and precise feedback. Though it is perhaps not able to produce as strong a sensation, the quick reaction times of the piezos were felt to have delivered a much more distinct feedback with sharp starts and stops, whereas the voicecoils had more noise associated with its edges, creating feedbacks that were not as well defined, feeling "mushier" to the touch. This, coupled with the very precise timing control provided by the feedback scripts, gave us hope that the 770T would have an increased expressive capability, making it easier to distinguish small differences between haptic stimuli, and generally giving greater discriminatory power to our haptic stimulus set.

Nevertheless, the 770T did have its drawbacks. As mentioned, amplitude of feedback given was generally less than the voicecoil vibrators, but in addition to this there was a strong audio component to any feedback given on the 770T due to the vibration of the screen within the casing. This sound required noise-cancelling headphones to be worn at all times during any testing of feedback on the device, with fairly loud white-noise having to be played in order to drown out the sound, which can be fairly intrusive and annoying to users. Another serious drawback was the hardware limitations on the number of commands per feedback file and the number of feedback files that can be active at any given moment. Though these problems could be worked around, they did create difficulties in the development process and somewhat hampered the controllability (and ease of programming) of the overall system.

Overall, we felt that the quality of the haptic feedback was well worth the switch to the new device. Furthermore, with an open-source operating system and a large (for its size) graphical display, it was felt that the 770T represented a strong platform on which to develop new haptic applications, and thus was worth choosing as a device to characterize and study.

5.2 MDS sorting program

As outlined in Chapter 1, two different MDS studies were required in order to both perceptually characterize our new rhythmic haptic stimuli set as well as validate our new method of gathering perceptual data. Both of these studies require gaining perceptual judgment data from users. Prior to this, we had used a simple PC setup, as in the pilot study. However, using a handheld, Linux-based platform for our studies necessitated a change in the stimuli sorting program that is used to collect perceptual dissimilarity data from the user.

For several reasons it was no longer feasible to use the box-sorting technique utilized in our pilot study (and developed by MacLean and Enriquez [18]); the format was too space-intensive to fit on a small screen, and it was felt that having the box-sorting GUI on a desktop PC while having the user still needing to hold the 770 and interact with it using a stylus would be needlessly complicated both from a usability and a technical standpoint. Usability-wise, it would require constant switching between two different tasks on two different platforms, and from a technical side, it would require detailed communication between handheld device and PC, as well as a complete re-write of the box-sorting GUI for Linux rather than the Windows platform. Consequently, we designed a new interface that would allow the stimuli to be sorted using strictly the 770 with no other devices necessary.

The main limiting factor in the design of the new interface was screen space. The 800x480, 90 x 54 mm screen does not provide enough space to simultaneously display buttons representing all stimuli as well as boxes which the buttons can be sorted in to. Especially when the user is expected to sort stimuli into a large number of groups, the box and buttons sizes required would be extremely small, something that is definitely a troublesome issue when we are relying on the hand-eye coordination of touch-screen interaction. The possibility of having multiple screens that the user must switch between in order to access all stimuli and boxes was raised, but it was felt that it was important that all stimuli and groups be present and accessible at all times, so that no stimuli were neglected and all groups were at the same level of visual saliency.

The method thus decided upon was to have a spatially static field of buttons that could be grouped by assigning different colours to the buttons, with each colour representing a different group. While adding colour does bring with it certain limitations, such as inaccessibility to colour-blind users and unavoidable affective responses to certain colours, we felt that this was the simplest technique that would achieve our goals of having all groups and stimuli present in the interface simultaneously. With this interface, users can feel any of the stimuli by pressing on any of the buttons, each associated with one stimulus. The user can then add a button to a group by selecting one of the colours along the bottom and assigning it to the desired button. In order to decrease confusion, an automatic sorting function is provided, which simply places all of the buttons, sorted by colour, at the top of the screen, with the un-coloured placed after it. In order to aid with the sorting task, users were provided with sheets of paper with coloured squares printed on them corresponding to all the grouping colours, where they could write descriptive names for each group if they so desired. This helped users conceptualize and remember the groups they were sorting, as well as providing insight to the experimenters about how users were sorting the stimuli.

5.2.2 Loading Haptic Feedback

We were successful in having all stimuli and groups equally accessible at all times both visually and physically, but hardware limitations forced us to introduce load times for playing some of the stimuli. As mention in Section 5.1, the 770 maps specific feedback files to types of GUI widgets, or specifically named widgets, but only provides space for 16 different user-defined feedback files to be loaded at any one time. As a result of this, only 16 buttons can play their particular stimuli immediately after being pressed. Any other stimulus has to be loaded first (a process that takes no more than two seconds), which, in turn, unloads one of the other 16 buttons that already had its feedback loaded.

To load an unloaded button, the user simply presses it once to load it, and presses it again to feel it. Loaded buttons are indicated with a "!" next to their numbers.

In order to minimize the amount of loading required by the user, both pre-fetching and a history queue were implemented for feedback loading. Thus eight out of the sixteen feedback slots were used as a history of the last eight buttons the user had pressed, while the remaining eight feedback slots were used to pre-load the nearest buttons next to any newly-loaded button. What this allowed the user to do is move from the top-left down to the bottom-right, having most of the buttons loaded ahead of time for him or her. Furthermore, since the immediate neighbours of any non-loaded button would also be loaded along with it, returning to feel what any given button felt like in a colour group would load all of the other buttons in the group provided the buttons have been sorted (see Figure 5.3 for an example). Though this does not remove all loading times, it does greatly decrease the total amount, making the sorting task less frustrating and time consuming for the user.

Chapter 6: Investigation of Rhythmic Haptic Stimuli (Gold Standard Study)

In Chapter 3 we developed our stimulus set using a novel application of rhythm to tactile stimuli. As always with haptic stimuli, developing them was one challenge, but determining how they were actually perceived by people was another. The intuitive understanding of haptic perception that guided our design process is by no means a guarantor of how the stimuli will be perceived by the broader public. Especially in the case of haptic rhythm, on which so little research has been performed, our knowledge is lacking. Thus in this chapter we seek a clearer picture of the important perceptual characteristics of our stimulus set, and how these relate to the design parameters we used to create the stimuli.

6.1 Purpose and Structure of Study

The purpose of this study was to produce a thorough description of the perceptual characteristics of our rhythmic haptic stimulus set through the use of an existing, verified experimental method. In this study, we wished mostly to learn what characteristics of the stimulus set define its perceptual space—that is, the dimensions along which people perceive these stimuli as a group, as opposed to the engineering parameters used to construct them. Moreover, by using a verified method we also aim to produce a "gold standard" result, which our modified, subset method of data gathering can be compared against.

To this end, we decided to use the sorting method of data gathering with the full set of 84 haptic stimuli; and in fact, this decision influenced the maximum set size we could test here. Since participants sorted the entire stimulus set, there were no concerns about participants making judgments based on only part of the total set, and thus the resulting aggregate dissimilarity matrix could be taken as a reasonable representation of the average perception of the total stimulus set. While the sorting technique is a validated and widely used technique [19], it is generally used with a much smaller number of stimuli. Sorting a full set of 84 stimuli is a much longer and more involved process, with participant fatigue becoming a major issue. At 2 hours, the experiment time is extremely long; realistically this was the absolute maximum set size that this sorting task can handle, and then only with carefully chosen subjects

In order to minimize worries of fatigue, as well as trying to guarantee that our results would indeed constitute a "gold standard," several slightly unusual alterations were made to the study. Primarily, participants were solicited directly, with the express aim of choosing people who were dedicated and trustworthy enough to be vigilant throughout the task, as well as having already had some experience with vibratory tactile stimuli. This would help ensure the quality of the data gathered, with the obvious proviso that it may not reflect entirely accurately the perceptions of the general public. Furthermore, this resulted in all participants, in fact, being acquaintances of the experimenter, another biasing factor. However it was felt that using unmotivated, inexperienced users for a fairly long and arduous study would almost certainly give results too noisy and inconsistent to interpret. Several other allowances were made in an attempt to minimize the strain put on participants running the study, as will be discussed in the "Method" section. We feel that these allowances, while deviating somewhat from the standard experimental method, actually help to guarantee that our results stand up in the face of such a large stimulus set.

6.2 Full-set MDS study

As a well established standard for the study of haptic stimuli, MDS studies have been shown to be a great tool for discovering perceptual characteristics of novel sensations [19]. What follows is a description of the first of the two major MDS studies performed in this thesis. It represents our best attempt to create a clear perceptual description of the rhythm-centered stimulus set we made.

6.2.1 Method

Six expert participants were solicited directly for the study. While relatively few, the demanding criteria set for the participants made recruitment difficult but meanwhile ensured a smaller amount of higher-quality data. Participants were 5 males and 1 female, all graduate students in computer science, with an age range between 24 and 40.

The experiment lasted two hours, but participants were given the choice of breaking the experiment up into two one-hour sessions in order to minimize the fatigue of running the experiment. Of the six, two chose to break the experiment up, and four chose to do it in one two-hour block. Participants were compensated $20 in total for the experiment.

Participants sorted the entire 84-stimulus rhythmic haptic stimulus set on the Nokia 770T tactile platform described in Chapter 5, using the program also described there. Each participant completed 3 sorting tasks on the same stimulus set. In the first sort, participants were told to group stimuli into whatever number of discrete, non-overlapping groups they felt was appropriate to describe the perceived dissimilarity between stimuli. For the remaining two sorting tasks, participants were required to sort the stimuli into a specified number of groups, either 3, 9 or 15. Of these three group numbers, the one closest to the number of groups chosen in the first sorting task was not used, with the remaining two numbers randomly assigned to the second and third sorting tasks. Having three repetitions of the sorting task performed on the same set of stimuli and varying the number of groups that the stimuli are sorted into as we have done has been shown previously in haptic MDS studies to yield good resolution for perceived differences [20].

Because of the auditory noise made by the 770T when playing haptic feedback, users wore Bose Quiet Comfort 2[1] acoustic noise cancelling headphones during the experiment. While the normal procedure is to play white noise during testing to drown out the sound, it was felt that for obvious reasons listening to two hours straight of loud white noise would itself be an impediment to making well-reasoned judgments. Consequently, and since participants had already been selected for their trustworthiness and dedication, participants were allowed to listen to music self-chosen according to stated criteria, and told to self-monitor to ensure that no sound from the device could be heard. The criteria were simply that the music was consistently loud enough to mask the noise made by the

[1] http://www.bose.com/controller?event=view_product_page_event&product=qc2_headphones_index

770T, and that it be emotionally neutral enough that it would not overly affect the participant's mood. The music played by the participants was periodically checked to ensure that it followed these criteria, and no violations were observed.

6.2.2 Basic Results

As per the methods of MacLean and Enriquez [19], similarity values were created for each pair of stimuli by taking each instance of co-occurrence in the same sorted group as an indication of perceived similarity, adding a similarity score to that pair proportional to the number of total groups used in that particular sorting task. Thus higher similarity values were given when two stimuli were placed together during the 15-group sorting task than the 3-group sorting task. These total similarity values were then subtracted from 1000 to create a dissimilarity matrix for each subject. These individual dissimilarity matrices, covering the total 84x84 (symmetric) comparisons between stimuli, were then added together and averaged, to create an overall average dissimilarity matrix for the group.

Analysis of MDS Plot

The resulting dissimilarity matrix (Appendix A) was then run through the SPSS ALSCAL algorithm for 1 to 6 dimensions. Graphing the resulting stress values (Young's s-stress Formula 1 was used) shows no clear elbow indicating a point of diminishing returns in terms of goodness of fit (Figure 6.1); instead both 2 and 3 dimensional results provide reasonable stress levels, while higher dimensions are somewhat decreased in improvement. Specifically, the 2D solution has an s-stress value of 0.36668 and an r^2 of 0 .47788, while the 3D solution has an s-stress value of 0.26630 and an r^2 of 0.58049. Both s-stress and r^2 values range from 0 to 1, with low s-stress values showing better fit, while low r^2 values show worse fit. These particular s-stress values are both relatively high, indicating the difficulty of fitting the data into the required dimensions; this is likely due to the number of stimuli used. Furthermore in both cases the r^2 values are fairly low, indicating that around half of the variance in the data set was not accounted for in the MDS model.

However, given our own experience with MDS plots of haptic stimuli, such levels of stress and r^2 are not necessarily signs that the plots themselves will not yield informative and trustworthy results [20]. Given this, and the diminishing returns of the s-stress plot, for the sake of parsimony as well as ease of interpretation, the 2D solution was chosen as the primary solution for analysis, though the 3D solution is given some consideration as a secondary tool for analysis. Despite the relatively poor s-stress and r^2 values, these solutions proved amenable to reasonable interpretation.

2D Results

Initial examination of the 2D plot (Figure 6.2) shows a clear circumplex arrangement in the data similar to that found in the pilot. Overall distribution of stimuli along this circumflex is fairly even. A few denser clusters are evident, but only as part of a general trend of dispersed stimuli. At best only three very large clusters could be said to exist, but they are almost too broad to be of any interpretive value. Instead, several grouping trends according to amplitude and rhythm are explored in the next section.

In analysis of MDS output, our main goal is to determine how the engineering parameters that were used to create the stimulus set map to the perceptual parameters that participants used to group the stimuli. In our stimulus set, three main engineering parameters were used: amplitude, frequency and rhythm type. Our main method of projecting engineering parameters onto the perceptual space is to average the values of all the stimuli in a given group as defined by some parameter, and treat the resulting point as a centroid representing the overall group that can then be compared against other groups. For example, one group might be all observations for a given amplitude, and another group all observations for another amplitude. Drawing a line between the centroids of two different groups (as done for amplitude and frequency in Figure 6.2) creates an axis from which we can further interpret the data. The length of this line can be interpreted as the strength of the effect of this parameter, because the length of the line is the distance between the two centroids of the two groups, and in MDS maps, distance in layout equates to magnitude of perceptual difference.

Thus projection of the design parameters of amplitude and frequency onto the perception space shows a very strong and very clean cut trend of amplitude, but almost no trend of frequency whatsoever (Figure 6.2). The dominance of amplitude in distinguishing stimuli is as expected, and agrees with the previous results in the study of haptic stimuli [19, 18, 20]. However the lack of effect of frequency is a unique result, counter to previous findings, and is discussed in the next section as well.

Analysis of Standard Deviation

Analyzing the standard deviation of the averaged values in the dissimilarity matrix provides encouraging results that broadly agree with the MDS output. The overall average standard deviation for all dissimilarity values for the matrix is 160.02, which, on scores of 0-1000 is fairly large, but not insurmountably so—prior research has shown positive results with comparable levels of SD [19]. More interesting is the distribution of the standard deviation over the dissimilarity matrix (Figure 6.4). Unlike the pilot study, where the spread of high and low points of standard deviation was, as far as could be distinguished, completely random, there are clearly two different areas of SD, one with a higher overall SD, one with lower overall SD. These can be seen as the rough square of low SD (light coloured squares) in the middle of the half-matrix displayed in Figure 6.4, and the two darker corners (high SD) of the triangle. As stimuli numbered 1-42 were high-amplitude, and those numbered 43-84 low-amplitude (see Table 3.3), the light square corresponds precisely to the area where stimuli with low amplitude are compared against stimuli with high-amplitude (or vice versa), while the darker areas are precisely where stimuli of the same amplitude level (either low or high) are compared against each other.

This distribution of standard deviation confirms the large role that amplitude played in how people characterized the stimuli, as evinced by examination of individual MDS results: almost everyone agreed that stimuli of different amplitude levels were indeed different, while there was much more disagreement about the similarity of stimuli of the same amplitude level, differing only by rhythm. Compared to the results of the pilot study, these results contain considerably more structure both in terms of MDS output and distribution of standard deviation. This gives us much more confidence to delve deeper into the MDS results in order to investigate the effect of rhythm on the perception of haptic stimuli (below).

Quality of Judgments from Participants

Since we are treating the results of this study as a "gold standard" truth of how our stimulus set is perceived, we need to feel confident that the judgments given us by our participants are trustworthy. The use of only six participants has been shown to produce consistent results (i.e. low between-subjects standard deviation, and similar overall structure in MDS result) for haptic stimuli before [20]. However, the use of six participants with a more tiresome task is a concern. Fatigue was potentially a problem for this experiment, given the large size of stimulus set the participants were being asked to sort. Mitigating this concern, we observed that four out of the six participants elected to perform the experiment in one large 2-hour block, preferring to "get it over with," though by their own reports the task was not overly taxing. Especially with being able to listen to music of their own choosing, most participants reported being fairly comfortable with making perceptual judgments for an extended period of time.

Furthermore, we find from analysis of the standard deviation that the level of disagreement between participants was fairly low. Aside from the trend of amplitude described in the above section, the level of SD is within a similar range for all values in the dissimilarity matrix. While we do not have a good threshold for a reasonable absolute value of standard deviation in a task of this type, consistency of the standard deviation values for the averaged dissimilarity matrix helps confirm that participants' results did not suffer from random noise introduced by fatigue. Along with the strength of the MDS results we actually obtained, we feel that our participant pool has been shown to produce trustworthy perceptual judgments.

6.3 Analysis of Frequency and Rhythm

The clearest result from our initial data analysis was that of amplitude, neatly bisecting the MDS plot, indicating a strong perceptual role for our stimulus set. The strength of amplitude is in accord with previous research [19], but other results require further examination, such as the absence of the effect of frequency, and the perceptual role played by the different rhythms. In particular, we will show that the frequency and rhythm results are intertwined, and thus discussed here in relation to one another.

6.3.1 Frequency

Unlike amplitude, in plotting the centroids from the two frequency ("tonal") levels, we find almost no trend whatsoever (Figure 6.2); the two centroids are extremely close together in the middle of the MDS map. Since the frequency of stimuli did not seem to play a large perceptual role in dividing the overall map, it would be expected that stimuli with the same rhythm and amplitude but different frequencies should occur very closely together, which is largely, but not exactly, the case.

Pairs of low and high frequency stimuli with the same rhythm are usually within at least the same quadrant of the map, if not much closer. For example, in the upper-left quadrant, high-amplitude and high-frequency stimuli 15, 16, and 17 all sit close to their low-frequency equivalents 36, 37, and 38, and in the lower-right quadrant stimuli 50 and 52 are quite near their counterparts 71 and 74. However, if frequency had absolutely no perceptual salience, then it would have been expected that the pairs of stimuli differing only by frequency would have been exactly co-located, which is almost never the case. The observed difference in placement could be due to noise in the data, but given the consistency of the rest of the results, it would seem odd that there might be pockets of such high noise that coincidently occur between stimuli of different frequencies. In Figure 6.4, there appear to be no distributions of standard deviation for between and within frequency level comparisons, as there are for amplitude (see 6.2.2 for discussion). Consequently it cannot be concluded that the frequency of stimuli had no effect whatsoever, merely that it did not have a consistent effect across all stimuli, and that in total magnitude its effect on distinguishing stimuli was less than that of amplitude, or indeed, certain types of rhythm.

Possible Explanation for Lack of Effect

This relatively small effect of frequency runs counter to the previous results of MacLean and Enriquez [19], who found that frequency dominated all other parameters, along with amplitude and waveform, among others. One potential explanation for the lack of effect is that the previous results were gained from continuous vibrations (in [19], of 2 second duration), while our results were gained from rhythms involving mostly quite short notes (e.g. a quarter note, the most frequently used, lasted for just 62.5 ms). Potentially, the short duration of the individual vibratory notes did not allow their overall frequency to be clearly perceived: a quarter note would allow just 12.5 repetitions at the lower frequency, 200 Hz (see Section 5.1.2 for frequency values used).

Pasquero *et al.* [20] have shown that performing MDS analysis on a sub-matrix of the total dissimilarity matrix can effectively unfold dimensions that were hidden in the perceptual map for the total stimuli set; essentially, a subsection of the stimuli might have a local dimensionality which is suppressed by a more dominant one that applies to the overall set. Thanks to this fact, we were able to look only at the rhythms containing longer notes, in hope that the longer vibration times would allow people more time to perceive frequency (Figure 6.5). However, when mapped and expanded, the stimuli behaved similarly to the larger group: they fell in a general circumflex arrangement, with amplitude being the largest distinguishing factor and frequency playing a considerably smaller role. Though the axis of frequency was slightly larger than in the full MDS plot, given the small sample size of long note stimuli (16 /84 stimuli contained only half-notes or longer, with an average on-time of 1312.5 ms allowing 26.2.5 cycles of the lower frequency level) it is hard to claim any clear effects.

With this explanation eliminated, only two other likely possibilities exist: either the two frequency levels were too similar to be consistently distinguished between, or rhythm dominated or masked frequency in a perceptual sense, essentially overriding any judgments that people might have made based on frequency. As outlined in Chapter 5, initial testing was done with different frequency and amplitude levels in order to determine values that would be perceptually quite different. These tests were performed only on continuous vibrations, and not on rhythms, because we wished to avoid the confounding factor of which rhythm or rhythms should be tested. Furthermore, participants could feel each vibration for as long as they wished, furthering differing the sensation from the time-limit rhythms. Nevertheless, the frequency levels selected created sufficient perceptual separation in the continuous vibration test, and the rhythmic stimuli themselves were tested informally to ensure that rhythms at different frequency levels could be distinguished.

So despite there being *some* a priori evidence of perceptual difference between stimuli played at different frequencies, it was seemingly not large enough to create a difference that people noted and used as a grouping criterion when combined with rhythm. Perhaps this lack of difference is because the two frequency levels chosen were dissimilar enough to create a difference, but only at the edge of creating a difference big enough to be perceptually important in classifying. This borderline condition might be an explanation for the inconsistent effect of frequency, but without a more concrete theory, we chose to largely ignore the effect of frequency throughout the remainder of this analysis. In future work, it will be relevant to consider a larger frequency differential than that made possible with the present hardware.

6.3.2 Rhythm

In the pilot study no obvious trends over rhythm were discernable. For every possible common-sense grouping applied to the data, there were enough exceptions that flew in the face of the trend that it was impossible to be certain that we were not simply imposing our view of the data onto what was essentially noise. In contrast, in the results from the full-set study the separation on amplitude was much more obvious and pronounced, and some clear clustering was evident, as opposed to almost no clustering in the pilot study results. This gave us greater confidence that our new hardware, as well as evaluation using trusted experts, had in fact tapped into the real perceptual characteristics of our stimuli—which, though displayed on different hardware, were made using the exact same parameters as in the pilot.

Analysis of 2D Solution

As is often the case for analysis of MDS solutions, the 2D map was used as a landmarking tool; its easy-to-conceptualize nature makes distinguishing the broad features of a solution a much more feasible task. Examining the 2D plot of our data (Figure 6.2), there appeared to be two breaks across from each other in the circumflex, and if we drew a line between these two breaks we split the perceptual maps into two halves roughly orthogonal to the two halves created by amplitude. This seemed very encouraging, pointing towards the existence of a second major perceptual axis in the definition of the stimulus set's space, and since frequency had already been ruled out as a potential factor, the next reasonable place to look for a cause of this separation was rhythm.

Inspecting the stimuli that fell along the left side of the map and the stimuli that fell along the right, it soon became clear that, barring a handful of exceptions, the stimuli along the left all involved rhythms that were comprised of entirely "short" notes, while the stimuli along the right had rhythms that contained "long" notes (see Figure 6.3). For our purposes, a "short" note was a either a quarter or an eighth of a bar long—thus stimuli from Groups 1 and 4 (see Section 3.3 for explanation) are rhythms containing only short notes. A "long" note is any note that is a half bar or longer; Groups 2, 3 and 5 all contain one or more long note. Plotting centroids for this split of "long" versus "short" note groups, we find an axis roughly as large in size, and orthogonal to, that of amplitude. The fact that such a strong grouping occurred according to pre-defined logical groupings of the rhythms was an encouraging result; these separations were *a priori* built into the stimulus set based on our intuitive understanding of how rhythms might be perceived. Finding these assumptions confirmed, at least in part, from our experimental results indicates that we are likely seeing evidence of peoples' true perceptual characteristics, rather than chance artifacts of the experimental and data analysis process. That we found evidence of the perception of parameters that we built into the stimulus set might seem to be a self-fulfilling prophesy; however, lacking knowledge of the overall possible space of rhythms, some assumptions and intuitions had to be used. That we have found these assumptions and intuitions confirmed experimentally indicates that we were justified in our original decisions, and nevertheless allowed us to notice some unexpected trends.

Analysis of 3D Solution

Attempting to find a different perspective on rhythm, the 3D MDS solution was examined. Arbitrarily assigning the names X, Y and Z to the three axes produced by the MDS algorithm, it was noted that the X-Y plane (Figure 6.6) was structured according to the same parameters as the 2D solution (though without the circumflex arrangement), with the Y axis differing along amplitude values, and the X axis differing along the "long note" to "short note" rhythms. However, examining the X-Z (Figure 6.7) and Y-Z (Figure 6.8) planes, it became evident that the Z axis was defined according to a different set of criteria.

As discussed in Section 3.2, the rhythms were constructed to fall into 5 major groups. Since the grouping of "long notes" and "short notes" already fell along these grouping trends, the 3D data was examined to see where each of these 5 groups was situated. What was discovered was that along the Z axis Group 2, the group containing only "long" notes, appeared at the very extreme end of the axis, with all the other groups spread fairly evenly along the rest of the axis. This seemed to indicate that there might be something special with Group 2, distinguishing it from the rest of the rhythms containing long notes. This makes intuitive sense, as Group 2 contains only long notes, while Groups 3 and 5 contain both long and short notes.

This difference manifests itself in an important perceptual characteristic of rhythm that will be discussed further below, namely the feeling of "evenness" or regularity, versus "unevenness" or irregularity of a given rhythm. At a high-level, rhythms that contain only notes of the same length feel even, while rhythms that contain notes of different lengths (or rest notes of different lengths) feel uneven. Yet given only the 3D solution, all that was evident was the different place of the long-note, even rhythm group; there was no clear evidence of evenness similarly affecting the short-note rhythms.

Amplitude-Independent Analysis

The fact that in the 3D solution the Y axis accounts for most of the effect of amplitude, presumably allowed the Z axis to account for certain perceptual features of rhythm that could not be fit into the 2D solution. As mentioned above, by taking only one section of the total dissimilarity matrix and analyzing it using MDS, factors that were hidden in the larger solution can appear [20]. Thus if we remove completely the dominant factor of amplitude, the more subtle characteristics that define rhythm have a chance to manifest themselves. To this end, two sub-analyses were performed on the two halves of the stimulus set that had the same amplitude level. Unfolding the data in this way, it became clear that there were actually two perceptual axes involved in the perception of rhythm. As initially noticed in the 2D solution one of the dimensions was the length of the notes in the rhythm (presence of absence of the longest notes). The other dimension, as hinted at in the 3D solution, was the "evenness" of the rhythms.

The 2D MDS output for all the high-amplitude stimuli is shown in Figure 6.9. As can be seen, the map can clearly be split into two halves of even and uneven rhythms; a large gap separates the two halves. Here we also see evidence of evenness of rhythms affecting both short and long note rhythms, which was hidden in the 3D solution. The split between the rhythms containing long notes and those containing only short notes is not quite as distinct, but still clearly observable. Initially this result may seem counterintuitive; the length of notes was the major discriminating factor in the 2D solution, so it would seem reasonable to assume that in the unfolded single-amplitude solution it would have the strongest effect. However, the likely cause of this is the fact that the "note length" axis actually contains a range of potential values, and can be somewhat ambiguously defined at certain points, while rhythms can be fairly unambiguously classified as either "even" or "uneven."

Note Length

Generally speaking, the definition of a "long note" rhythm is any rhythm containing at least a one half, three-quarters or whole note. Seemingly the longest note present in a rhythm defines how it is perceived along this perceptual axis. Rhythms with three-quarters or whole notes tend to fall near the ends of the axis, while rhythms containing half notes fall closer towards the middle, and those containing only short notes fall towards the opposite end, all as would be anticipated. Furthermore, if short notes are also present in a rhythm, the more short notes there are relative to long notes, the further towards the middle of the axis the rhythm will fall. This is especially evident for rhythms 11 and 19, which consist of a half note followed by two quarter notes or four eighths respectively, and both of which fall roughly in the middle between the long note and short note groups. This placement only serves to reinforce that note length is indeed the trend that is being displayed here, as the rhythms contain long and short notes in equal measures; their placement directly in the middle of the axis is exactly where one would expect them to be.

The trend of note length is not perfectly consistent throughout. Given the position of rhythms 1, 2 and 3 in the map, it may be claimed that the *number* of notes in a rhythm is also confounded somewhat with overall note length. Seemingly by having multiple short notes, these rhythms have moved towards the center of the axis. Under this explanation, it might actually be more accurate to describe the trend as one of overall time spent with notes playing versus not playing: if we add up the total duration of all notes played in the rhythms near the center of the axis, they come to a similar total, though the *number* of notes might be quite different for each rhythm. But this description is not strictly true either, as the three rhythms that are the equivalent of rhythms 1, 2, and 3, but with two eighth notes replacing each quarter note (and therefore with the same amount of total playing/not playing time), are placed much farther towards the "short" end of the note length axis. It is sufficient to say, then, that increasing the number of notes present can have an effect of moving rhythms more towards the "long" end of the note length axis, but that effect is not stronger than the overriding effect of the longest note present in the rhythm.

Evenness of Rhythm

As opposed to the note length axis, the even/uneven perceptual axis is very clearly delineated, with essentially no middle ground between the two groups. This can be felt quite distinctly when the stimuli are actually displayed on the haptic device. Even rhythms have a regular repeating nature in which each part of the rhythm feels the same as every other part, throughout the duration of the stimulus. Uneven rhythms have an irregular, lurching feel to them; even with our monotone, same-amplitude stimuli, there is an emphasized portion of the rhythm and a deemphasized portion, such that the rhythm has an overall two-part structure, with a perceived emphasis on the first part of the rhythm. The most obvious examples of uneven rhythms are those in Groups 3 and 5, which consist of one long note followed by a number of shorter notes. Thus the longer note draws the emphasis, while the smaller notes are deemphasized, creating a skipping, *one*-two emphasis within the rhythm.

By only looking at the structure of the rhythms (as shown in Table 3.3), it is easy to conceptualize that Groups 2 and 5 might be perceived as uneven given the above description, yet it far less intuitive as to why rhythms 2, 3 and 15 also feel uneven, despite containing only notes of the same length. Yet upon feeling these rhythms, the sensation of unevenness is distinct. What appears to create the feeling of "unevenness" in these stimuli is actually the rest that occurs after the notes; the initial set of notes played thus creates the emphasized portion of the rhythm, with the blank occurring as the deemphasized portion. A caveat to this is that the rhythm must contain more than one note before the rest in order for it to be perceived as uneven. In the case of rhythms like 5, 7 and 8, which all contain only one note and then a rest for the remainder of the bar, the rhythm is seemingly recontextualized into one longer, slower pace rhythm containing a single bar consisting of a note played four times, instead of a bar repeated four times containing one note per bar. What appears to be causing this are the different sizes of the blank periods in the rhythm: in 2, 3 and 14, there are the blank periods that separate each note, as well as a longer rest note at the end of the bar. Thus what appears to define an "uneven" rhythm, in terms of how our subjects have placed them here, is that it either contains notes of two different lengths, or blank periods of two different lengths. The blank space between pairs of eighth notes, however, does not seem to count towards this effect. Consequently rhythms such as 16 and 17 are perceived as even.

As a last, confirmatory point, each of the four groups, from the four perceptual quadrants found in the amplitude-independent data (long-even, long-uneven, short-even, short-uneven, as seen in Figure 6.9), were individually run through the MDS algorithm (Figure 6.10). The resulting outputs were fairly similar in layout to the full 2D solution, with amplitude playing the largest defining role, but now with stimuli more spread out. No further insights were gained into how tactile rhythms are perceived. The most significant result of this sub-analysis was the stress and r^2 values produced by these solutions (Table 5.1). As can be seen, stress values are lower and r^2 values higher than the overall 2D solution. While some of this improvement can be attributed to the smaller number of stimuli, it should be noted that the long-uneven group has a lower stress value that the short-even group, even though it has 8 more stimuli. Consequently, we can see this as further evidence that the stimuli in these groups naturally "fit" together, as it is fairly easy algorithmically for MDS to deal with them.

Rhythm Groupings in 2D Solution

Returning to the 2D solution, we can see how these two axes manifest themselves when forced to contend with the overriding factor of amplitude (Figure 6.11). Plotting the mid-points of the 4 groups (short-even, short-uneven, long-even, long-uneven), several features can be noted. Firstly, the mid points all fall roughly in a line orthogonal to the line of the axis created by amplitude. Secondly, it is clear that of the two factors, note length has a stronger effect than evenness of rhythm, such that stimuli are grouped first by note length, and then within that group they vary according to evenness of rhythm. However, by introducing unevenness as a criterion, it explains the position of several stimuli whose placement was somewhat counter to the trend using strictly note length. For instance, without considering evenness stimuli 44 and 45 appear to wrongly be positioned with the long-note rhythms, despite consisting only of quarter notes. With evenness considered, it becomes evident that 44 and 45 are uneven rhythms, and are actually positioned in a group containing long-note rhythms *as well as* uneven rhythms. This result gives further weight to the claim that these two dimensions of rhythm are truly being perceived by people and that this is not a case of over-analysis of the data.

6.4 Summary

In Chapter 3 we created a set of 84 different haptic stimuli by varying three design parameters: amplitude, frequency and rhythm. In this chapter, we set about studying how these stimuli were actually perceived when presented to users. Since the other main challenge this thesis deals with is how to present such a large number of stimuli to users, certain concessions had to be made in order to successfully examine these stimuli. Nevertheless, the results we achieved were extremely encouraging, especially in regards to the effect of rhythm within tactile stimuli. Our analysis showed clearly that different aspects of rhythm could be distinguished, and that coupled with amplitude could create a very wide range of perceptually different haptic stimuli.

Our study consisted of asking six expert users to sort all 84 stimuli into groups, using the standard sorting method of data gathering for MDS. Since sorting 84 stimuli using a small handheld display is quite a tiresome task, only devoted and diligent participants were solicited. An elite subject pool is not always possible to recruit, and nor are its results necessarily reproducible by the general public. Yet despite the challenges in gathering participants, their willingness to accept a more difficult commitment and their prior experience and knowledge in the field made certain that data quality was high, and gave it credence in establishing ground truth.

After gathering the subjective perceptual data and running it through the MDS algorithm, we performed analysis, primarily on the 2D solution. We found amplitude to be the strongest perceived differentiating factor, while frequency was almost completely absent from the picture. The strength of amplitude agreed with previous findings [19], but the lack of frequency did not, an effect that can mostly be explained by the strong role of rhythm (which was conversely *not* present in the earlier analyses).

From our analysis, it appears that the two primary characteristics on which our rhythms are distinguished between are the length of the longest note present in a rhythm, and the "evenness" of the rhythm ("even" rhythms only have notes and rests of the same length, "uneven" rhythms have notes or rests of different length). Controlling for the effect of amplitude, these two criteria are perceived roughly orthogonally, with note length being the slightly more dominant of the two. Though the rhythms that we tested in our stimulus set do not by any means cover the range of all possible rhythms, their simplicity should make these trends quite generalizable. Indeed, the consistency with which these two criteria were used to judge our rhythms is very encouraging, and should prove extremely useful to those wishing to use tactile rhythms in the future.

Our results have provided interesting new insights into how tactile stimuli are perceived, and specifically how haptic stimuli can be designed in order to maximize both perceptual differentiability and grouping. Furthermore, using an established, validated technique with committed, expert users has provided us with a clear "gold standard" as to what constitutes ground truth for the human perception of this stimulus set. Thus we now have an empirically derived standard that can be used to compare the results from our as-yet unvalidated novel data gathering method, a goal we pursue in the next chapter.

Chapter 7: Subset Method Validation Study

In Chapter 6, we examined our haptic stimulus set through the use of the sorting method often used to gather MDS data on haptic stimuli [19] [11] [18]. However, using this method with the full set of 84 stimuli created an extremely arduous task which was demanding even for skilled participants—problems outlined in depth in Chapter 6. In fact, if our total stimuli set had been any larger, we likely could not have performed the full-set study at all, sorting 84 stimuli at once being about the absolute maximum that could be done by a single participant. Consequently, this makes the full-set study described in Chapter 6 a unique, one-off experiment. With the goal of creating a general, easy-to-use method for evaluating large numbers of haptic stimuli, a less arduous technique must be developed.

As proposed in Chapter 4, by combining several existing methods that deal with large stimulus set sizes in MDS, we devised the subset method of data gathering for MDS that allows users to sort only a subset of the total stimulus set, thus greatly shortening experiment times, loosening restrictions on potential participants, and yet still producing a total picture of the perceptual space of a given stimulus set by averaging over a series of overlapping subsets. The cost of this method is in requiring a considerably larger number of participants for a given set size (to obtain sufficient overlap and reduce noise due to between participant variability), as well as the increased complexity of experiment design and analysis.

This chapter is concerned with validating the accuracy of this new method, by testing the hypothesis that the subset method of data-gathering for a perceptual MDS analysis can produce results comparable to the normal, full-set sorting method, but with a considerably shorter and less taxing experimental task. Thus, in addition to providing new results about rhythm stimuli, the full-set study also played the role of "gold standard" in this scheme.

7.1 Validation Overview

As discussed in Chapter 4, our main concern with collecting dissimilarity data from subsets of the total stimuli set is that the specific composition of each subset might affect how participants judge each stimulus. As an analogy, consider a quiet library in which everyone is whispering: if someone were to talk at a normal voice level, they might be considered "loud" compared to everyone else being "quiet." But if someone were then to start yelling, the person talking at a normal level might also be considered "quiet" compared to the "loud" yeller; conversely, the yeller might be considered "very loud" while the normal talker would still be "loud." Thus a subset without the yeller might produce different results across the board compared to a subset with the yeller. This is essentially a question of relative versus absolute judgments, and how great their effect might be on judgments within a subset.

At a slightly higher level, we are also concerned that participants might miss some of the larger patterns existent in the stimuli due to their lack of representation in the particular subset that participants are presented with. Especially if there is a small but highly distinct group of stimuli, there is a definite chance that some subsets might not have any of these stimuli, causing the user to completely miss their existence. Missing these stimuli would, in turn, create problems when averaging together the results from the different users, as different subsets would highlight different aspects of the stimuli, creating noisy averages that cover over incongruent pieces. On the other hand, it is possible that this will not be an issue, because the particular trends are only noticed when the stimuli that manifest these trends are present in a subset, thus making each subset fit together like a jigsaw, with different subsets providing coverage for the particular trends that are most evident in their stimuli. In fact, this situation could even serve to highlight subtle aspects of the stimuli that might be obscured in a full-set analysis. What we hope is that the relative difference values assigned to stimuli by participants stays at least roughly the same regardless of the composition of the subsets. The presence or absence of particularly "loud" stimuli would thus function in a way similar to a fish-eye lens on the MDS plot—distorting and compacting the positioning of all the stimuli around it, yet keeping their relative positioning. This kind of mild distortion can then be dealt with in a convergent manner by averaging together observations collected from different randomly-generated subset comparisons.

Our criteria for validation of the subset data-gathering method are thus as follows. Firstly, stimuli that occur together in different subsets multiple times (such that dissimilarity values for that pair of stimuli will come from more that one subset) must still be given comparable dissimilarity ratings by users. "Comparable" in this case will mean that the level of standard deviation between ratings from overlapped stimulus pairs is no larger than the overall level of noise in users' dissimilarity ratings. Secondly, the averaged dissimilarity matrix must produce MDS results that are reasonable and logically sound, standing alone. Thirdly, the results must be similar to the gold-standard study, both qualitatively and quantitatively. Criterion one checks for the effect of subset-relative judgments, while criteria two and three check whether the averaged result in fact reflects the real nature of the stimulus set. Criterion two is thus a lighter version of criterion three, assuming the accuracy of the gold-standard result.

7.1.1 Criterion 1: Consistency of Results Obtained from Different Subsets

Inter-subset consistency is checked largely through examination of the uniformity of standard deviations of the averaged dissimilarity matrix elements. By looking at the standard deviation of values in the matrix where multiple subsets contributed to the dissimilarity rating, and comparing them to those points that have been covered only by a single subset (provided multiple participants judged the subsets, so that standard deviation can be calculated), we can see whether the points of overlap have higher variability compared to the points of non-overlap. There is always some disagreement among different users when it comes to perceptual judgments (and, indeed, users can often disagree with themselves on different repetitions). However, if different subsets do indeed produce highly different results for the same stimulus pairs, then it would be assumed that there would be a much higher level of disagreement (and thus standard deviation) for those areas of overlap, compared to the normal level of noise (disagreement) between ratings given by participants.

In order to be able to perform this particular test on the data, a slight variation on the proposed experimental method had to be made. Our original design called for complete randomization of the stimuli in each subset, such that each participant would be presented with a unique subset. Randomization was to be performed in order to minimize any possible effect due to subsets, so that the damage of any particularly unfortunate grouping of stimuli in a subset would be covered over by the bulk of reasonable subsets. It was also done to ensure an even coverage of the dissimilarity matrix with as few participants as necessary.

However, if we wished to compare the standard deviation of matrix points that average over multiple different subsets versus points that are averaged only over the *same* subset, then subsets must be repeated in order to develop a baseline level of noise/standard deviation that is to be expected when different participants are presented with the exact same set of stimuli. In this way we can compare the baseline level of standard deviation from individual difference to the level of standard deviation that occurs from individual differences *plus* differences due to participants experiencing different stimulus subset. To make this comparison possible, the minimal number of subsets needed to cover the entire dissimilarity matrix with one observation was created, which in our case took 5 subsets. These five subsets were used multiple times, such that each of the five subsets was sorted by several participants, allowing us to determine our baseline level of standard deviation while getting multiple observations per stimulus pair. This baseline level could then be compared against the standard deviation of areas where the five subsets overlapped, giving us a measure of how much comparisons from different subsets disagree with each other compared to the overall level of disagreement.

7.1.2 Criteria 2 & 3: Overall Accuracy of Results

Criteria two and three both pertain to the resulting MDS output map: they seek to determine whether the output has real-world traction, and specifically whether it compares favorably to our gold standard. We perform much of the analysis required for these critieria in an *ad-hoc* method similar to the analysis performed in Chapter 6 on the output of the full-set study. Yet calling our analysis *ad-hoc* is not meant as a slight to its efficacy. Computerized clustering algorithms have yet to consistently attain the results of detailed human analysis; their lack of semantic reasoning and "common-sense" appreciation of the dataset is usually their downfall. Nevertheless, since this type of analysis can be considered as quite qualitative, a statistical means of testing similarity has been considered as well. As the accepted statistic for the comparison of MDS results, the coefficient of alienation, *K*, is also used to determine statistical similarity, using the empirically-derived values presented by Borg and Leutner [4] to determine similarity at the $p = .05$ level (as discussed in Section 2.2.1). However, statistical significance is not always practical significance, so it is important that our qualitative analysis—our assessment of reasonableness of result—agrees with the statistical measures on the similarity of the results of the two studies; both measures of analysis are required if we are to consider criteria three to have been met.

7.1.3 Strengths and Weaknesses of Validation Process

Overall, the main weakness of this validation process is the degree of bootstrapping involved in the creation of the stimuli and the design of our studies: we are testing a new study methodology on a new stimulus set. Furthermore, the gold standard that we are comparing the results of our new study against comes from stretching an already established technique potentially to its breaking point.

However, these two studies were carefully designed to minimize any circularity in their reasoning and to minimize the amount of bootstrapping. The full-set study uses an already valid technique to gather data, and its main weakness is the potential fatigue of its participants, which we watched closely for. The subset study uses an unvalidated technique, but its potential weaknesses are in the logic of the study itself, and it is, in fact, designed to minimize the issue of fatigue that troubles the full-set study. So each study's weaknesses are designed to counteract the weaknesses of the other, with the full-set study providing the solid ground-truth, produced at a heavy cost, while the subset study can be compared to this ground-truth with data much more easily gleaned from the user.

7.2 50-Stimulus Subset MDS Study

Our study using the subset method of data gathering for MDS takes place in two parts. In the first part we ran a study on 15 participants using the subset method, specifically designing our experiment to allow us to analyze several different characteristics of our new methodology in order to test for validity. While this experiment did produce several very important insights into the strengths and weaknesses of our new method, it also failed to produce an MDS plot that was sufficiently similar to our gold standard due to the experimental design decisions we made (specifically, it was derived from non-uniform number of observations across the full-set dissimilarity matrix). Thus in the second part of our study, we ran an additional 7 participants with the specific aim of gaining a better coverage of perceptual data across the entire stimulus set. By adding these supplementary data points, we were able to increase the quality of the resultant MDS plot such that it was both qualitatively and quantitatively similar to our gold standard. At the same time, we gleaned an important methodological insight, i.e. the importance of uniform coverage.

7.2.1 Method (Study Part One)

Fifteen participants, 5 female, 10 male, ages ranging from 22 to 35 were recruited to run this experiment. All were graduate students at UBC. The experiment lasted approximately one hour, and participants were compensated $10 for their time.

As in the full-set, gold standard study, participants sorted a set of haptic stimuli on the Nokia 770T using the program described in Chapter 5. Each participant sorted a particular stimulus set three times during the experiment session. In the first sort, participants were told to group stimuli into whatever number of discrete, non-overlapping groups they felt was appropriate to describe the perceived dissimilarity between stimuli. For the remaining two sorting tasks, participants were required to sort the stimuli into a specified number of groups, either 3, 9 or 15. Of these three group numbers, the one closest to the number of groups chosen in the first sorting task was not used, with the remaining two numbers randomly assigned to the second and third sorting tasks. Participants wore Bose Quiet Comfort 2 acoustic noise cancelling headphones during the experiment, which played white noise loudly enough to mask the sound made by the haptic feedback on the device. White noise was substituted for music, which was used for the gold-standard study, because (a) the experiment time was shorter (and so the noise less tiresome) and (b) we had recruited 'normal' rather than especially trustworthy participants, and were not comfortable allowing them to self-monitor their own music choice. The use of white noise is a much simpler and more realistic experimental setup.

Unlike the full-set study, participants were not presented with the full 84 haptic stimulus set, but instead with a subset of 50 haptic stimuli. Subsets of size 50 were chosen as the target size as, in initial testing, it was found to be the largest number of stimuli that could be consistently sorted in approximately an hour, using the technique described above. Using the subset algorithm described in Chapter 4, and employing the modifications to our method described in Section 7.1.1, we produced 5 randomly distributed subsets (see Appendix C for specific subsets used) that we could use multiple times in order to test if judgments differed from subset to subset. Our algorithm ensured that every two stimuli appeared together at least once in one of the subsets, and guaranteed that a dissimilarity value would be present for each combination of stimuli. While our algorithm attempts to minimize the amount of overlapping coverage, certain points in the dissimilarity matrix are overlapped by as many as four different subsets, though most points are covered by only one or two subsets. This overlap is unfortunate but some amount is unavoidable due to the nature of the sorting task. Each one of the 15 participants performed their sorting task on one (and only one) of the five subsets, meaning that each subset was sorted by three participants, with 3 participants x 5 subsets giving the total 15 participants, as shown in Table 7.1.

7.2.2 Results

Dissimilarity values for each participant were calculated in the same manner as described in Section 6.2.2. Full 84x84 symmetric dissimilarity matrices were then created for each participant, containing the dissimilarity values for those stimuli present in the subset they were tested with, and a value of -1 for all stimuli not presented, to mark them as missing. These dissimilarity matrices were then averaged over all users, with only the non-missing values used to create the average value for each point within the matrix (many of the final values were thus averaged over different numbers of individuals).

This averaged dissimilarity matrix was then run through the SPSS ALSCAL algorithm for dimensions 1 to 4. The resultant stress values were plotted (see Figure 7.1), and a marked elbow was looked for, but no obvious candidate was forthcoming. The most likely candidate for an elbow was the 2D solution, though the overall curve of the graph was fairly even. The 2D solution had S-Stress = 0.38949 and r^2 = 0 .23463, while the 3D had S-Stress = 0.28216 and r^2 = 0.35597. These stress values are reasonable, though the r^2 values are very low, indicating that a large amount of variability in the data was not accounted for in the solutions. However, this trend occurs across all dimensions, so we were forced to use the data as it was. Therefore, as in our full-set study, for parsimony as well as ease of interpretation, the 2D solution was selected for analysis, and the 3D solution was consulted for clarifying purposes.

Upon an initial analysis of the 2D solution, several features were evident (see Figure 7.2). Firstly, the strength of the amplitude axis is still quite evident, which is encouraging if we are concerned about the results being realistic: the subset technique has at least captured this, the strongest trend in the data according to our gold standard. Additionally, essentially no effect of frequency was found, just as in the gold standard. However, applying the rhythm trends as established in our gold standard, we see that the placement of the four groups has shifted. In the gold standard, the 2D solution was split, orthogonally to the trend of amplitude, first according to the note length of the rhythms, short to long, and then within those two halves, from even rhythms to uneven rhythms. In the 2D solution for the subset study, however, the solution is split first according to evenness of rhythm, and then by note length. The major manifestation of this is that the group of stimuli that has long notes and an even rhythm has shifted over to the far extreme left of the map, pushing the other three groups towards the right.

Comparing the subset study MDS output statistically with the gold standard, we find a result contradictory to our visual inspection: the coefficient of alienation, K, is 0.4485, which, at $NP = 84$, $ND = 2$, is less than K critical = 0.55, and is significant for $p = .05$, according to the work of Borg and Leutner [3] and described in detail in Section 2.2.1. This result means that the similarity between the MDS maps of the subset and full-set studies is statistically similar, at a 95% confidence interval. Here we are presented with a case where statistical significance does not seem to agree with our practical analysis. The reason for these troublesome results is discussed in the following section

7.2.3 Reasons for Difference in MDS Results

The discrepancy between our statistical and practical analysis was a major concern to us. The placement of the "long-even" group in a different position as opposed to the gold standard study was a potentially fatal result for our new study methodology. This result is the primary reason why we determined that we needed to run additional participants, as described in Section 7.2.4—a choice that would result in a successful validation of our new technique. First, however, we will describe how a limited number of observations caused this group of stimuli to be placed differently, the key insight leading us to run additional participants.

Analysis of Standard Deviation

As visual inspection and statistical comparison differed in their conclusions, greater importance was placed upon our third means of analysis, comparison of the standard deviation values of the averaged dissimilarity matrix. The average standard deviation of all values in the dissimilarity matrix is 346.27, which is considerably higher than the gold standard's average SD of 160.02, so right away we were presented with a potential explanation for the difference in the two MDS outputs (gold standard and subset study) as being some source of additional noise in the subset data.

Next we observed the distribution of the SD values over the dissimilarity matrix for the subset study, and noted a marked difference in the distribution of high and low SD values as compared to the gold standard (see Figure 7.3). The gold standard generally contained high SD values for points in the matrix where two stimuli of the same amplitude level were being compared, and low SD values for points where stimuli of different amplitude levels were compared. By contrast, the subset study has distinct "stripes" of high SD values running through the dissimilarity matrix. These stripes occur along groups of four or five stimuli, and extend through comparisons with other stimuli of both different and the same amplitude level. No overall trend of between and within amplitude comparisons can be seen.

As noted in Figure 7.4, many of these stripes of high SD occur along stimuli from the "long-even" group —the same group whose placement in the MDS map is the primary difference between the full-set and subset study's results. Though there are two other stripes that correspond to stimuli in parts of other groups (the high-amplitude and frequency members of Group 4 from Table 3.2, and the high-amplitude, low-frequency members of Group 5), the long-even group has by far the largest number of stimuli that are part of these high SD stripes. Certainly it can be noted that not all of the stimuli in the long-even group correspond to areas of high standard deviation; indeed several of the stripes are off by one or two stimuli from the actual stimulus groups. However, as we will argue later, this is because such stripes of high SD are the result of a combination of certain hard-to-judge groups with areas that received low numbers of observations, and so this lack of exact correspondence is to be expected.

Nevertheless, a high degree of variability would explain why the long-even group appears in a different position in the subset study's results compared to the full-set study. What requires an explanation is the source of this high degree of disagreement among participants. Comparison of individual MDS plots is not overly fruitful, as most participants saw different sets of stimuli than the others and thus have, by definition, different MDS plots (see Appendix B for individual plots). Consequently we continue to rely on standard deviation as our main method of analysis.

Hypothetical Explanations for Divergent Long-Even Group Results

There are several possibilities for the observed strips of high standard deviation associated with the long-even group in the subset MDS result. It could be that by a fluke, these stimuli only ever occurred in one of the subsets, thus biasing their results

(Hypothesis 1); it could be that this is the result of different subsets producing different results for the stimuli in this group (Hypothesis 2); it could be that there was simply not enough data gathered for the stimuli in this group to gain a statistically reasonable average (Hypothesis 3); or it could be that the long-even group is inherently harder to judge than the other groups (Hypothesis 4). Hypothesis 4 is hard to prove definitively; instead it becomes the default conclusion by elimination if the other explanations are actively disproved.

In the following, we will address Hypotheses 1-3.

Hypothesis 1: Fluke Distribution of Stimuli

As the subsets were randomly created, it would be expected that the stimuli in the long-even group would appear fairly evenly throughout all five different subsets used, and this is indeed the case. All five subsets contained between 7 to 12 out of the 16 total stimuli in the long-even group (see Appendix C for subsets used). Thus all of the subsets would have contributed values to describing this group, so Hypotheses 1, the 'fluke' uneven distribution of stimuli, is eliminated.

Hypothesis 2: Subset-Relative Judgments

We know that long-even note rhythm stimuli occurred at roughly the same frequency in all 5 subsets; however, we do not know if one or more of these subsets had a distribution of stimuli that would cause the judgments in the set to be skewed and/or noisy. If such outlier subsets existed in our study, it could be that they contributed to the long-even group being placed differently in the MDS plot (hypothesis 2). If this were the case, we would notice this most distinctly for dissimilarity values that were averaged using data from different (conflicting) subsets.

A simple way of determining whether the averaged dissimilarity values for the long-even group came mostly from overlapping subsets or just from single subsets was to look at the number of values used in the average of each dissimilarity value in the group. If we plot the number of observations for each dissimilarity value in a similar way to the standard deviation values (see Figure 7.5), we can see which values were produced only from a single subset (evaluated by three participants, and shown in dark purple), and which values were produced with data from more than one subset (run by some multiple of three participants (shown as light purple or white cells, i.e. lightest means highest number of both observations and distinct subsets used).

By lining up the columns and rows that contained the long-even group (orange) with the plot of the number of observations, we can see that for the most part, the dissimilarity values for the long-even group have been aggregated from single subsets (dark purple; though the particular subset that has contributed to each value does differ). Given the high standard deviation that the long-even group is correlated with (as illustrated in Figure 7.4), a possible explanation is that the noise was due to judgments for different subsets being distinctly different as a consequence of between-subset variations--yet this appears not to be the case. These levels of high SD seem to be occurring despite the values being averaged from only one subset, so we can claim that Hypothesis 2, noise from conflicting subsets, does not appear to be a convincing explanation for the source of the higher overall noise exhibited in the subset analysis compared to the single set analysis.

It should be noted that there is another way that subset-relative judgments could have affected the placement of the long-even group, but it is not an effect that would have produced the distinctive , long-even group associated stripes of high SD that are evident. If there were any subsets that produced judgments for the long-even group which were distinctly different from that of the other subsets (i.e. these stimuli substantially rearranged on the MDS output, as opposed to their relative positions 'stretched' a little), then stitching together the results from these subsets could create a contradictory picture of the entire stimulus set, as evinced by high noise associated primarily with members found in the idiosyncratic *subset* as opposed to just the problematic stimuli group. However, the stripes of high SD occur across all five different subsets (since the long-even group that corresponds with them appears in all five subsets)--so either all the subsets produced idiosyncratic results for the long-even group, or none of them did. If each subset skewed its judgments consistently, we would *not* notice a trend of higher SD for the long-even group, a consistent skew should produce similar results for each of the subsets which in turn produce a low SD when aggregated. In fact the only way in which we would be able to determine such a skew would be in comparison to the gold standard.

Alternatively, if the subsets caused judgments to skew unpredictably for each *participant* (between-participant variations rather than between-subsets, but in a subset-specific way) then we would expect to see levels of high SD across all stimuli and not just in the long-even group (Hypothesis 4 already accounts for there being something particular about the long-even group that tends to create noise).

Hence it is not the case that our observed noise resulted from the fact that in some instances, dissimilarity values in the long-even group were averaged from multiple-individual evaluations of a single subset, as we can see no evidence that any of the subsets produced 'bad', outlier results. In Section 7.3.2 we further discuss how we found no evidence of subset-relative skewing of judgments overall, but for our current argument pertaining to the long-even group it suffices to say that such subset-relative judgments do not appear to have caused the high SD exhibited by this group. Thus we are left with either Hypothesis 4 (long-even group is inherently hard to judge) or Hypothesis 3 (this group did not receive sufficient observations).

Hypothesis 3: Insufficient Observations

While one can view the plotted observations (Figure 7.5) as a means of determining how many different subsets contributed to an average value, we can also simply consider the number of observations as a raw value in itself, disregarding how many subsets these observations came from. Performing this mental switch, we notice that the long-even group largely corresponds to areas with the minimum number of observations (three, represented by dark purple). A low n value in the calculation of standard deviation allows outliers to more strongly affect the value, and so the high standard deviation that the long-even group corresponds to could well have been caused by have an n of 3 for many of its values. It is thus possible that outliers from such a small sample of data (as evidenced by the high SD) caused the long-even group to be placed differently in the subset study's MDS output, as compared to the gold standard. Given that the subsets were randomized, we can only conclude that it was simply an unfortunate distribution that caused the long-even group to have so few observations for many of its dissimilarity values.

Thus far, our data analysis allows us to make this claim specifically about the long-even group, but we have not yet presented a general analysis of how the number of observations (as well as the number of subsets) can affect the quality of the MDS results. In Section 7.4 we discuss this topic in much greater detail. Nevertheless this analysis is key to understanding why we decided to run additional participants: i.e. we needed to distinguish between Hypothesis 3 (insufficient observations) and Hypothesis 4 (inherent difficulties in judging) to explain the placement of the long-even group. Thus, we chose to obtain additional data to rigorously test Hypothesis 3.

To bring the current analysis to closure, we will thus make a forward reference to the conclusions derived from this augmentation of our study in Section 7.3, where we do indeed find that increasing the number of observations (apparently independently of the number of subsets) to a level that is relatively uniform across all stimulus pairs, diminishes the stripes of high SD. That is, low observation numbers seem to correlate to high SD, and this compounded with the fact that the long-even group, by chance, appeared to particularly suffer from receiving a low number of observations. Further, the augmented subset study produced an MDS result in which the long-even group is placed consistently with the gold-standard result. We therefore will conclude that Hypothesis 3 is upheld.

Hypothesis 4: Long-Even Group is Inherently Hard to Judge

Our results to this point do not allow us either to firmly accept nor refute Hypothesis 4, that the long-even group was inherently harder to judge. Compounded with the lack of observations, this inherent difficulty could well have been an additional source noise. We therefore must conclude that it may also have contributed to the stripes of high standard deviation seen associated with this group in Figure 7.5.

Summary of Long-Even Results

Overall, these findings are very encouraging: we were initially worried that subsets might produce poor results, but have found instead that they can produce strong results quite similar to the gold standard. Along the way, we discovered the importance of maintaining uniform coverage, of at least 5 observations per point, across the whole dissimilarity matrix. Below we will further disambiguate the role of the subsets themselves in the result (7.3.2).

7.2.4 Study Part Two: Additional Participants with New Subsets

The fact that many values in the averaged dissimilarity matrix for the initial subset study only came from a single subset, tested three times on three different participants, was due to a particular choice in the study design of using only 5 subsets for 15 participants. We made this choice so we could gather a baseline level of variance due to individual differences, and compare it against the level of variance between different subsets. However, this choice had the negative side effect that there was a fairly large range in the number of observations that a given value in the aggregate dissimilarity matrix could be averaged over. Values where subsets overlapped were replicated three times, meaning that while some values had as few as three observations, others had as many as twelve. Since it appeared that these values with a low number of observations might be causing the MDS output to differ from the gold standard (Hypothesis 3), it was decided that we should run more participants, using new subsets designed to "fill in the gaps" left by the first subsets, thus evening out the number of observations across the entire dissimilarity matrix.

Method

Seven additional participants were run through the same procedure as described in 7.2.1, but this time each with a *unique* subset of 50 stimuli designed to even the coverage provided by the first subset study. Participants were all graduate students at UBC, ages ranging from 22 to 29. The seven additional subsets ensured that each point in the aggregate dissimilarity matrix had a minimum of 5 observations (filled from a minimum of 3 different subsets), while the majority of points had between 6-10 observations and some points had as many as 17.

Results

Adding the dissimilarity matrices produced from the 7 additional participants to those produced from the original 15 participants, we created a new aggregate dissimilarity matrix, which we then ran through the SPSS ALSCAL algorithm as before. Figure 7.6 shows the stress plot for the new MDS solutions, from dimensions one to six. Though the stress curve is very similar to Figure 7.1, again with no marked elbow, there is a significant increase in r^2 values for both the 2D and 3D solutions at 0.32848 and 0.47007 respectively, indicating a greater amount of variability in the data has been accounted for in the solution.

Graphing the MDS output with the additional participants' data, and applying again the grouping of stimuli from the gold standard, we find a much more encouraging result (see Figure 7.7). The trend of amplitude is just as strong as before; but now the trend of rhythms, from long note to short note with uneven to even nested within, is present in the exact same order as the gold standard (Figure 6.10), though mirrored left-right. As relative, not absolute, position is what is important in MDS plots, mirrored results are equivalent. Mapping an axis along the centroids of all four of these groups creates a line almost perfectly perpendicular to the axis of amplitude, precisely as it does in the gold standard. Additionally, idiosyncratic placements of stimuli such as 44 and 45 (from the short-uneven group) amongst a generally long-uneven cluster are replicated quite similarly to the gold-standard solution. Further qualitative similarities and analysis are described in Section 7.3.1.

Two quantitative values also point towards an increase in similarity to the gold standard. The average standard deviation is down from 346.27 to 245.78, which is still higher than the gold standard's value of 160.02, but greatly decreased from the initial subset study, indicating that these results are more internally consistent. Furthermore, the new, lower K value of 0.3534, a roughly 21% decrease from the previous value of 0.4485, is consistent with the theory that the additional participants run have increased the similarity between the subset study's results and the gold standard—though as discussed before and further elaborated in 7.3.1, K cannot be taken as a complete guarantor of similarity.

Running additional participants was done in order to address our analysis in Section 7.2.3. The results of this addition back up Hypothesis 3, i.e. that insufficient observations can explain altered placement of the long-even group in Part 1 of this study.

7.3 Validation of Subset Technique

We set out to prove the validity of the subset method of data gathering for MDS by ensuring that it met three criteria: that different subsets did not produce significantly different results for the same stimuli; that the resultant MDS plot was reasonable and believable in terms of interpretability; and that the MDS output compared favorably to that of the gold standard. Below, Section 7.3.1 describes how the output of the MDS algorithm is structured and how it compares to the gold standard, thus validating our method in terms of the second and third criteria, given sufficient data as collected in Part Two of this study. After this, Section 7.3.2 describes how the standard deviation of the dissimilarity values shows where discrepancies between individuals arose, disproving the theory that these discrepancies arose from the use of subsets, thus satisfying the first criterion. The analysis of standard deviation (7.3.2) is made easier by first considering the shape of the MDS output, which is why it is discussed second.

7.3.1 Criteria 2 and 3: Reasonableness of Results & Comparison to Gold Standard

The initial MDS results of our subset study could be said to have met the second criterion of reasonable and believable results, but failed on the third criterion of similarity to the gold standard. The strength of amplitude and the lack of effect of frequency were as expected, and grouping according to certain aspects of rhythm on an axis perpendicular to that of amplitude were also evident. Without a gold standard referent, we could have concluded that these results in fact represented ground truth as to the perceptual characteristics of the rhythmic haptic stimulus data set. Even with the gold standard, according to our statistic of similarity, *K*, the two results were similar. However, according to our visual, *ad hoc* analysis of the MDS outputs, the two trends of rhythm were without a doubt different: the four main groups differed in order of appearance along the rhythm axis, which would lead do different conclusions about which aspects of rhythm were more perceptually important in differentiating between stimuli.

Happily, running additional participants closed the gap between the subset study and gold standard, greatly increasing their similarity even to detailed visual inspection. Though rotated roughly 45 degrees clockwise, and mirrored along the rhythm axis (both simply products of random variations within the MDS algorithm itself, and therefore inconsequential), the two MDS maps maintain the same order of grouping along the rhythm axis, and have an even stronger and cleaner separation along the amplitude axis. *K* was similarly more positive, approaching more closely its ideal value of 0. Thus at a broad level, the subset results did seem to resemble those of the gold standard to a reasonable and practically useful degree (in the absence of other objective measures).

Sub-Group Analysis for Higher Resolution

However, these trends were fairly high level, and so a more detailed analysis was performed in order to determine how well the subset method captured the more nuanced characteristics of the stimulus set. In the analysis of the gold standard, a sub-analysis of all the stimuli with a high amplitude level was performed in order to examine more closely what effects rhythm had on the stimuli's perception, regardless of amplitude; we performed a similar sub-analysis on the data produced by the subset method, to see if it yielded the same insights.

In Figure 7.8, the high-amplitude sub-group is analyzed in isolation and graphed. Applying the same groupings as in the gold standard and plotting their axes, we see that they are almost exactly the same length and in similar directions. Indeed the general layout of the two graphs, Figures 7.8 and 6.9, is strikingly similar. The evenness of a given rhythm makes up one axis, going from even in the top right to uneven in the bottom left, while note length makes up the second axis, going from long notes in the top left to short notes in the bottom right.

However, there are some differences between the two MDS maps, most notably that the short-even and short-uneven groups are not totally separated, as they are in the gold standard. The two stimuli with a four quarter-note rhythm, and the two stimuli with an eight eighth-note rhythm are situated amongst the short-uneven group, which is counter to their placement in the gold standard. This placement may indicate that, at least for short-note rhythms, the number of notes in a rhythm is sometimes considered as the same thing as the perceived evenness of the rhythm. However, at this level of detail, we are entering into a realm of very precise pronouncements about how very small numbers of stimuli are perceived, from a study that involved the judgment of a very large number of stimuli. Insights at this level are probably better served by studies run on small sections of the data, looking for particular characteristics of individual stimuli. Thus at a secondary level of detail (with amplitude removed) the results from the subset study can be said to be similar to that of the gold standard, though at an even further level of detail, discrepancies begin to appear. This level of correspondence is likely greater than we can expect between any two experiments run on the same stimuli, so for our purposes, the MDS results of the subset study and the gold standard can be said to be both qualitatively and quantitatively similar.

Questioning the Statistical Analysis

One last question that might be asked is why the K statistic failed to account for the differences in MDS outputs that were observed through our own analysis. A potential explanation for this is due to dependence of K on the distances between each data point on the map, as K is calculated by comparing the distances between each data point in the first map against the same distances in the second map. Since both 2D results were arranged in a circumflex, most points in the MDS map have large distances between them, across the circumflex. Consequently, the similarity of these large distances may have had a large enough effect on the K statistic that the change in position from one side of the circumflex to the other, of a small number of stimuli, created too small a difference to greatly change the overall *K* value. Since *K* does not encode the relative importance of any particular stimuli, it could not reflect the significance that the change in position of those particular stimuli had. Indeed, a similar change in position (distance-wise) of a different but similar number of stimuli could likely have produced more-or-less the same ordering of rhythm groups, which we would have then used as an argument for the similarity of the two MDS results. This result only serves to confirm to us the importance of cross-checking conclusions using several means and of looking for practical significance as well as statistical significance; which our results, in the end, have indeed demonstrated.

7.3.2 Consistency of Results: Do Subsets Introduce Too Much Noise?

The standard deviation of each averaged dissimilarity value can be used as an indicator of the degree to which different participants disagreed on how dissimilar a pair of stimuli appear: the higher the standard deviation, the higher the disagreement between participants. If all the participants were tested with the same subset, then the level of disagreement can be attributed solely to individual differences, and/or variability in repeated observations by the same individual, in their perception of the given stimuli. If the participants were judging the same stimuli, but in different subsets, then an additional potential source of disagreement is the relativizing effect of different subsets on perceptual judgments. Thus if we are concerned about whether splitting the stimuli up into subsets will cause too much variability in judgments (Criterion 1 in Section 7.1.1), analysis of standard deviation is where we need to concern ourselves.

Noise Due to Subset-Relative Judgments: Between-Subsets Analysis

The need to check for this effect drove our initial 3 participant x 5 subset study design: we required replicated data for a small number of unique subsets, as opposed to a larger number of non-replicated unique subsets, with their more complex overlapping pattern. The data from the additional participants is discussed in the next section, but this particular analysis requires the 3x5 structure of the first part of the study. As we had three participants sort each subset in the first round of our study, we can create an average for the dissimilarity values for each subset, and examine the standard deviation of those averaged values to determine the baseline level of noise that comes solely from individual differences. The baseline SD gives us a reference point to compare against the SD of dissimilarity values averaged with data from multiple, overlapping subsets—values indicating the level of noise from individual differences *plus* the effect of different subsets. If the SD for these overlap values is substantially greater than the baseline, then we would have strong evidence for there being an effect of subset on the judgments given by participants.

This comparison is performed by plotting the SD of all values in the dissimilarity matrix, aside by side with the number of subsets involved in the average of each dissimilarity value, as is shown in Figure 7.9. By comparing back and forth between the two halves of the matrix, various trends can be discerned. Through choice of colouring we highlight that the darker areas of high SD (and specifically the distinct "stripes") generally occur where there are darker areas indicating a single subset – i.e. non-overlap areas; and that the lighter areas of low SD generally occur where there are lighter areas of high numbers of subsets. This result is in contrast to our original concern that increasing the number of subsets in play would increase SD for observations from overlapping subsets, although it is countered by the fact that these points *also* have more observations overall. Nevertheless, it does appear that dissimilarity values from overlapping subsets converge towards an appropriate value for this stimulus set.

If some particular subsets affect peoples' judgments by consistently skewing them a certain way (for all individuals), then we would expect to see the result of overlapped dissimilarity values having higher SD, a result that we did not see. However, if some subsets by chance contain combinations which generate confusion or disagreement and simply make everyones' judgments noisier, then we would expect to see that some subsets exhibit overall levels of noise higher than others. This too, is not evidenced by our data, most strongly by the instances of stripes of high SD. These stripes occur across values from all five of the subsets used originally, and as noted above, *generally only have single subsets contributing to each of its dissimilarity values.* The consistency of these stripes of SD across all five subsets seems to indicate that no one subset was noisier than the other.

The fact that the levels of high SD found in the first round of the study cannot seemingly be attributed to the negative effects of subsets is strong support for the subset method meeting the first criterion of validity (no effect of subset-relative judgments).

Evidence from Additional Participants

Further evidence for the lack of subset-relative effects is the results from running additional participants, each with their own, unique subset. If each unique subset did tend to produce judgments that were unique for the stimuli contained in the subset, then adding in seven new subsets to a data set already built up of five different subsets should increase the overall noise level in the aggregate matrix. Yet the net effect was to *reduce* standard deviation and increase the MDS result's similarity to the gold standard. This reduction confirmed our initial hypothesis (Section 4.2) that the best way to counteract any effect of subset, large or small, is to completely randomize the selection of each subset used and to use overall a 'reasonably' large number of subsets relative to the size of the complete stimulus set. In this way, if any one subset did have a strong relativizing effect on judgments, its effect would be minimized due to its data being mixed in with many other subsets, which should, on average, contain a reasonable cross-section of stimuli. Another way of saying this is that we wished to have as many different subsets as possible, to minimize the effect of each one; randomized subset construction maximizes this effect. By giving multiple participants the same subset (Part One of this study), we were able to observe this trend, but this technique is not recommended for regular use: instead complete subset randomization, as originally specified, will minimize overall noise levels in the data, as well as the number of participants needed to obtain a desired number of observations for each data point.

In summary, we cannot claim that there will be *no* effects of subsets; and to some extent, the low impact of subsets observed here could be a function of characteristics of the particular overall stimulus set which we have explored in the present research. Other sets, e.g. those containing small groups of highly salient stimuli, could potentially be more vulnerable to such problems. However, it appears that (a) potential subset effects can be mitigated by using *more* and randomly created subsets, as opposed to fewer; and (b) the effect of subsets handled in this way are likely to be small, or even negligible (as observed here) in comparison to that of individual differences. Since the problem of individual differences is one that is never going to be removed completely from an experiment, we can assume that the effect of subsets, if any, will manifest itself very infrequently if handled properly. Individual differences are a problem that any MDS data gathering technique suffers from, so we feel we can conclude that our new subset technique suffers from no problems worse than those confronted by any other method known to this author.

7.4 Reflections on the Design of the Subset Data Gathering Method

With a strong case made for the validity of the subset method of data gathering, we turn next to a reflection on the overall nature of the technique, its strengths, weaknesses and peculiarities. Especially in our analysis of the standard deviation of the dissimilarity matrix, we found many interesting features indicating where our experimental technique succeeded, and where it struggled. As we wish this technique to be taken up by other researchers in the field, we outline here several important features or the subset method that any experimenter who wishes to use it should be aware of.

7.4.1 Observations vs. Subsets

In validating our method, we showed that subset-relative judgments did not appear to have a strong negative effect on our results (7.3.2). If judgments did differ from subset to subset, this difference was not evident to us. Furthermore, in our analysis of the long-even group (in Section 7.2.3), we concluded that its improper placement in the MDS plot from the first part of our study was due to it receiving an insufficient number of observations in conjunction, perhaps, with it being an innately difficult group to judge. These two results together seem to indicate that to ensure the quality of MDS results using the subset method, one should concern oneself most with gathering enough observations, and not be too concerned about the effect of subsets; and potentially to even employ *more* subsets to reduce the skewing influence of 'outlier' stimulus groups. We pursue this argument to its conclusion here.

Importance of Having Sufficient Observations

A low n value in the calculation of standard deviation allows outliers to more strongly affect the value. Thus analyzing Figure 7.9, we cannot be too surprised to find that having high SD in dissimilarity values seem to consistently occur where there are a low number of observations, though the converse is not always the case. It is important to note that a low number of observations has not *in all cases* resulted in high standard deviation, especially the noticeable "stripes" of high SD. Referring back to Figure 7.5, we see that these stripes also largely correspond with several different groups of stimuli from particular types of rhythms. Thus a second condition for creating high SD seems to be that the stimuli being averaged are from either the long-even group, or the two additional small subgroups of stimuli that also have high SD.

The nature of these two conditions is largely quite encouraging. By design, the areas of the dissimilarity matrix with the highest number of observations are also the areas with the greatest amount of overlap between subsets. Yet these areas of high overlap in fact generally correspond to values with *low* standard deviation; and even with the noise-reducing role of increased observations, if subset overlap was a source of truly discrepant data, we would not expect to see this. What this correspondence seems to indicate is that having observations coming from multiple different subsets has not been a noticeable source of noise in our data—the judgments from different subsets have generally converged. Instead, our major source of noise appears to be simply the effect of individual differences, regardless of subset, causing havoc within an average only when it contains too few observations. Furthermore, adding in more observations (as was done in the second part of the study) served to decrease overall standard deviations levels (see Figure 7.10) while simultaneously adding in more subsets.

Value of Having Many Subsets

However, a potential confound to the claim that increased observations brought about better data is that, in adding in seven more participants we also added in seven more subsets. Thus it could be argued that it was adding in subsets and not observations that increased the quality of the data. Though we had previously been concerned that different subsets would produce markedly different judgments, we had also tried to mitigate this problem by creating random subsets that would, on average, not suffer too heavily from this problem. Thus it could be argued that adding in more subsets simply allowed for the differences between subsets to be covered over more evenly (indeed, a positive role which we originally hypothesized in our argument for subset randomization).

We did not find evidence of the subset-specific effect which the problem subset randomization was meant to mitigate. Furthermore, the only positive effect we would supposedly be gaining from adding in more different subsets would be to counter this effect. It would almost then seem that our entire attempt to create randomized subsets was of no value, given that the main effect that it was meant to counter was not found – at least for this stimulus set.

Yet it would be premature to conclude that randomization is unnecessary, for two main reasons. First, we have not shown that there will *never* be subset-relative effects, merely that they were not evident in the current results. Indeed, it seems intuitive to us that there must be at least *some* variation due to subsets occurring, if only at a fairly small level, and that randomization would still be the best means of handling this problem. Thus we would argue that randomization acts as a sort of "safety net" that should help guard against effects of subset, should any manifest themselves; and further, could help to identify when larger subset effects do occur through a simple subset-overlap SD analysis.

The second point is efficiency: our subset randomization algorithm also helps to minimize the number of subsets needed in order to have a certain number of observations for all values in the dissimilarity matrix. This helps cut down on the number of participants needed to run a study using the subset method and at the same time optimizes uniformity of number of observations across all stimulus pairs, which is very important given that one of the major tradeoffs of the subset method is the number of participants needed to gather data. In fact, if we had used unique randomized subsets from the start, we would only have required 17 participants to gain the same number of observations (as opposed to 22), and the distribution of observations would have been considerably more even, with a range of 5 to 12 rather than 5 to 17 observations per dissimilarity value.

Types of Outliers and Method to Deal with Them

Another way viewing the argument put forward above is in terms of outliers. In our analysis of standard deviation, it was noted that there were two main possible sources of outlier data points that could cause noise in the data: individual differences and idiosyncratic subsets. Furthermore it was noted that these two sources produced different types of outliers, and could be dealt with separately using different methods.

The common source of outliers in psychological experiments is from individual differences in perceptual, physical or cognitive ability. We found this most evident in the five groups of three participants that each judged the same subset. Despite all experiencing the same subset, they still exhibited a high degree of standard deviation, even more so than where subsets overlapped, though not at a level that their results were entirely different (see Appendix B for individual plots). Nevertheless these differences in opinion must be attributed to individual differences. The standard method of dealing with such individual differences is to ensure the participant pool is representative of the overall population and to use enough participants to gain a representative sample. While we made our best effort to ensure participants were representative, we found that we had initially gathered too few observations for many of the data points. Thus we used increased observations to guard against outliers from individual differences.

The second source of outliers was subsets, though we did not find strong evidence of this being a large source of outlier data in the present data set. Nevertheless, we have not disproven that subsets could exist that would greatly skew any judgments given from it. Thus it is useful to consider the fact that the outliers caused by such subsets would occur not at the level of individuals, but at the level of entire subsets. Instead of one individual producing outlier results, one subset could produce outlier results over and over again, each time it was used by a new participant. To guard against this form of outlier, we used unique randomized subsets, so that the cost of any one outlier subset is mitigated by, on average, having many more subsets that do not cause strong subset-specific effects.

In essence, adding in more randomized subsets and adding in more observations both help reduce the effect of outliers, but at different levels. More subsets reduces the effect of outlier subsets, more observations reduce the effect of outlier individuals.

7.4.3 "Striping" of Standard Deviation

One last particular facet of the standard deviation results that does bear further examination is the "stripes" of high standard deviation in the aggregate dissimilarity matrix. As can be seen in Figure 7.3, the stripes are very distinct and generally occur in groups of four or five consecutive stimuli, though several single-stimulus stripes are also evident. While these stripes were observed to occur due to a combination of a low number of observations with certain groups of stimuli, adding more participants and subsets removed the stripes of extreme high levels of SD. Yet if we observe the distribution of SD within the new aggregate dissimilarity matrix (Figure 7.10) we find striping again, though at a much lower absolute level of standard deviation. Some of these stripes are in similar places as before, but many of them are not. Furthermore, their correspondence with areas of lower numbers of observations is less marked than before.

Thus we are forced to conclude that the striping at least somewhat comes from the algorithm that creates the subsets (see Chapter 4, Section 4.2 for details). The most likely explanation is that those stimuli which are placed into subsets made first by the subset algorithm (as is the case for many, but not all, of the stimuli that are part of stripes), end up not being used very often in later subsets, since they have already gotten the requisite number of observations assigned to them via the subset creation algorithm. This in turn causes other stimuli that occur more in the latter subsets to have a much more randomly distributed number of observations, since it becomes harder and harder for the subset algorithm to come up with stimulus pairings that do not create overlap. Thus stimuli used in early subsets will likely appear in fewer different subsets, receiving fewer total observations, and thus higher standard deviation. This potential weakness of the subset algorithm was admitted early on, and it could possibly be dealt with through continued iterations on the subset algorithm. However, once lower than a certain threshold of SD, there seemingly appears to be no longer a great effect of the striping on the actual output of the MDS algorithm. Moreover, if we had used completely randomized subsets (rather than re-using the first five) this effect may have been even smaller to begin with.

7.5 Summary

With all 22 participants run through our subset study, the results we produced were very similar, both qualitatively and quantitatively, to the gold standard which we produced using an established, validated data gathering technique. Furthermore, the addition of more subsets was found to increase rather than decrease the quality of the MDS output. Thus our three criteria for validating our new experimental method were met.

The effect of different subsets on judgments made in the sorting task was found to be negligible compared to individual differences in judgments, as evidenced by the relative distribution of standard deviation for within-subset averaged dissimilarities and between-subset dissimilarities. The fact that we saw areas of high SD generally being associated with values that only a single subset contributed too, and that adding in more new subsets resulted in lower SD values, all point towards subsets not having a negative effect on overall agreement within the data. Thus criterion one was met.

Criteria two and three were met due to the reasonable and interpretable results of running the dissimilarity matrix gathered by our subset method through the MDS algorithm, and its significant similarity to our gold standard. The 2D MDS output from the subset study exhibited very similar trends of amplitude and rhythm as those found in the gold standard, full-set study. Furthermore, the two outputs were statistically similar in layout according to the alienation coefficient K, the best statistical measure available for judging similarity of MDS results [4]; although we point out that this statistic must be used with caution in the current context, and paired with other means of analysis.

With all criteria met, and with results that are clearly similar to the gold standard, we can feel confident that our new subset method of gathering data for MDS will indeed produce valid results in future studies. Thus validated, we can recommend the method for use with any similarly large stimuli set, as a means of gaining perceptual dissimilarity ratings from users quickly and accurately.

Chapter 8: Conclusion

We set out in this thesis to accomplish two main goals. Firstly we wished to create a large and diverse set of haptic signals, by using rhythm as a parameter that could potentially increase a set's expressive range; and to ascertain the perceptual dimensions by which users actually categorize these signals. This first goal brought about the second goal, which was to develop a new means of evaluating the perceptual characteristics of such a large set of haptic stimuli. This we accomplished through the development and validation of a novel technique for allocating stimuli to participants, allowing smaller subsets to be tested separately on different users and greatly easing the task of data gathering.

In each of these goals we produced contributions to the field. The use of rhythms as a design parameter opened up a huge design space that easily allowed for the creation of a large set of different haptic stimuli. Prior work on haptic rhythms [24] [5] had only shown some evidence of this promise. Furthermore, people responded well to the use of rhythms in haptic stimuli; they were able to discern different aspects of rhythm within the stimuli, and distinguished the stimuli accordingly. Not only was the stimulus set successful, but our new experimental methodology proved valid as well. By producing results that are comparable both statistically and practically to an established gold standard we showed that breaking the data gathering task into subsets of the total, and then building the overall picture out of the pieces, is indeed a valid way to gather perceptual data about a stimulus set. The particular contributions of each of these successfully met goals are described in the sections below.

At a high level, these contributions mean that we have crossed a major hurdle in what we can do with haptic icons. No longer confined to small numbers used in restricted laboratory studies, haptic icons can now be produced and analyzed in set sizes of more broadly practical utility, given human perceptual abilities. Larger scale haptic icon production can ensure that a designer wishing to use haptic icons can find the types of stimuli that he or she wants, can find enough of them, and can know how each of them will feel relative to each other. This is a strong step towards mainstreaming the use of haptic icons, and bringing them into the world of practical application development.

8.1 Conclusions on Rhythms for Haptic Icons

When we were trying to find a means of enlarging the number of haptic icons that could be made, rhythm intuitively seemed like it might have depth enough to allow this. Our intuition was amply repaid by the results we found. We made a simple first trial at creating haptic stimuli using rhythm, attempting to use as few parameters as possible in order to allow for easier interpretation. Thus we did not even tap all of the potential aspects of rhythm—not to mention melody—that could be used with haptic stimuli. Nevertheless we easily created a set of 84 haptic stimuli that we intuitively believed, and later confirmed, to be distinct.

As always with haptic stimuli, designing them was one problem, but determining how people actually perceived them was a second, and in many ways more difficult problem. By using an established data gathering technique, we were able to build a dissimilarity matrix for all stimuli which could be applied to MDS. Analyzing the resultant output map, we were able to gain great insight into how these rhythm-based stimuli were perceived.

Perceptually, our stimuli were distinguished first by their amplitude, though not by frequency. This result counters results of previous studies that found both amplitude and frequency to be of importance [17]. This difference is perhaps because after amplitude, it was the previously untested parameter of rhythm that appeared to be the most important distinguishing feature. Rhythms shortened the overall time people were exposed to the frequencies of the vibrations, the quick succession of relatively short notes overwhelming the effect of frequency. This strong role of rhythm, and the conclusions we were able to make about how haptic rhythms are perceived, is a major contribution of this thesis.

Our analysis showed that our haptic rhythms were perceived according to two orthogonal axes. Though these axes may be particular to the types of rhythms we chose to study, we hope that the simplicity of our rhythms should make our insights a solid grounding for more complex instances. The first axis was what we termed "note length," while the second was "evenness" of the rhythm. How a stimulus was perceived along the "note length" axis depended on the longest note present in the rhythm. If a rhythm contained a half note or longer, it would fall on the "long" end of the note-length axis; if it contained only notes shorter than a half note, then it would fall on the "short" end of the axis. The "evenness" of a rhythm was determined by whether there were notes or rests of differing lengths within a given rhythm. If so, then the rhythm will feel distinctly uneven or unbalanced; conversely if a rhythm has only the same length notes and rests, it will feel even. This was a consistently reported perception that showed itself clearly in the data, yet was an unexpected perceptual classification which was not part of our original rhythm creation scheme. Nevertheless, these two axes of perception for rhythm appear to be strong and robust. Not only that, but they provide useful tools for future designers to predict how the stimuli they make will be grouped perceptually.

Compared to previously created sets of haptic icons, we succeeded in making an icon set larger than any yet created. The next largest, the tactile melodies created in [24], was 53 icons, but these icons were collected at random from a database of real-world musical melodies. More systematic icon sets like those created in the work of MacLean and Enriquez [17] or Brown *et. al.* [16] are generally even smaller, at 36 and 27, respectively. Thus we have roughly doubled the size of any prior icons sets. Furthermore, the perceptual axes our icon set exhibited—such as "note length" and "unevenness"—have a much larger and more interesting space for growth compared to such prior axes as amplitude or waveform; as they are not strictly ordinal in nature there is much more room for creative design.

The different perceptual axes found for rhythmic haptic stimuli, even when using a fairly simple set of rhythms, already show interesting and novel ways that stimuli can be distinguished. Our stimulus set has shown very promising results, with interesting perceptual features leading to an easily diversified set of rules about perception of haptic rhythms. The detailed insight into how these haptic stimuli are perceived has made them of great use to designers who wish to use haptic icons in their application, or to create more haptic icons themselves.

8.2 Validation of MDS Data Gathering Technique

Our new method of gathering perceptual judgment data also proved successful. By splitting the data gathering task into smaller subsets, we were able to greatly reduce the amount of time it takes for a user to perform a complete set of judgment tasks. This sub-sectioning allows for much more feasible experiment times, and ensures that fatigue is considerably less of an issue for the judgments given. Fatigue in particular is a very large issue, as other methods of data gathering such as pair-wise comparisons [20] can be heavily affected by drifts in judgment criteria caused by fatigue or loss of attention over time. Our new technique allows experimenters to select whatever size stimulus set they would like, giving them complete control over how hard they wish to push the participants in their study. The only major tradeoff of this technique is that the smaller the subset of stimuli presented to the user, relative to the superset, the more participants will have to be run in order to gain the same amount of data. For practical purposes, we suggest that a subset / superset size ratio less than one third will require an impractical number of participants; in our case, with a subset size of 50, this would allow for a superset of 150, which is almost twice again the size of our current—quite large—stimulus set. Thus we still have considerable room for growth before our data gathering method reaches its capacity.

To review the work we accomplished in designing and validating this new method of data gathering, we first proposed a simple study design based on combining two existing methods of gathering data for MDS. By using the sorting method as used by MacLean and Enriquez [17] among others, as well as the incomplete-set design described by Spence and Domoney [20], we developed the subset method, whereby each participant in the study was presented with a subset of the total stimulus set, with the average dissimilarity matrix being created out of the patchwork coverage of the various overlapping subsets.

We then validated our new technique by running a study using the new method on the same rhythmic haptic stimulus set that we had already studied using an established, but more cumbersome technique (the "gold standard"). What we found was extremely encouraging. Using 12 different subsets and 22 participants, we produced MDS results that both visually and statistically highly resemble the MDS results from the gold standard. The same trends of amplitude and rhythm that we found in the gold standard study were also found in the results of the subset study. Furthermore, the statistic of similarity K [3], also showed the two results to be statistically similar at p=.05.

Perhaps even more convincing, was that when presented initially with somewhat unclear results, it was running additional participants with *more* different subsets that improved the results to a point at which they were clearly similar to the gold standard. It was seen as a potential stumbling point of our method that participants would make their judgments completely relative to the stimuli present in their subset, and thus each set of judgments would be highly dependant on the subset they were from. However, we have shown that by randomizing subset selection, adding in more subsets (along with more participants) actually *increases* the accuracy of the averaged data.

Thus our main concern was allayed, and our resultant MDS output was confirmed as similar to the gold standard. Consequently we feel confident that our new subset data gathering method will produce valid results, and can be used in cases where the size of the stimulus set that needs to be tested is larger than any one user can be reasonably expected to make judgments on in a single sitting.

8.3 Future Work

There are two main areas of future work that start where this thesis ends off. The first is a further refinement of the haptic stimulus set using the insights we gained from our studies, and the second is a larger goal of applying these haptic icons in more in-depth applications.

Given what we found in our twin studies on the rhythmic haptic stimulus set, there are clear indicators of which design parameters we used were most important to people perceptually, and which were not. Certainly amplitude, length of notes present in rhythm and evenness of rhythm are among the former, while frequency (in the ranges tested, and in the presence of the more salient rhythmic variation) is among the latter. Thus if we wish to improve upon our stimulus set, making each stimulus more distinctive from the others as well as providing logical grouping for the stimuli, we need to take our findings into account and redesign the stimulus set accordingly. This redesigned stimulus set would have to be tested again with users (most likely using our new data gathering method), so that we could feel confident that our stimulus set actually exhibited the perceptual characteristics we anticipate.

With a well-designed stimulus set in tow, our next task, and the true aim of all the work that has gone into this, would be to apply the stimulus set as haptic icons in an interesting application and test it with users. Specifically, our hope is that with such a large stimulus set, we might be able to study the use of a haptic icon-enabled application over a longer period of time, in order to determine the upper maximum of how many different haptic icons a user can reasonably handle in an application. Due to the novel nature of the sensation haptic icons deliver, it is our belief that users have to struggle considerably with overcoming the novelty and unfamiliarity of the feeling before they are able to use haptic icons successfully. Yet if users were exposed to haptic icons for a longer period of time, to the point at which these sensations became normalized, then we might be able to determine such things as just how prevalent the use of haptic icons can be in an application, how many haptic icons people can learn to use, and how useful haptic icons can be for designing usable interfaces.

It is our sincere hope that the work done in this thesis will provide a key piece in answering these questions.

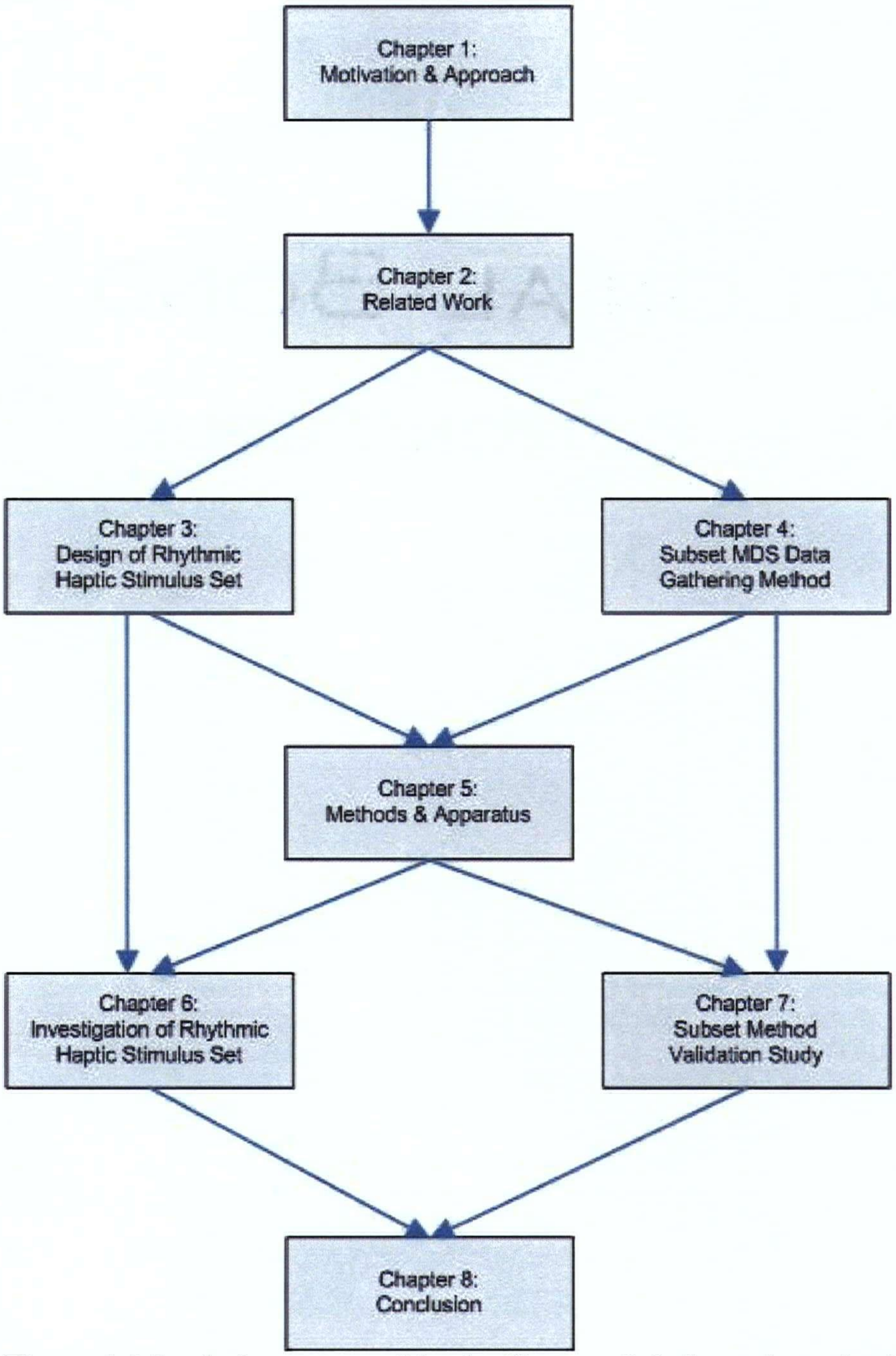

Figure 1.1 Logical structure of thesis. Chapters 3 & 6 pertain to the design and evaluation of the rhythmic haptic stimulus set. Chapters 4 & 7 pertain to the experimental design and validation of the new data gathering method. Chapter 5 describes the methods and apparatus that are shared by the studies described in Chapter 6 and 7.

Table 3.1. Note Types Used in Rhythms. Though at its smallest level of granularity there are 16 different slots in which vibration can be played, logically the notes are arranged either according to whole, three-quarter, half, quarter and eighth notes. Each note consists of both the time in which the vibration is played as well as the off-time where no vibration is played that is needed in order for one note to be distinguished from the next. Rest notes are referred to in the same manner as normal notes, except no vibrations are played.

R#	Note Type															
	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16
	Whole Note															
	Three-Quarters Note															
	Half Note								Half Note							
	Quarter Note				Quarter Note				Quarter Note				Quarter Note			
	Eighth		Eighth		Eighth		Eighth		Eighth		Eighth		Eighth		Eighth	

Table 3.2. Rhythms Used in Stimulus Set. Each row represents one "bar" that is repeated four times over a 2 second interval to make a rhythm. Within each bar, a note is demarcated by a pair of bold black lines. Notes contain both the on-time of the vibration plus the off-time that allows each note to be distinct from the next. Thus within each note there is a grey area indicating a time period where vibrations are playing and a white area indicating no vibrations are playing. This is except for rest notes, which are all white. See Table 3.1 caption for explanation of types of notes.

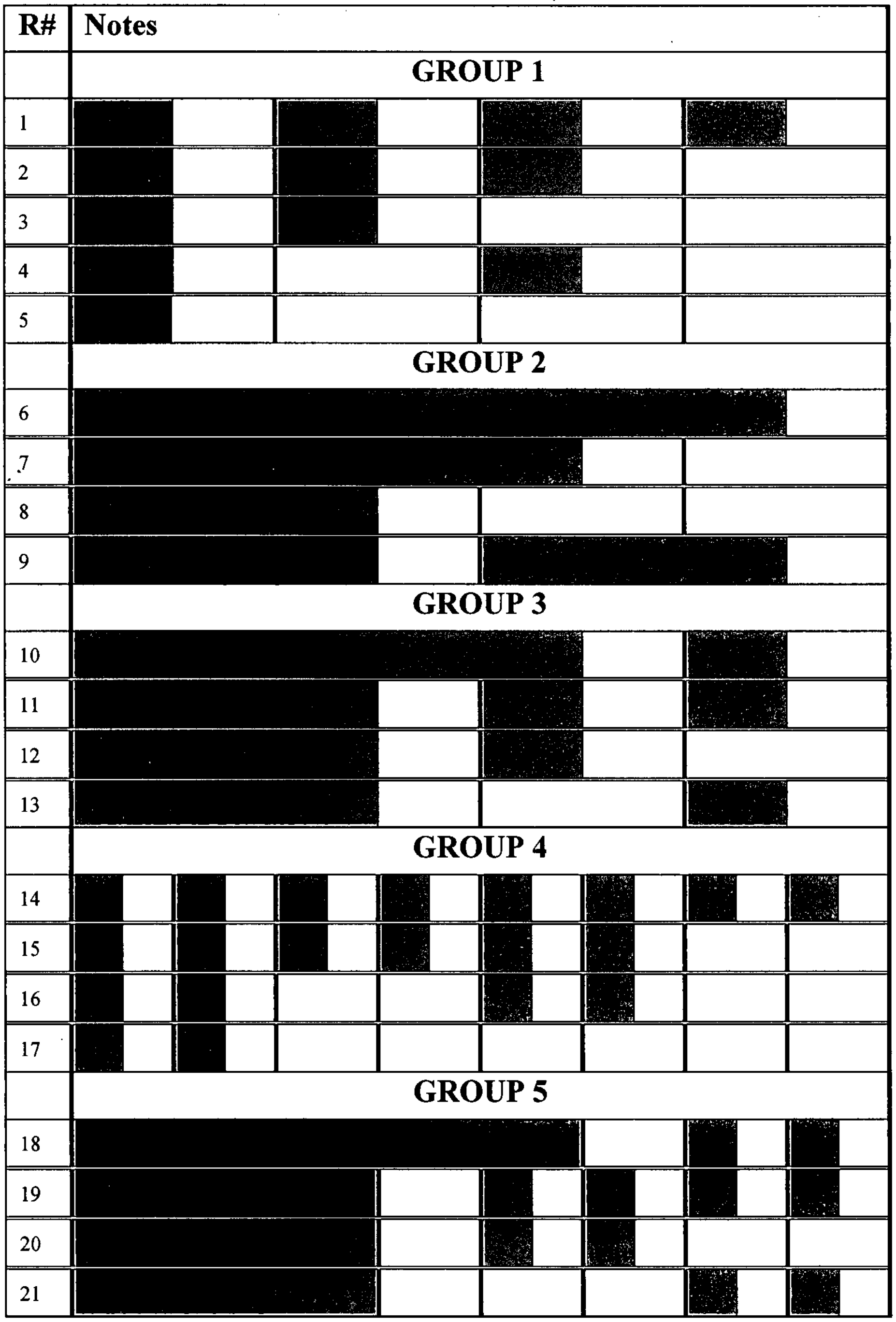

Table 3.3 Lookup Table for Stimulus Set. Stimulus numbers are used to refer to individual stimuli throughout the remainder of this document. We used a total of 84 stimuli, which consisted of 21 rhythms which varied as described in this chapter, combined with 2 amplitudes and 2 frequencies, distributed as described here.

	High Amplitude		**Low Amplitude**	
Rhythm #	**High Frequency**	**Low Frequency**	**High Frequency**	**Low Frequency**
1	1	22	43	64
2	2	23	44	65
3	3	24	45	66
4	4	25	46	67
5	5	26	47	68
6	6	27	48	69
7	7	28	49	70
8	8	29	50	71
9	9	30	51	72
10	10	31	52	73
11	11	32	53	74
12	12	33	54	75
13	13	34	55	76
14	14	35	56	77
15	15	36	57	78
16	16	37	58	79
17	17	38	59	80
18	18	39	60	81
19	19	40	61	82
20	20	41	62	83
21	21	42	63	84

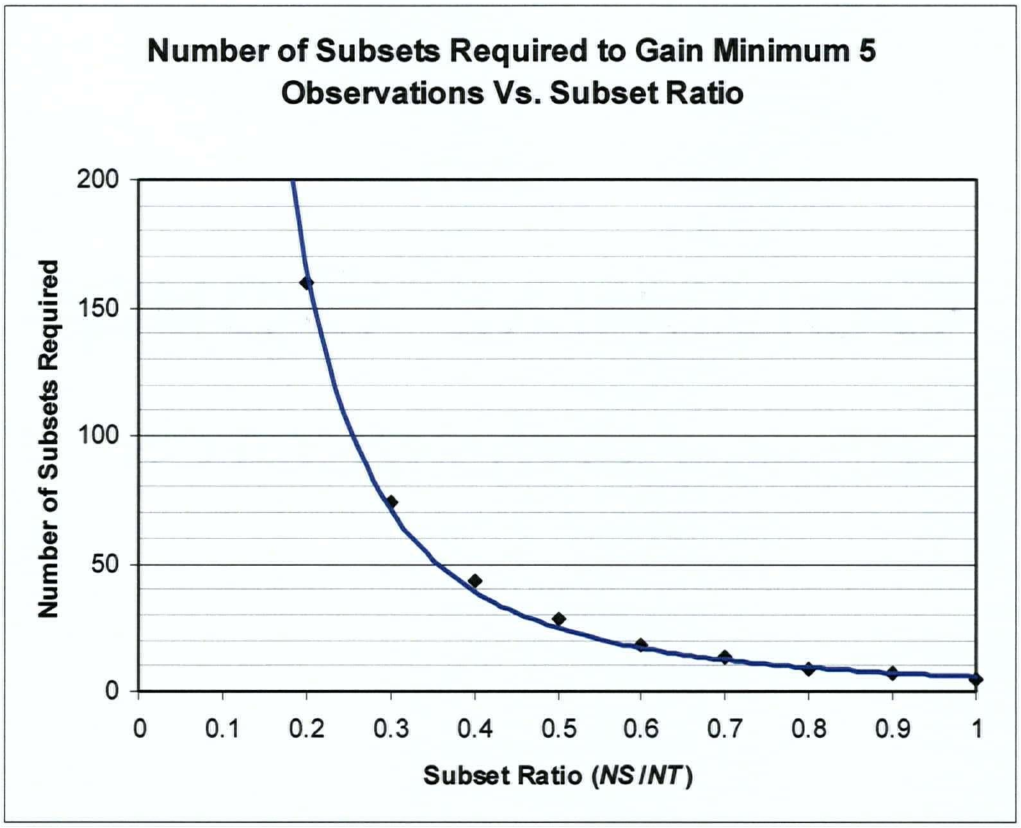

Figure 4.1. Graph of subsets versus subset ratio, for a desired minimum of five observations per dissimilarity value. For any ratio of subset size (NS) to total set size (NT), between 0 and 1, our subset creation algorithm attempts to create the minimal number of subsets that will ensure that at least the required number of observations are present for each point in the dissimilarity matrix. The resultant curve is shown above. As can be seen, anything lower than a ratio of roughly a third requires a number of subsets that is quite unreasonable for practical purposes.

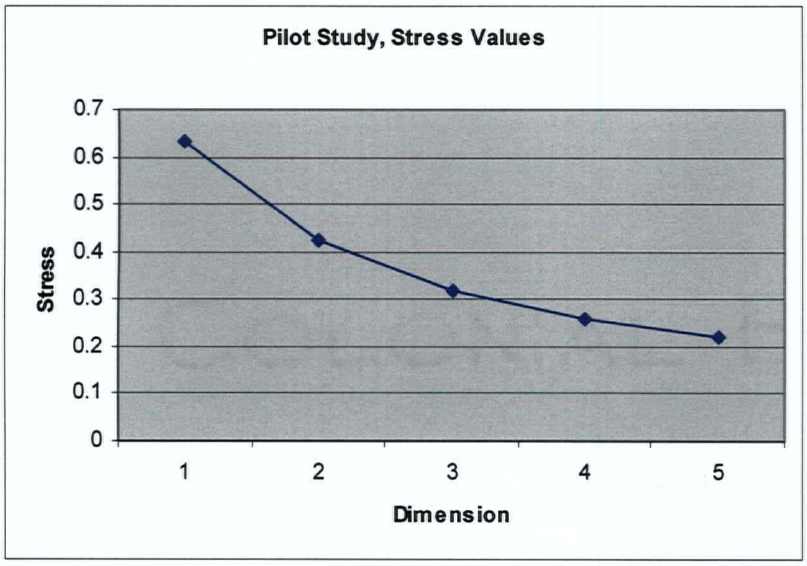

Figure 4.2. Stress values for the first five dimensional MDS solutions. No distinct elbow in the curve can be seen

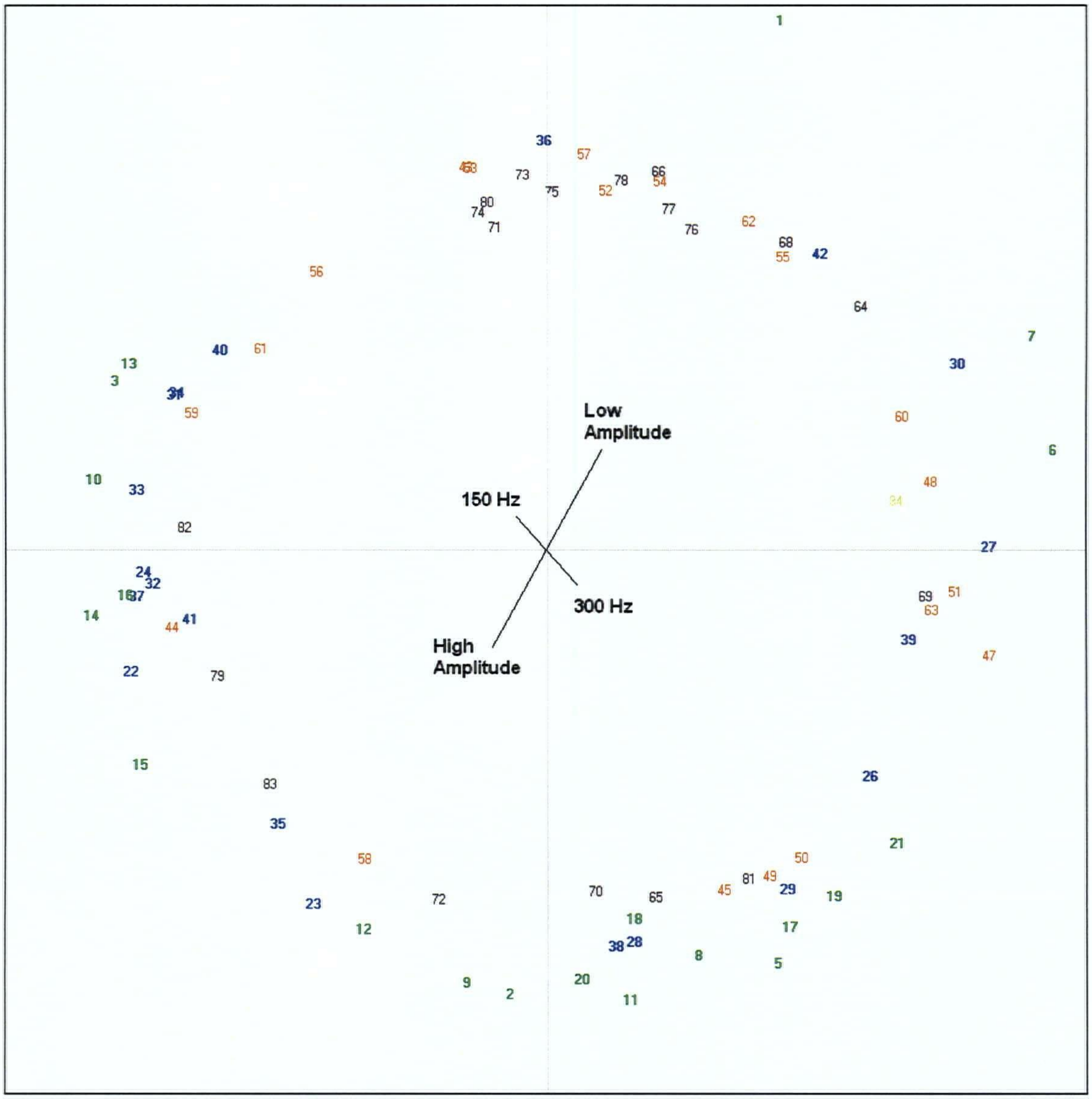

Figure 4.3. 2D Map of MDS Results. Trends of frequency and amplitude are as displayed. Green and blue stimuli are high amplitude, grey and orange stimuli are low. Green and orange stimuli are high frequency, blue and grey stimuli are low frequency. Icon numbers are as in Table 3.3. As can be seen, rhythms do not appeared to be grouped together, and even frequency and amplitude do not appear to have an overly strong grouping effect.

Figure 5.1. The Nokia 770

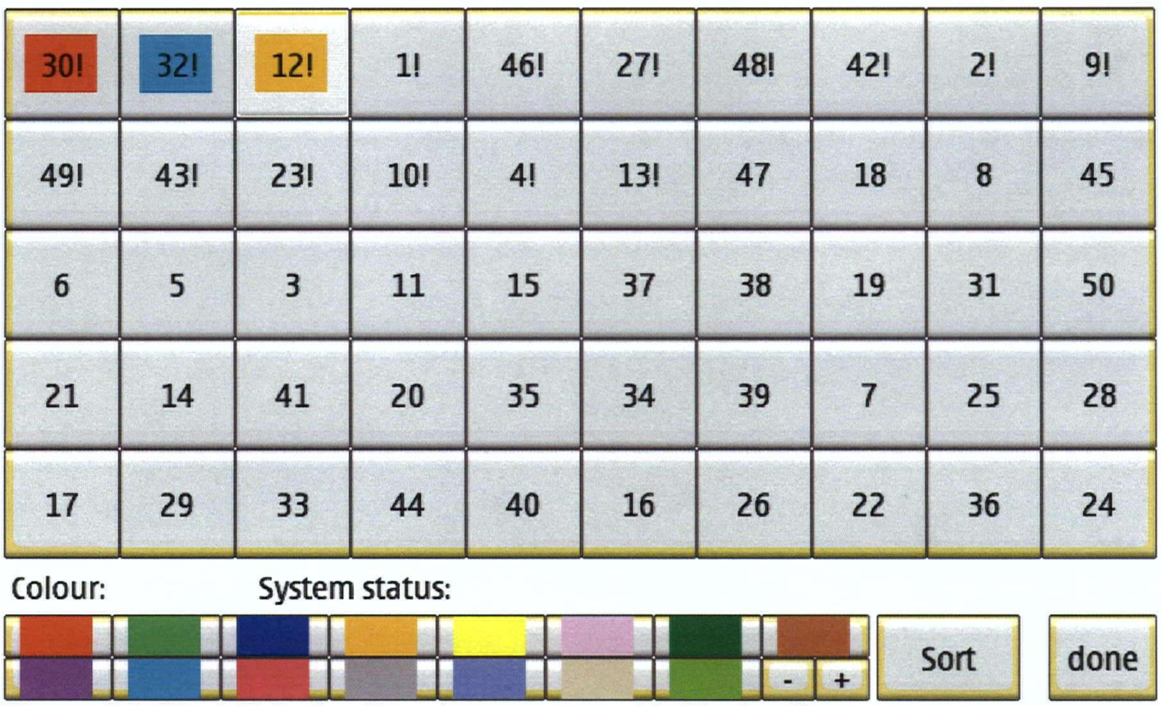

Figure 5.2. The MDS stimuli sorting program, with 50 stimuli.

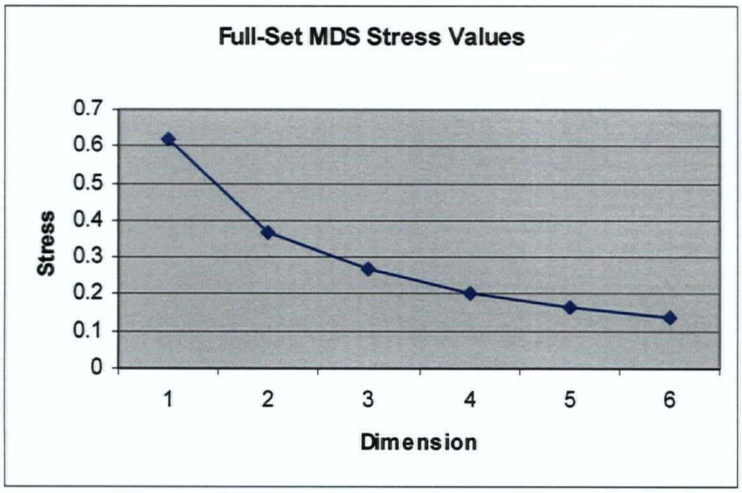

Figure 6.1. Stress values for the first six dimensional MDS solutions. No distinct elbow in the curve can be seen.

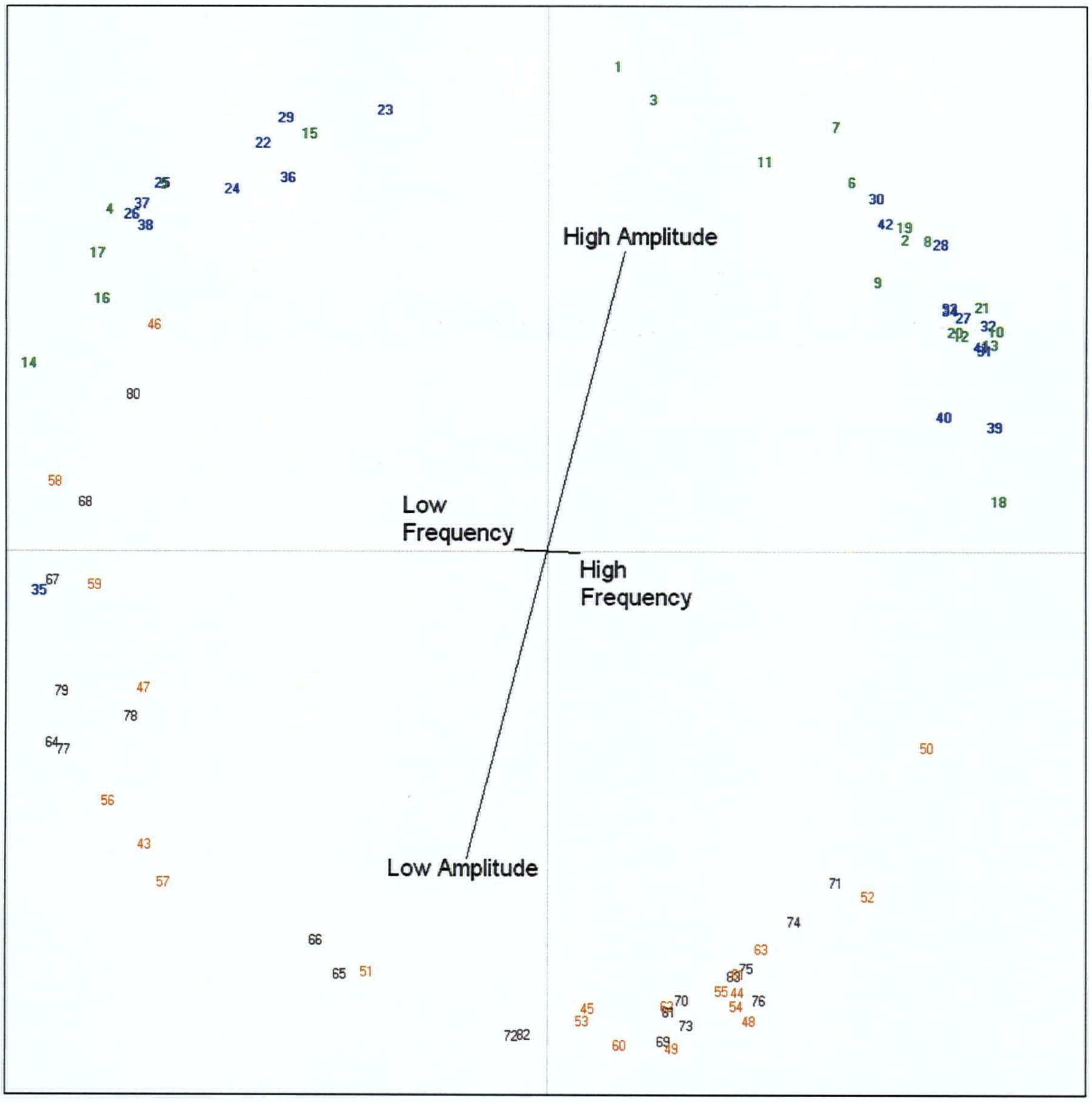

Figure 6.2. 2D MDS output, with all 84 stimuli plotted. Green and blue stimuli are high amplitude, grey and orange stimuli are low. Green and orange stimuli are high frequency, blue and grey stimuli are low frequency. Projected axes are labeled accordingly. See Table 3.3 for a lookup table of individual stimulus numbers, and in particular to identify their rhythm.

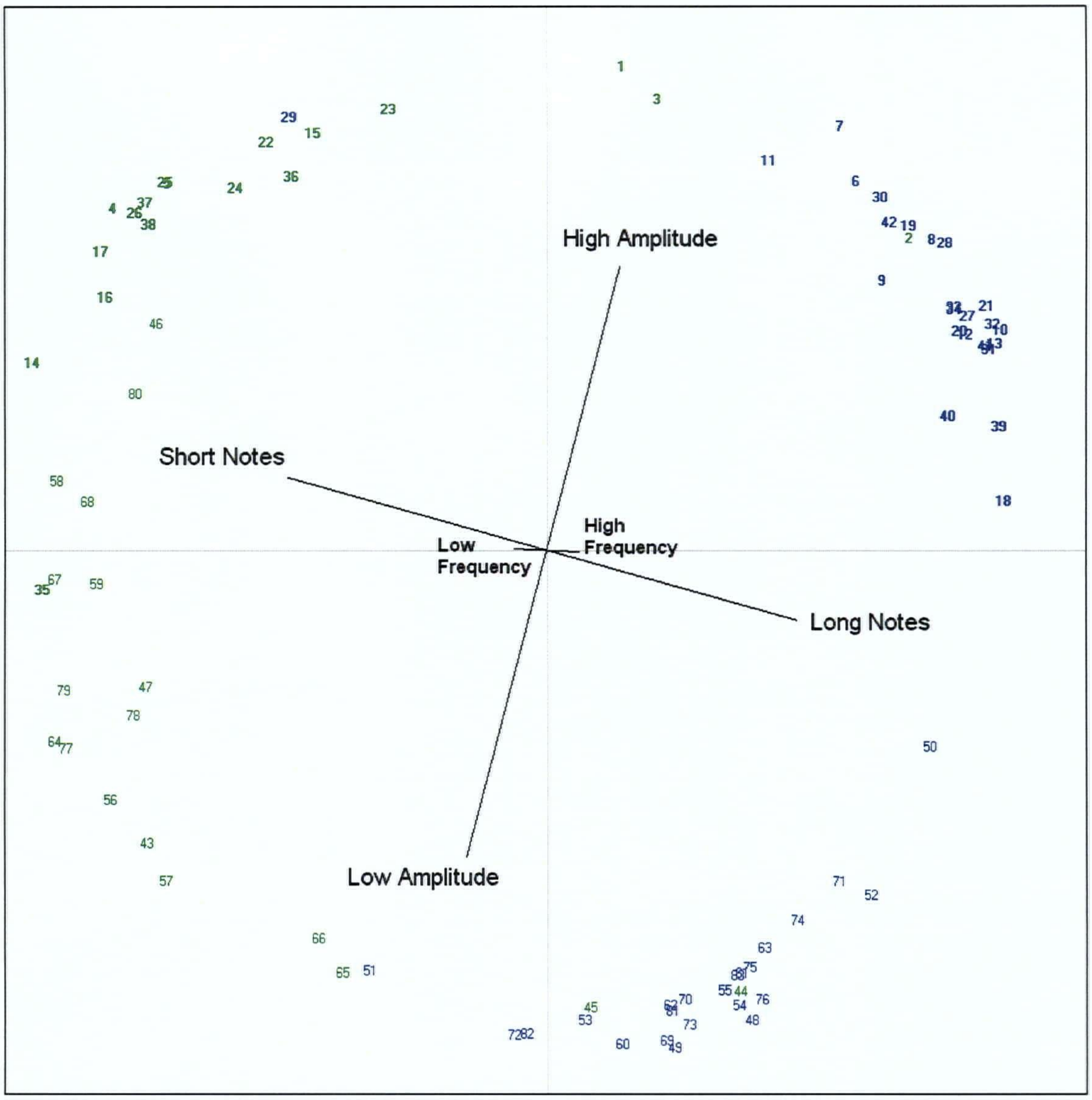

Figure 6.3. 2D MDS output, with all 84 stimuli plotted. Projected axes are labeled accordingly. Green stimuli are from Groups 1 and 4, containing only “short” notes; blue stimuli are from Groups 2, 3 and 5, containing “long” notes. See Table 3.3 for a lookup table of individual stimulus numbers and Table 3.2 for a lookup of rhythm groups.

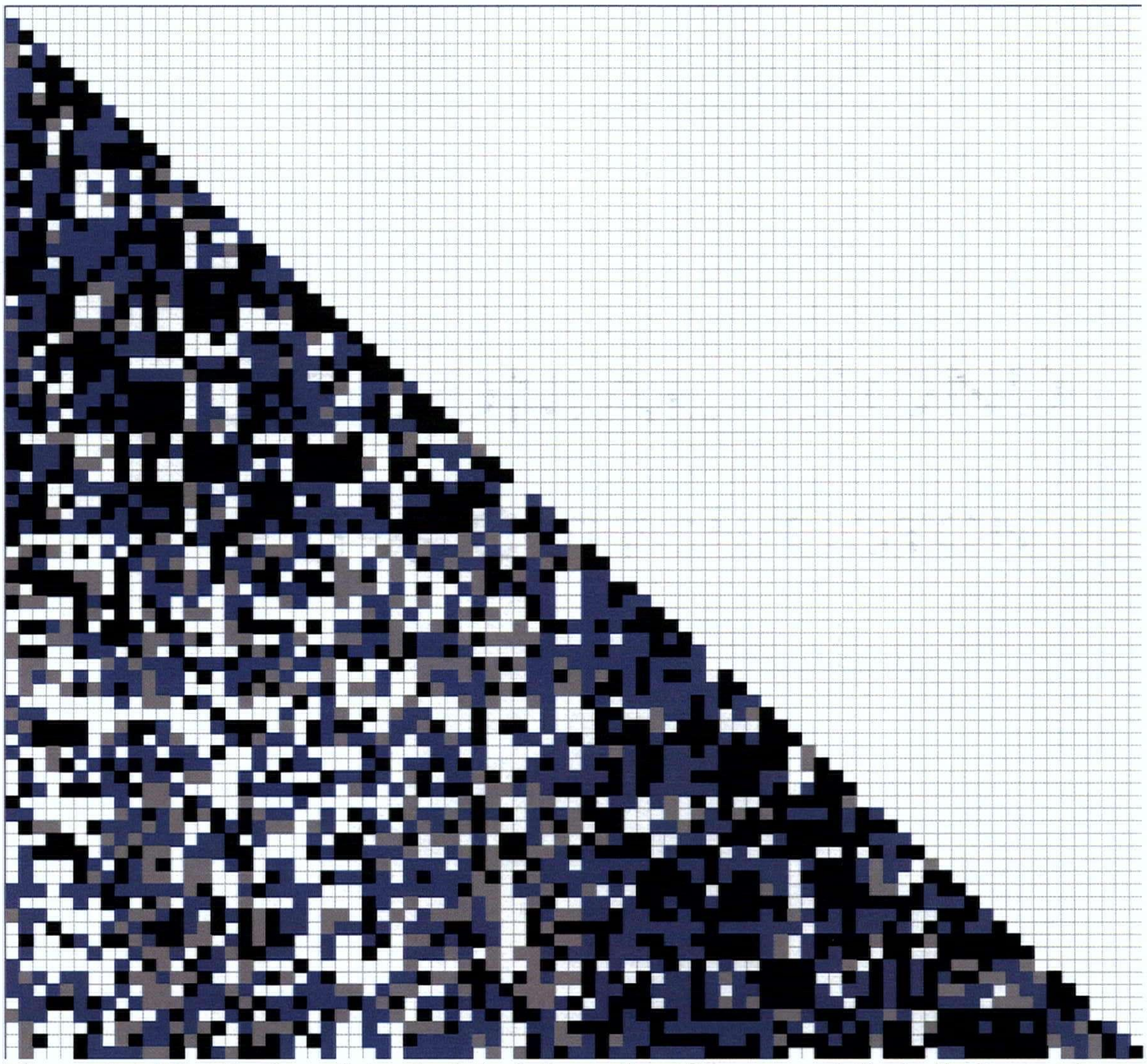

Figure 6.4. Distribution of standard deviation values for averaged dissimilarity matrix. Black squares indicate high SD, grey-blue medium-high, grey medium-low, and white squares have lowest SD.

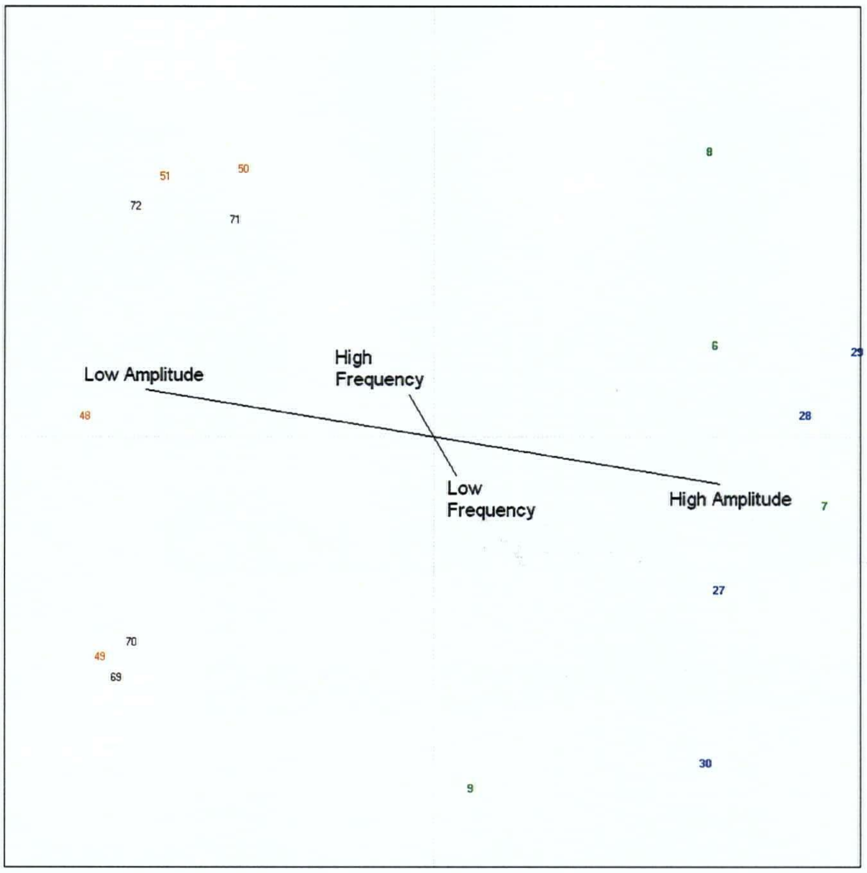

Figure 6.5 MDS plot for stimuli only containing long notes. Green and blue stimuli are high amplitude, grey and orange stimuli are low. Green and orange stimuli are high frequency, blue and grey stimuli are low frequency. Projected axes are labeled accordingly. See Table 3.3 for a lookup table of individual stimulus numbers.

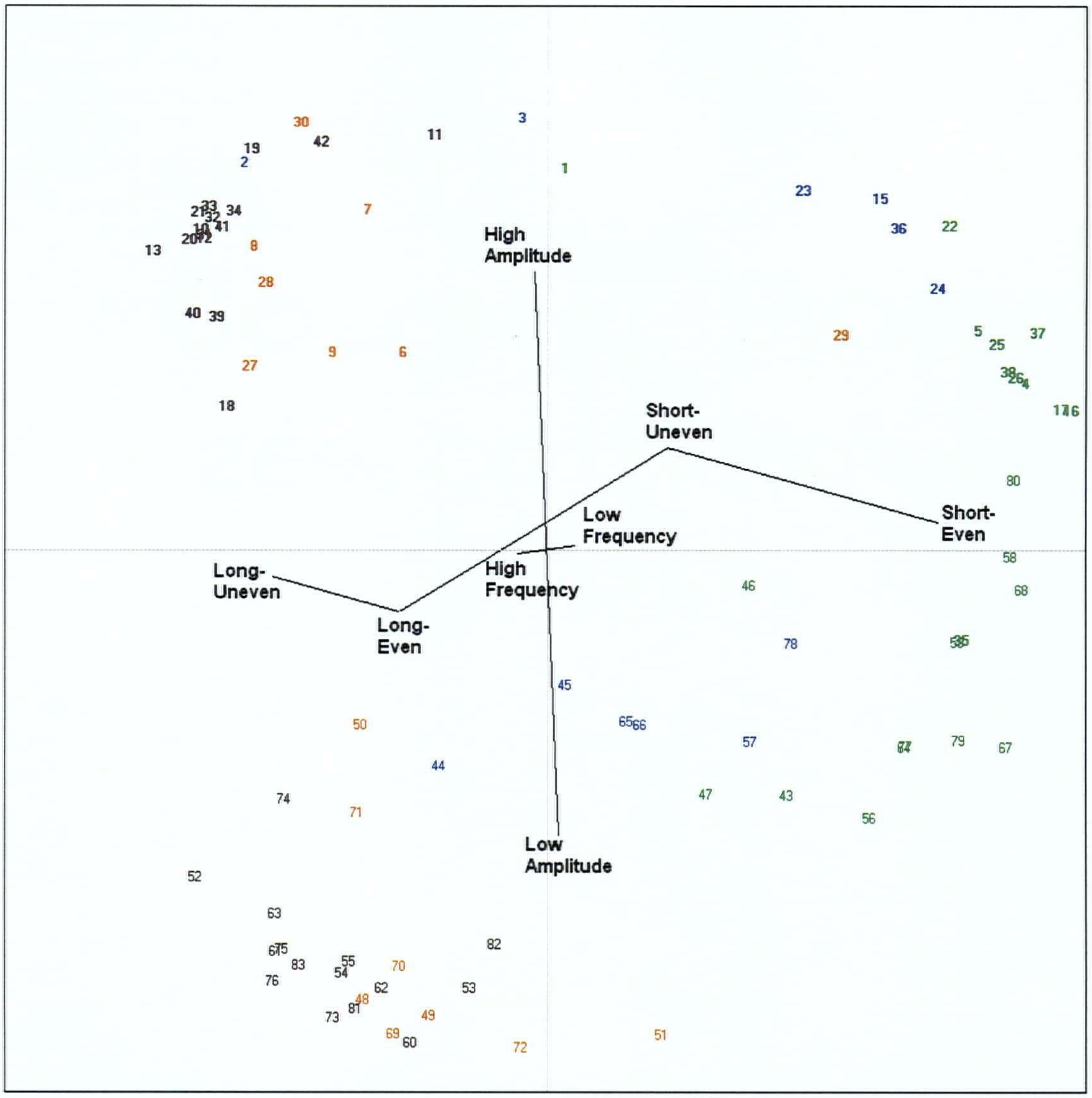

Figure 6.6 X-Y plane of 3D MDS for all 84 stimuli. X is the horizontal axis, Y the vertical. The green stimuli are from the short-even group, the blue are the short-uneven, the orange the long-even, and the grey are long-uneven (see text for explanation of group names). Projected axes are labeled accordingly.

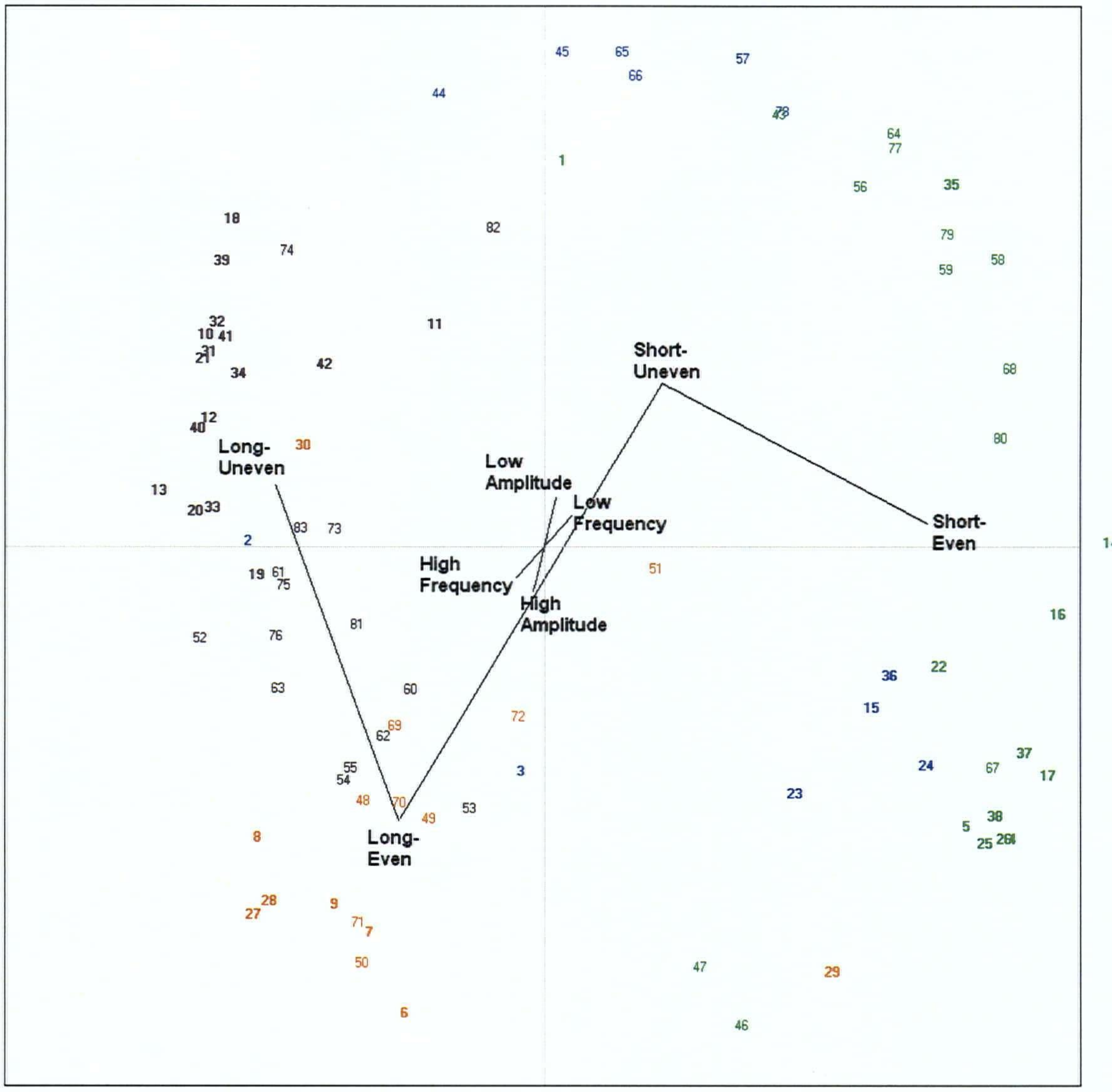

Figure 6.7 X-Z plane of 3D MDS for all 84 stimuli. X is the horizontal axis, Z the vertical. The green stimuli are from the short-even group, the blue are the short-uneven, the orange the long-even, and the grey are long-uneven

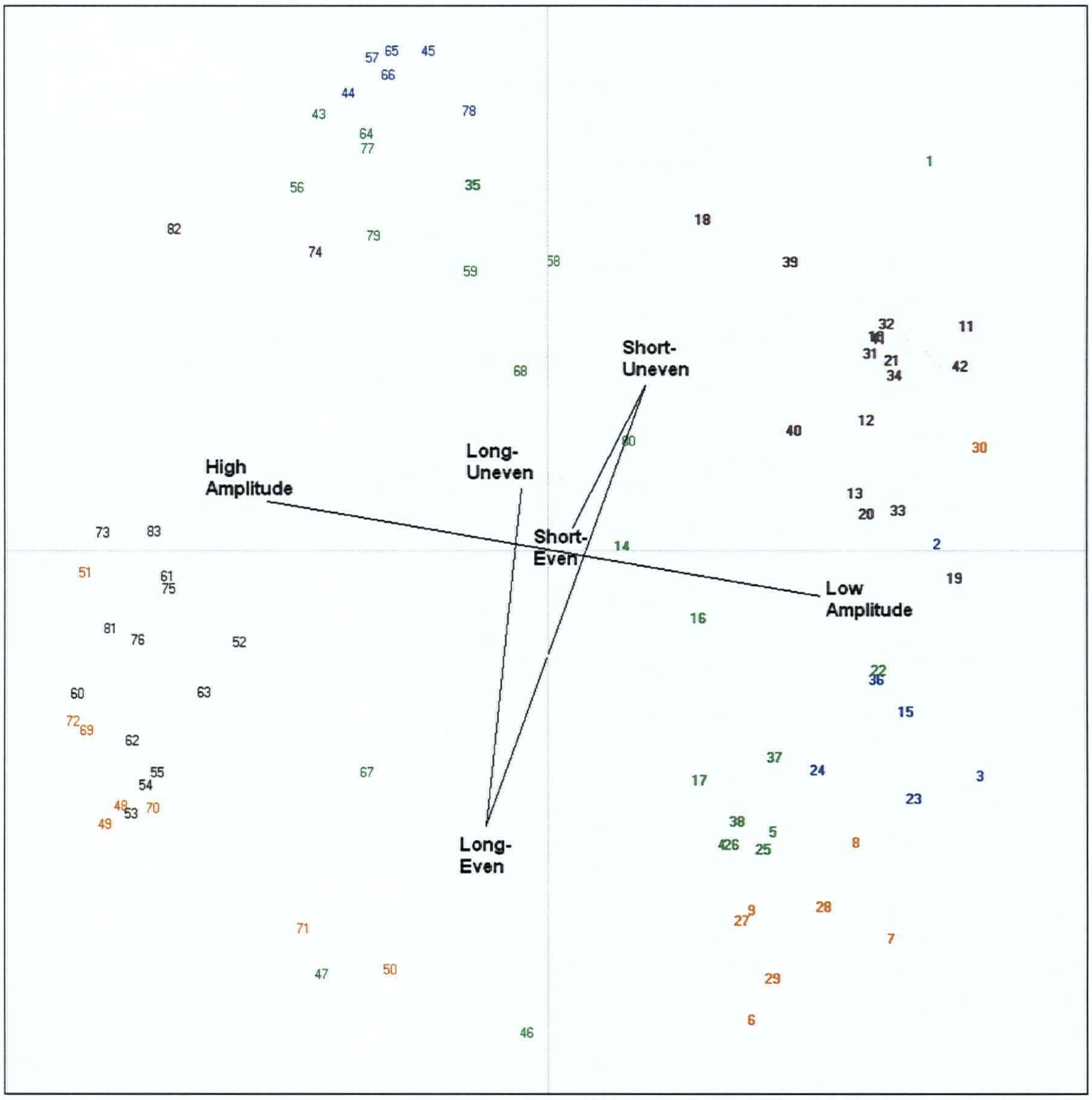

Figure 6.8 Y-Z plane of 3D MDS for all 84 stimuli. Y is the horizontal axis, Z the vertical. The green stimuli are from the short-even group, the blue are the short-uneven, the orange the long-even, and the grey are long-uneven. Frequency axis is omitted due to space constraints, but is similar in size to Figures 6.6 and 6.7.

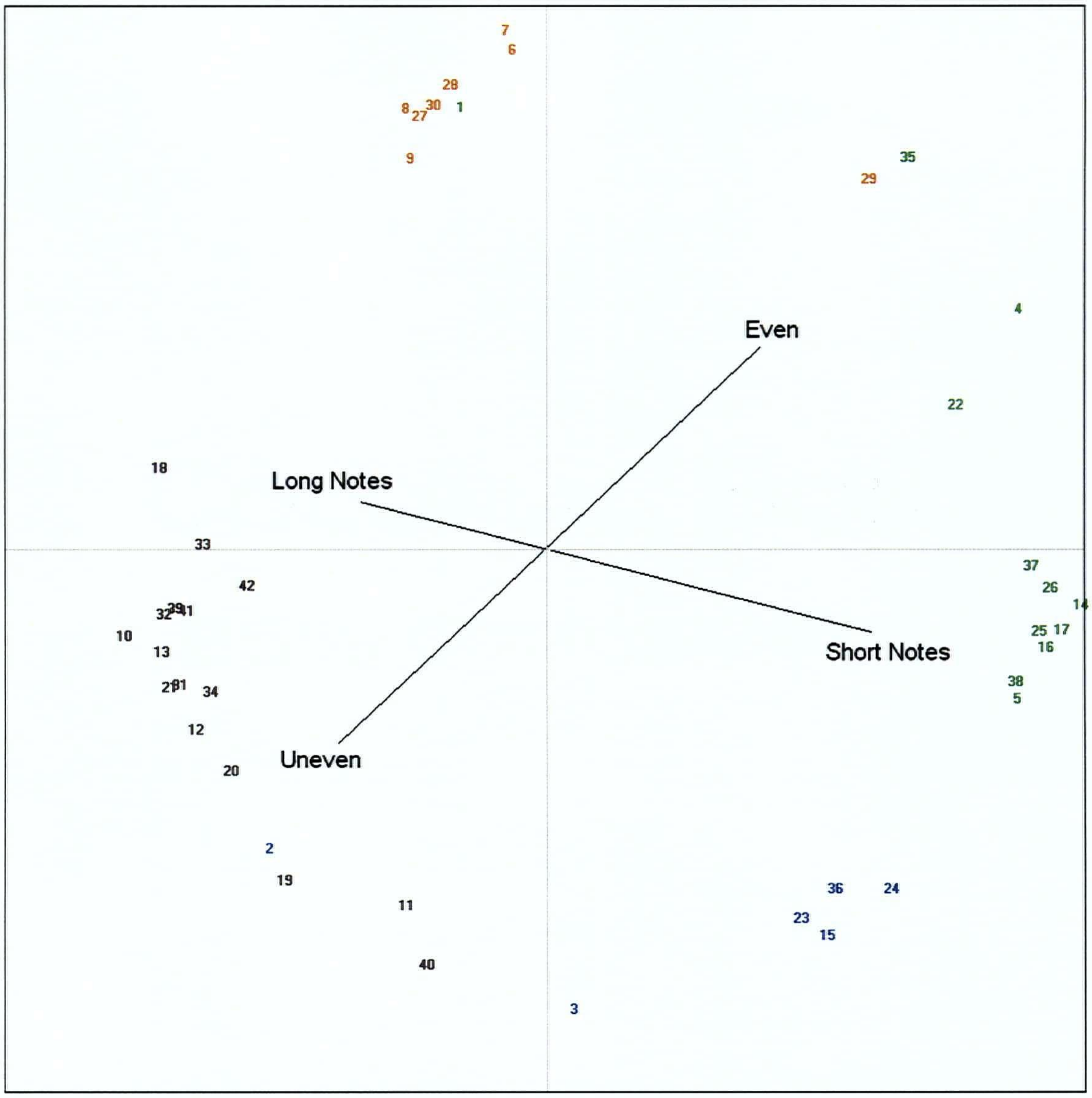

Figure 6.9. High-amplitude only MDS sub-analysis. The green stimuli are from the short-even group, the blue are the short-uneven, the orange the long-even, and the grey are long-uneven. Projected axes are labeled accordingly.

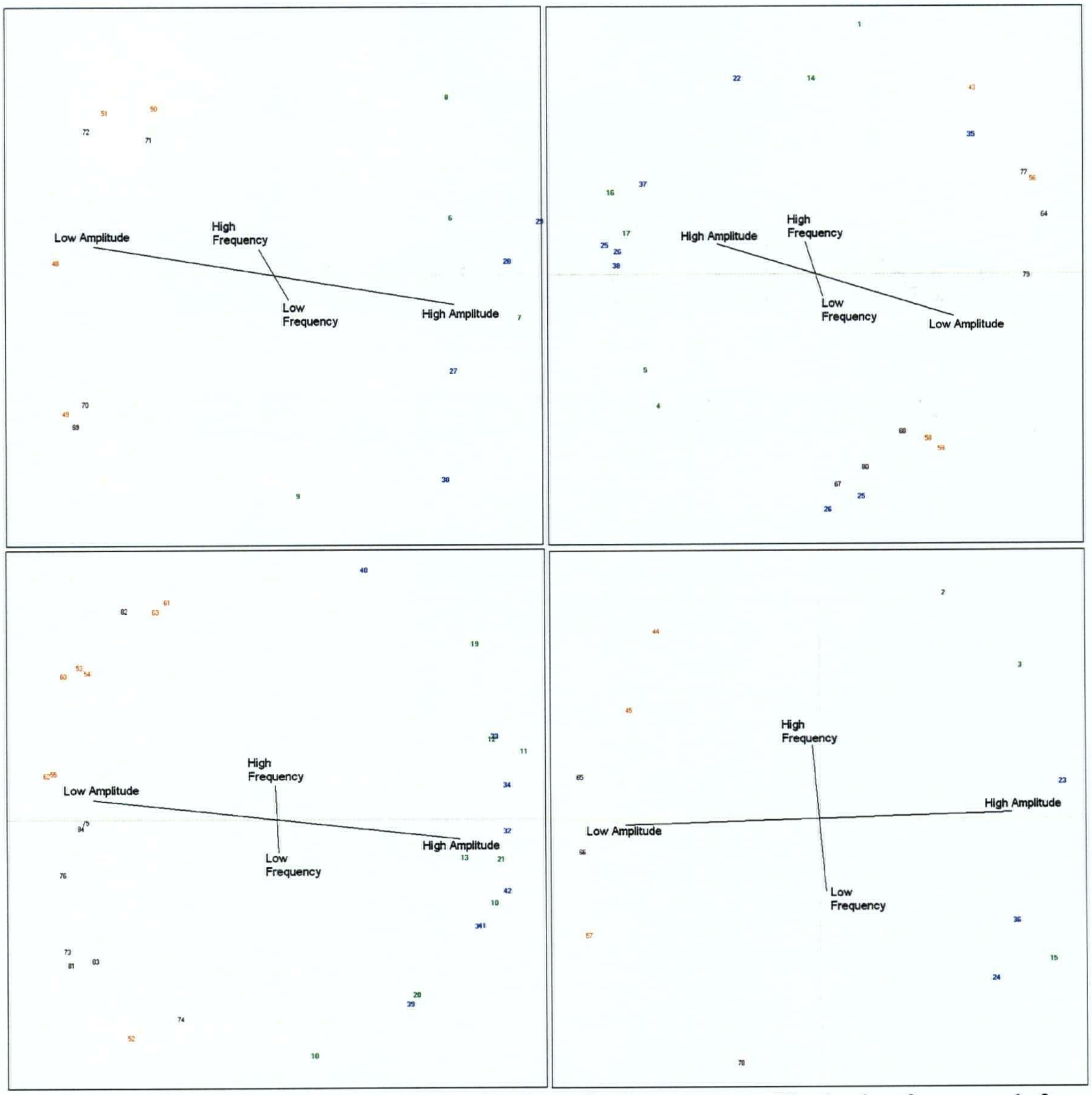

Figure 6.10 Individual MDS plots for the four rhythm groups. Clockwise from top left: long-even, short-even, short-uneven, long-uneven. Green and blue stimuli are high amplitude, grey and orange stimuli are low. Green and orange stimuli are high frequency, blue and grey stimuli are low frequency..

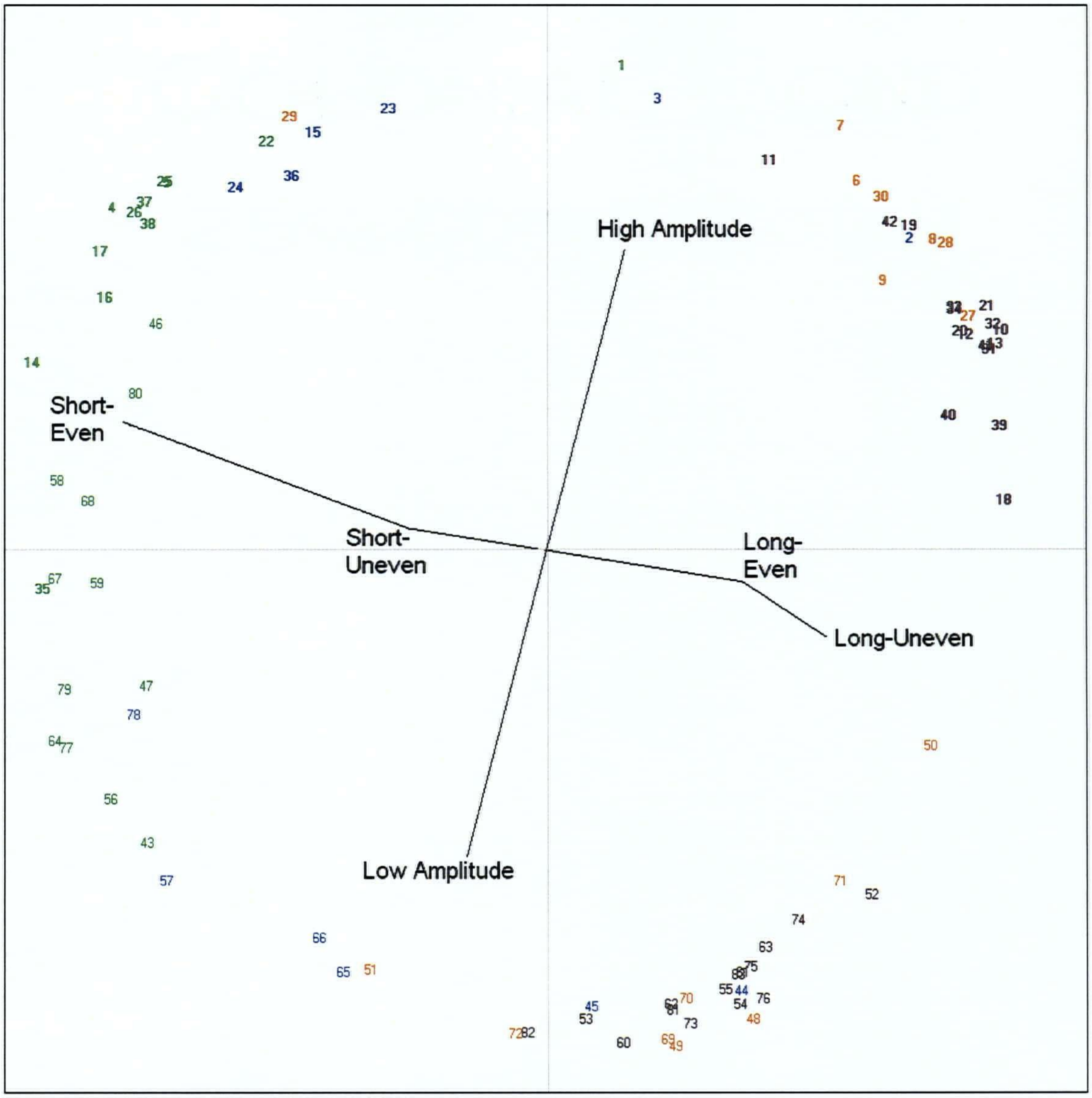

Figure 6.11. 2D MDS output with all stimuli. The green stimuli are from the short-even group, the blue are the short-uneven, the orange the long-even, and the grey are long-uneven. The middle axis is made up of the plotted centroids of the 4 groups, as labeled. We consider this to be the gold standard MDS map for our stimulus set.

Table 6.1. S-Stress and r2 values for individual groups 2D MDS output

Group	S-Stress	r^2
Long-even	.23096	.82260
Long-uneven	.27128	.73703
Short-even	.27397	.71022
Short-uneven	.22747	.79722

Table 7.1. Mapping of Subsets to Participants. Subsets are defined in Appendix C.

	Subset 1	Subset 2	Subset 3	Subset 4	Subset 5
Participants	1,6,11	2,7,12	3,8,13	4,9,14	5,10,15

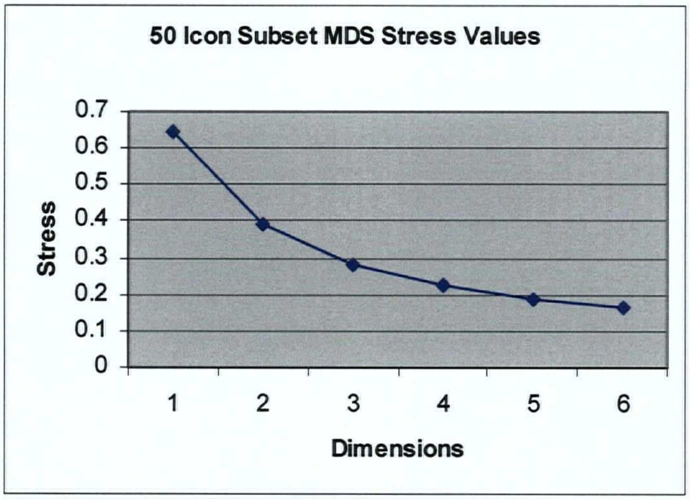

Figure 7.1. For first round of subset study, stress values for dimension 1 to 6 of the MDS solutions.

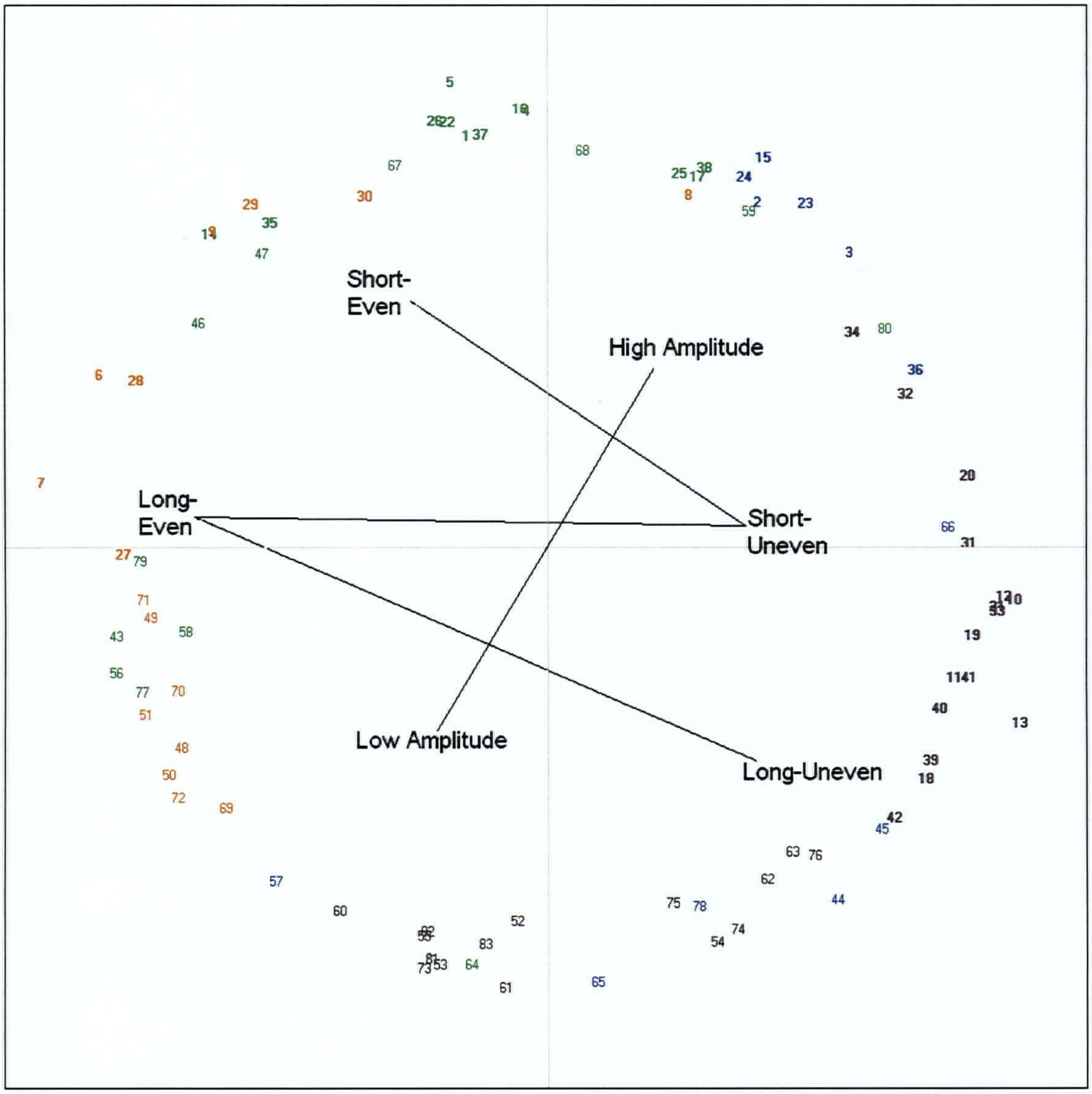

Figure 7.2. For first round of subset study, 2D MDS output map of results. Grouping is the same as in Chapter 6. Green is the short-even group, blue is short-uneven, orange is long-even, and grey is long-uneven. As can be noted, the orange long-even group is significantly out of place from the ordering in the gold standard, out on the far right of the map.

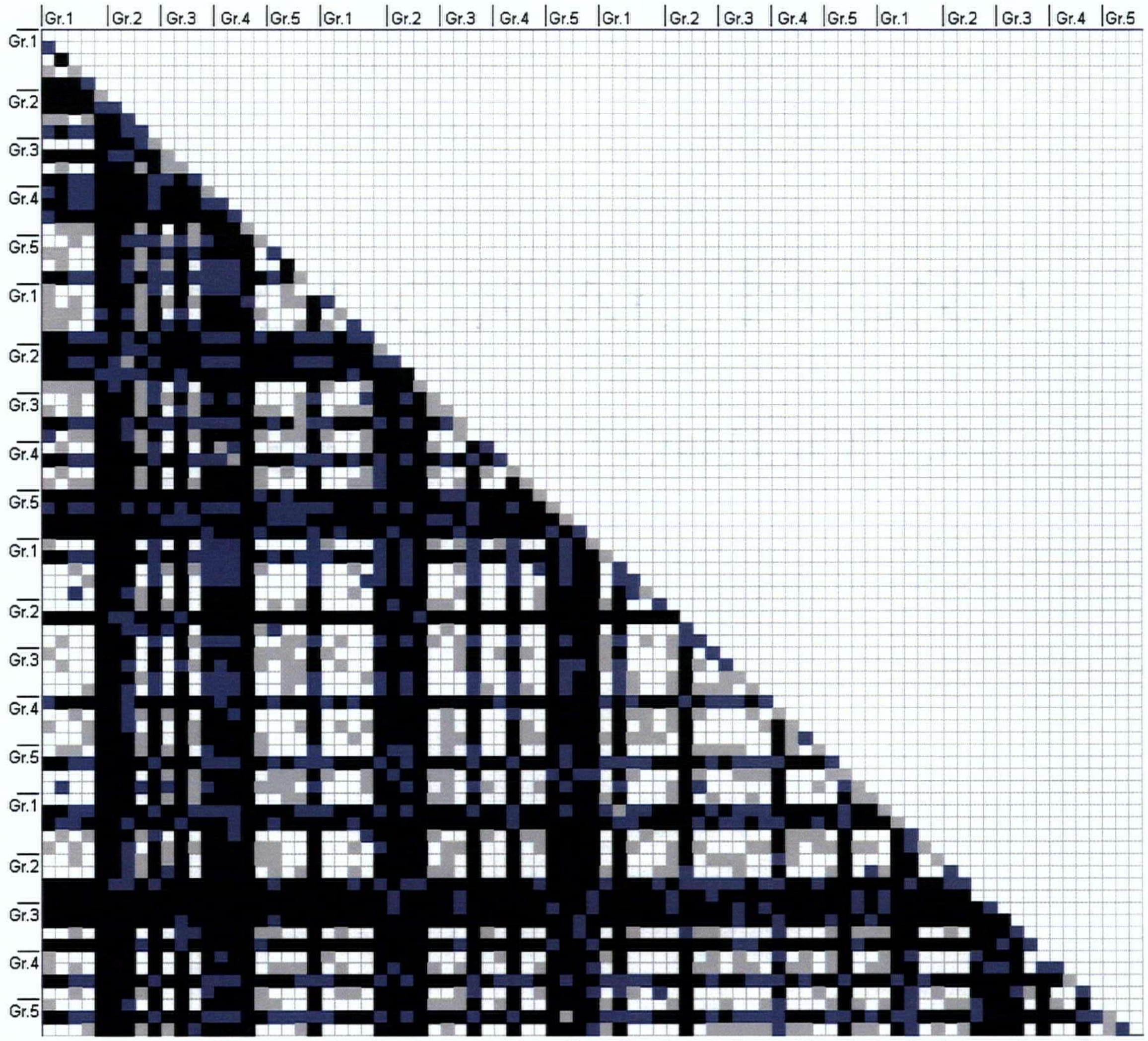

Figure 7.3. For first round of subset study, distribution of standard deviation values for averaged dissimilarity matrix. Black squares have the highest levels of standard deviation, blue-grey squares the next-highest, light-grey lower still, and white the lowest. Note the distinct stripes of darker (higher standard deviation) values, running along various columns and rows.

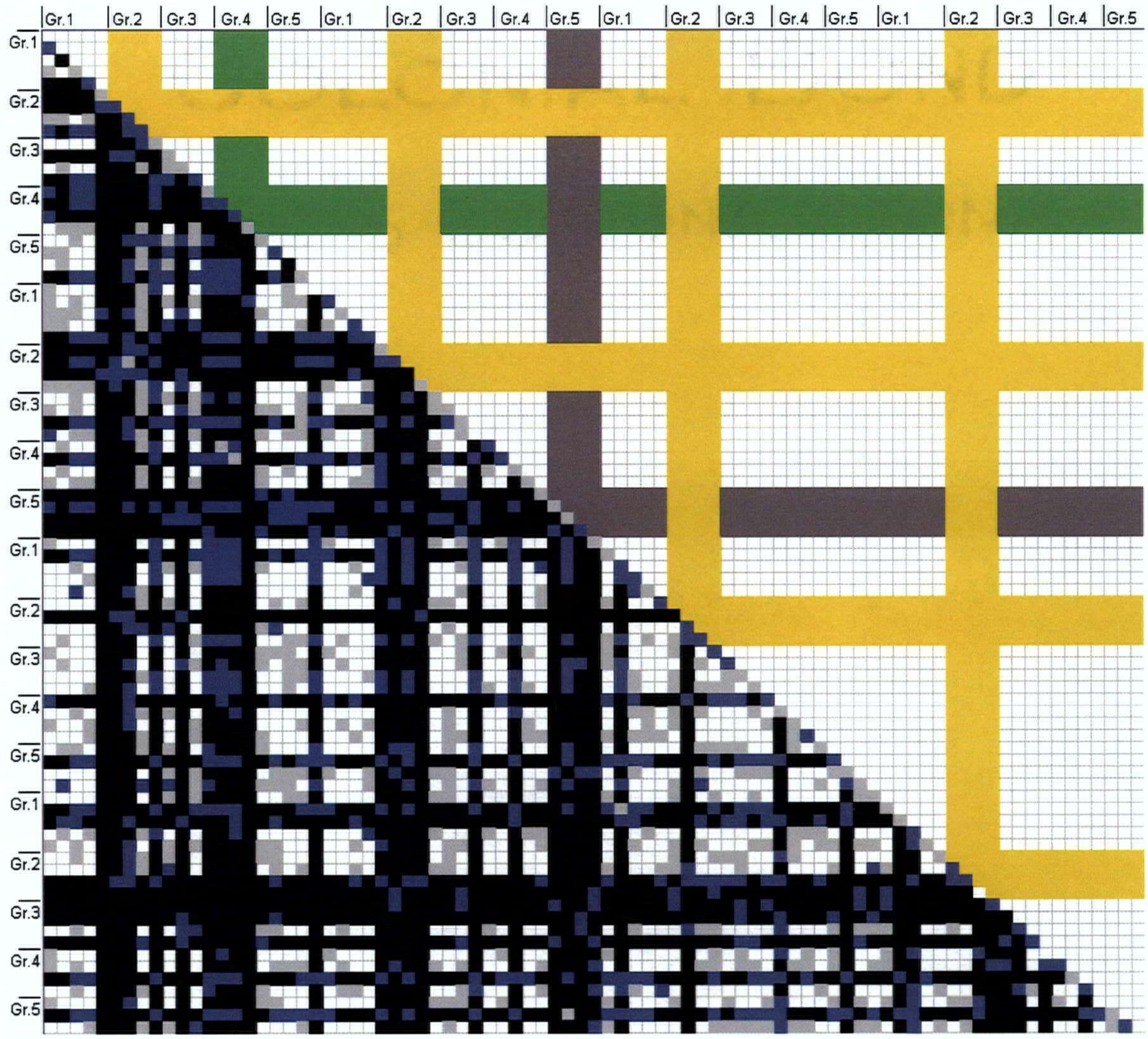

Figure 7.4. For first round of subset study, distribution of standard deviation values for averaged dissimilarity matrix with stimuli groups marked in opposite half of matrix. Stimulus groups are labeled along the side and top; note that each is spread across 4 places in the matrix, once for each combination of amplitude and frequency. The four orange columns/rows correspond to the "long-even" group. The green columns/rows correspond to stimuli 14-17, members of Group 4 in Table 3.2, played at high amplitude and high frequency. The grey columns/row correspond to stimuli 39-42, members of Group 5 in Table 3.2, played at high amplitude and low frequency.

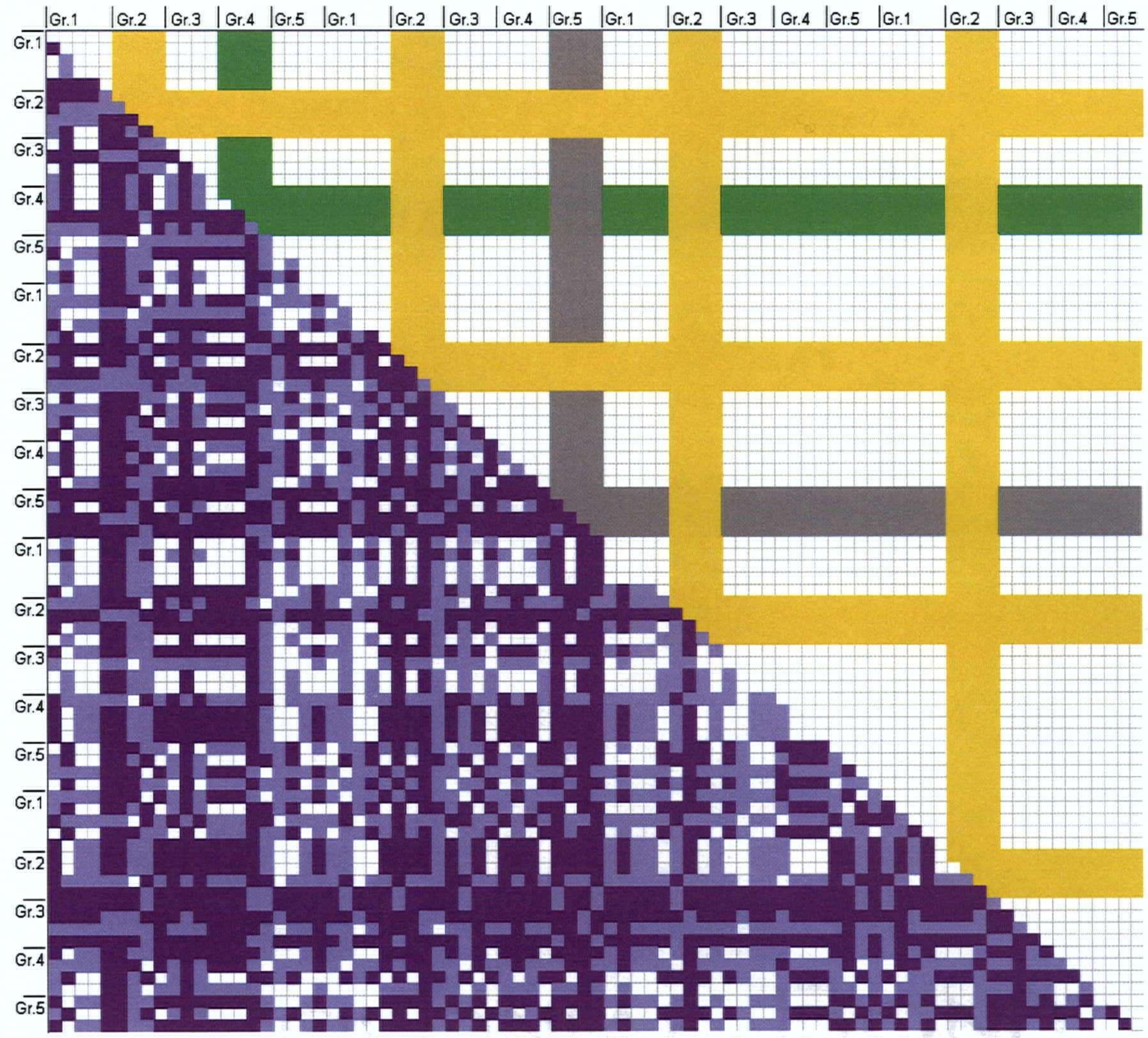

Figure 7.5. For first round of subset study, distribution of number of observation per value of averaged dissimilarity matrix with stimuli groups marked in opposite half of matrix. Stimulus groups are labeled along the side and top. The colour coding for the number of observations is that dark purple values have 3 observations, light purple have 6, white values have 9 or greater. Coding of stimuli groups is the same as in Figure 7.4

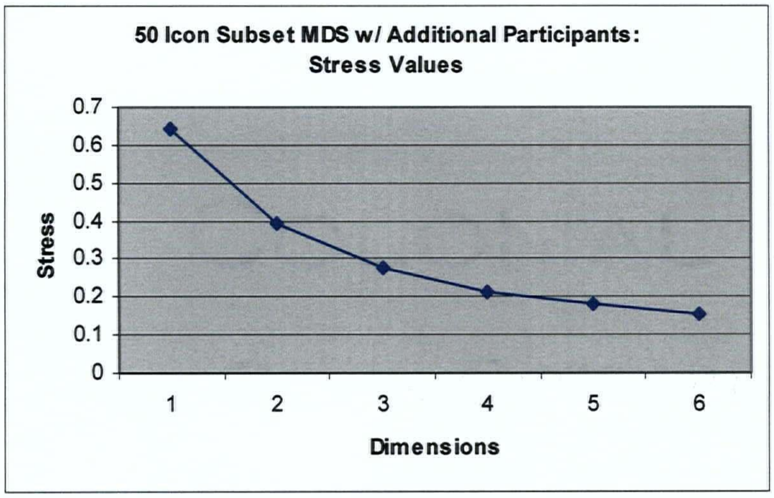

Figure7.6. Stress values for dimension 1 to 6 of the MDS solutions for the subset study with additional participants (second round of subset study).

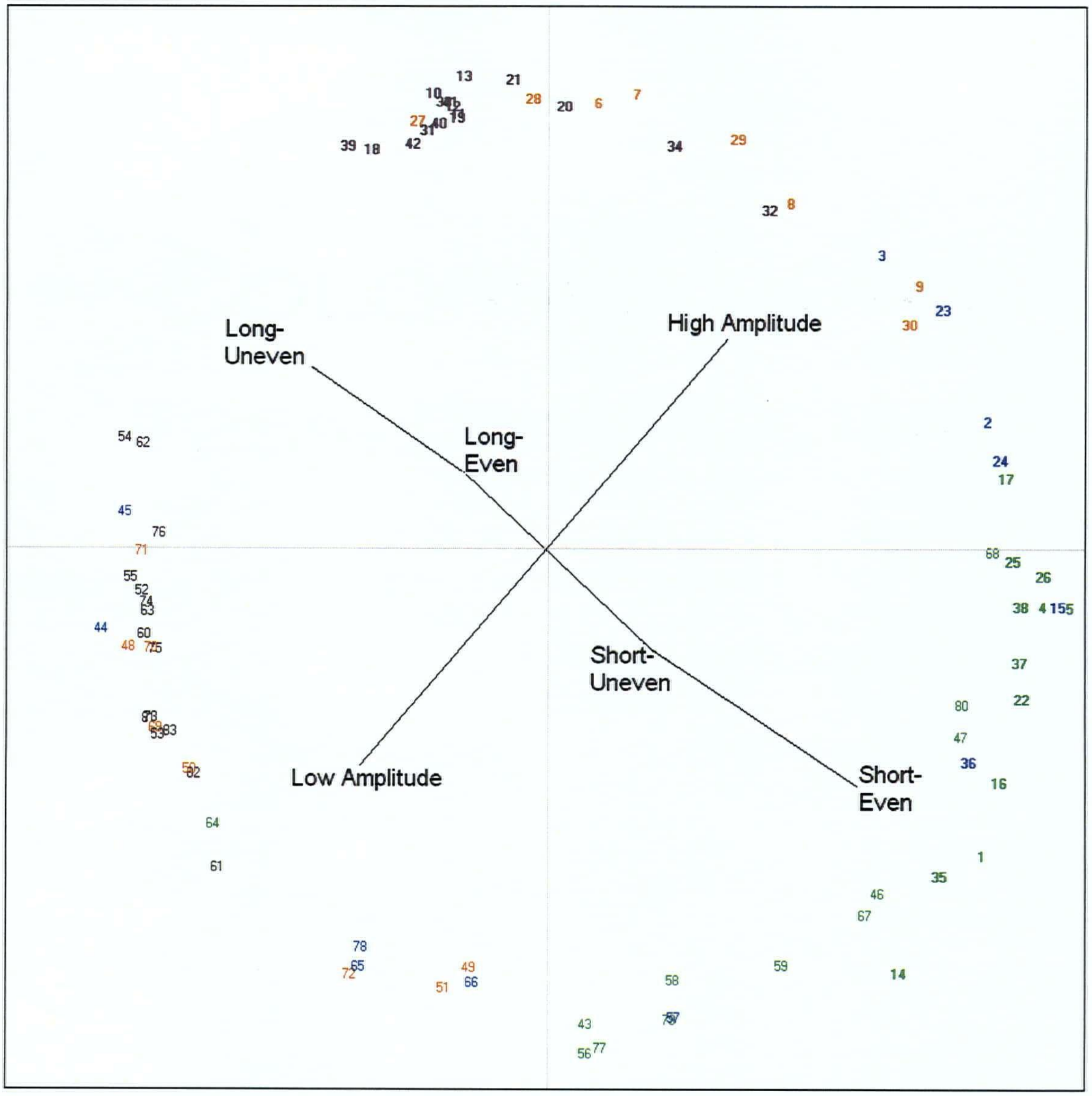

Figure 7.7. 2D MDS output map for subset study with additional subjects (second round of subset study). Green is the short-even group, blue is short-uneven, orange is long-even, and grey is long-uneven.

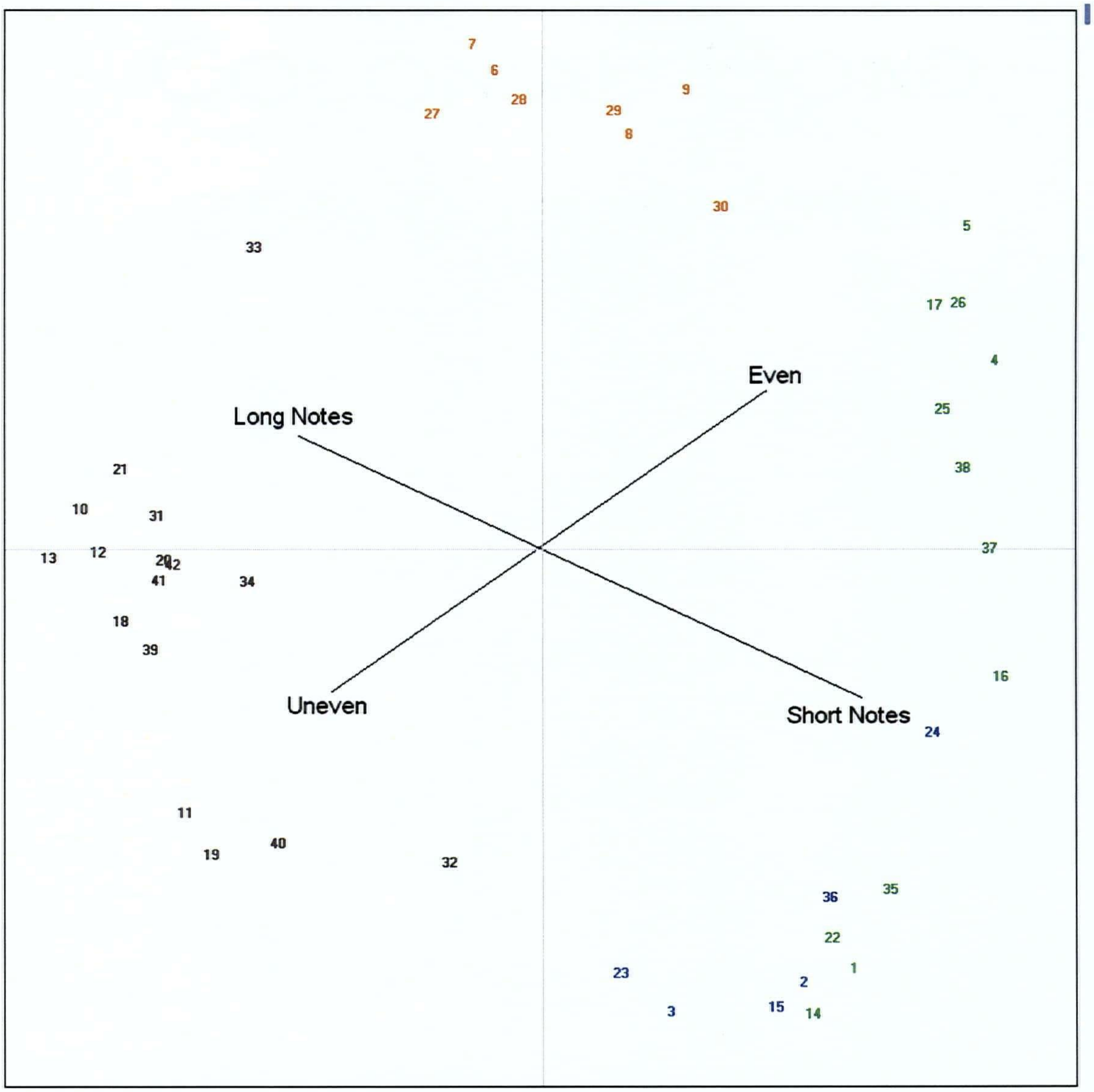

Figure 7.8. 2D MDS map of only high amplitude stimuli (second round of subset study). Green is the short-even group, blue is short-uneven, orange is long-even, and grey is long-uneven. Two perceptual axes are labeled accordingly.

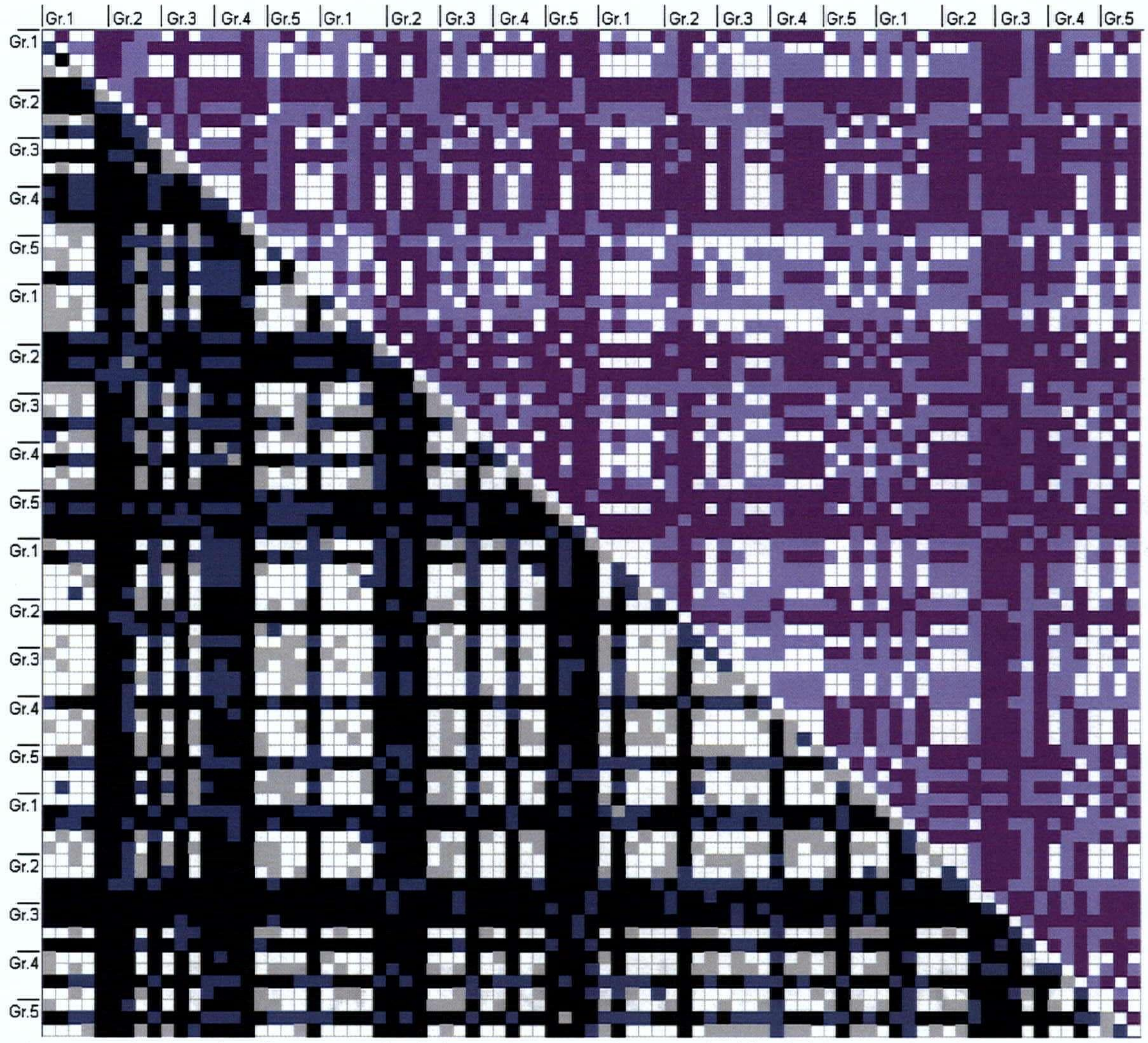

Figure 7.9. For the first round of the subset study: one half of this matrix shows standard deviation (bottom left triangle) while the other half shows the number of observations (top right triangle). Stimulus groups are labeled along the side and top. Color coding of SD: Black squares have the highest levels of standard deviation, blue-grey squares the next-highest, light-grey lower still, and white the lowest.. Colour coding of number of observations: dark purple values have 3 observations, light purple have 6, white values have 9 or greater. It can be seen how the (darker black) stripes of high SD correspond to the (darker purple) areas of low numbers of observations. There are however many areas of low observations that do not correspond to areas of high SD; thus we claim that both low numbers of observations plus more "difficult" stimuli are needed to create stripes of high SD.

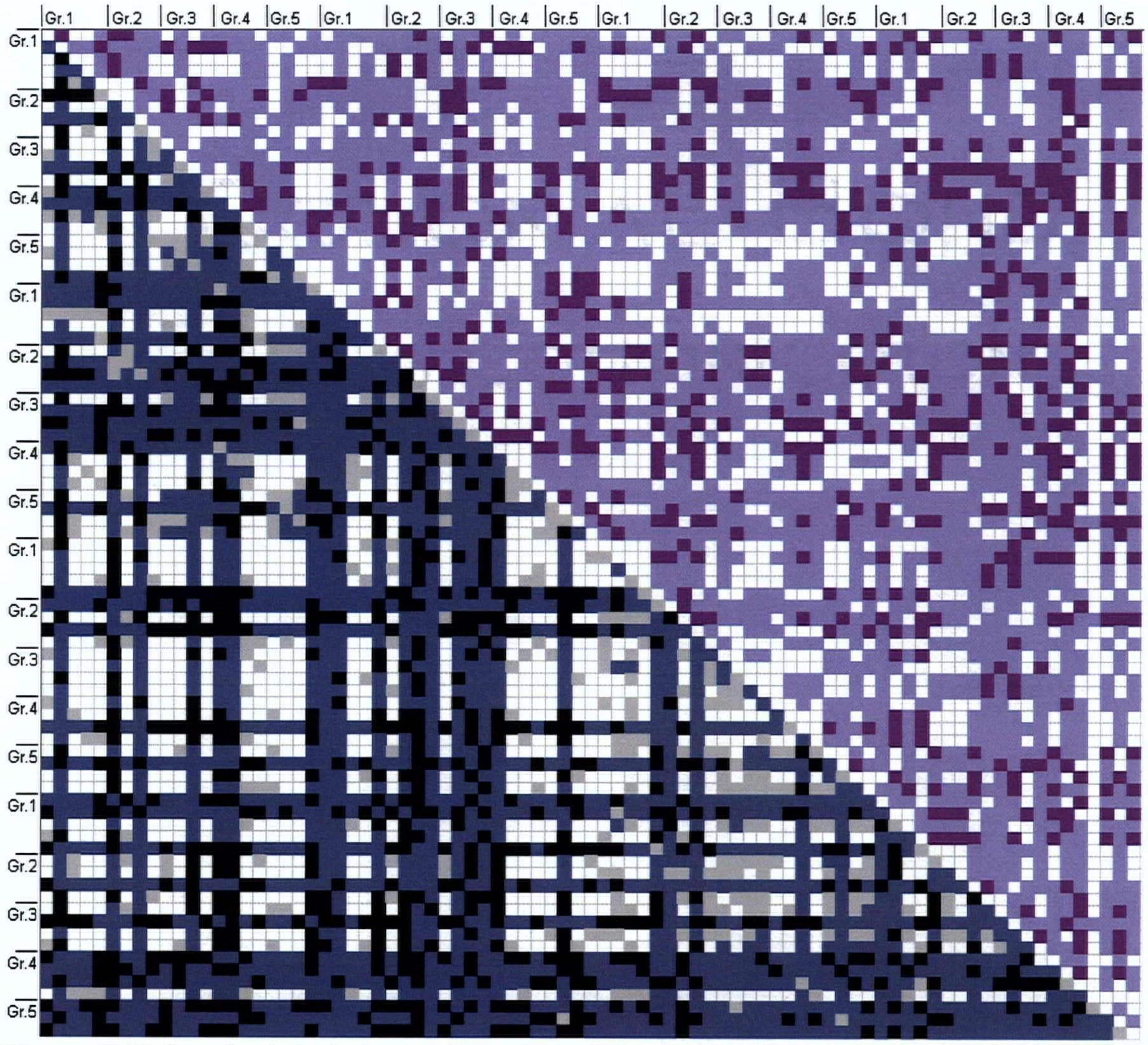

Figure 7.10 Standard deviation and number of observation for subset study with additional participants (second round of study). Format is same as in Figure 7.9. Stimulus groups are labeled along the side and top. Color coding of SD: Black squares have the highest levels of standard deviation, blue-grey squares the next-highest, light-grey lower still, and white the lowest. Colour coding of observation values differs slightly, dark purple values have 5 observations, light purple between 6 and 9 observations, white values greater than 9 observations. As can be seen, overall level of SD is much lower, and overall level of observations is much higher and more evenly spread

References

[1] Bijmolt, T.H.A, and Wedel, Michel. (1995). The effect of alternative methods of collecting similarity data for Multidimensional Scaling. *International Journal of Research in Marketing*, 12. pp 363-371.

[2] Bonebright, T.L. (2001). Perceptual Structure of Everyday Sounds: A Multidimensional Scaling Approach. *Proc. Of the 2001 International Conference on Auditory Display, Espoo, Finland*. pp. 73-78.

[3] Borg, I. and Groenen, P.J.F. (1997). *Modern multidimensional scaling*. 2nd edition. New York: Springer.

[4] Borg, I. and Leutner, D. (1985). Measuring the Similarity of MDS Configurations. *Multivariate Behavioral Research*, 20. pp 325-334.

[5] Brewster, S. and Brown, L.M (2004). Tactons: Structured Tactile Messages for Non-Visual Information Display. *In Proc of 5th Australasian User Interface Conference (AUIC2004), Dunedin.*

[6] Brown, L.M., Brewster, S.A., and Purchase, H.C. (2005). A First Investigation into the Effectiveness of Tactons. *First Joint Eurohaptics Conference and Symposium on Haptic Interfaces for Virtual Environments and Teleoperated Systems (WHC'05).*

[7] Brown, L.M., Brewster, S.A., and Purchase, H.C. (2006a). Multidimensional Tactons for Non-Visual Information Presentation in Mobile Devices. *MobileHCI'06.* pp 231-238.

[8] Brown, L.M., Brewster, S.A., and Purchase, H.C. (2006b). Tactile Crescendos and Sforzandos: Applying Musical Techniques to Tactile Icon Design. *CHI '06 extended abstracts on Human factors in computing systems,* Montreal, Canada. pp 610-615.

[9] Brown. L.M. and Kaaresoja, T. (2006). Feel Who's Talking: Using Tactons for Mobile Phone Alerts. *CHI '06 extended abstracts on Human factors in computing systems,* Montreal, Canada, pp 604-609.

[10] Chan, A. (2004). *Designing Haptic Icons to Support an Urgency-Based Turn-Taking Protocol.* M.Sc. Thesis. Univ. of British Columbia.

[11] Chan, A., MacLean, K., and McGrenere, J. (2005). Learning and identifying haptic icons under workload. *Haptic Interfaces for Virtual Environment and Teleoperator Systems, 2005. WHC 2005.* pp 432-439.

[12] Enriquez, M., MacLean, K. E., and Chita, C. (2006). Haptic Phonemes: Basic Building Blocks of Haptic Communication. *in Proc. of the 8th International Conference on Multimodal Interfaces, ICMI'06*, Banff, Alberta, Canada, November 2006.

[13] Grey, J. M. (1977). Multidimensional perceptual scaling of musical timbres *J. Acoust. Soc. Am.*, 61:5. pp. 1270-1277.

[14] Hoggan, E.E., and Brewster, S.A. (2006). Crossmodal Icons for Information Display. *CHI '06 extended abstracts on Human factors in computing systems,* Montreal, Canada, pp 857-862.

[15] Hollins, M, Faldowski, R., Rao, A., and Young, F. (1993). Perceptual dimensions of tactile surface texture: a multidimensional scaling analysis. *Perception & Psychophysics*, 54(6). Pp. 697-705.

[16] Klatzky, R.L., and Lederman, S.J. (1999). Tactile roughness perception with a rigid link interposed between skin and surface. *Perception & Psychophysics*, 61 (4). pp. 591-607.

[17] Laitinen, P., and Maenpaa, J. (2006). Enabling mobile haptic desing: Piezoelectric actuator technology properties in hand held devices. In proc of. *IEEE International Workshop on Haptic Audio Visual Environments and their Applications (HAVE2006).*

[18] Luk, J., Pasquero, J., Little, S., MacLean, K., Levesque, V., and Hayward, V. (2006). A Role for Haptics in Mobile Interaction: Initial Design Using a Handheld Tactile Display Prototype. *in Proc. of ACM Conference on Human Factors in Computing Systems (CHI '06),* Montreal, Canada. pp. 171-180

[19] MacLean, K., and Enriquez, M. (2003). Perceptual Design of Haptic Icons. *in Proc. of Eurohaptics, Dublin, Ireland.* pp. 351-363.

[20] Pasquero, J., Luk, J., Little, S., and MacLean, K. (2006). Perceptual Analysis of Haptic Icons: an Investigation into the Validity of Cluster Sorted MDS. *In Proc of Symposium on Haptic Interfaces for Virtual Environment and Teleoperator Systems (HAPTICS'06).* pp. 67-74.

[21] Shepard, R. N. (1962). The analysis of proximities: Multidimensional scaling with an unknown distance function. Part I. *Psychometrika, 27.* pp 125–140.

[22] Spence, I, and Domoney, D. W. (1974). Single Subject Incomplete Designs for Nonmetric Multidimensional Scaling. *Psychometrika*, 39 (4). pp. 469-490.

[23] Tan, H.Z., Durlach, N.I., Reed, C.M., and Rabinowitz, W.M. (1999). Information transmission with a multifinger tactual display. *Perception & Psychophysics*, 61 (6). pp. 993-1008.

[24] Tikka, V., and Laitinen, P. (2006). Designing Haptic Feedback for Touch Display: Experimental Study of Percieved Intensity and Integrations of Haptic and Audio. *HAID 2006.* pp 344-360.

[25] Tsogo, L., Masson, M.H., and Bardot, A. (2000). Multidimensional Scaling Methods for Many-Object Sets: A Review. *Multivariate Behavioural Research*, 35 (3). pp. 307-319.

[26] Van Erp, J.B.F (2002). Guidelines for the use of vibro-tactile displays in Human Computer Interaction. *In Proc. Of Eurohaptics 2002.* pp 18-22.

[27] Van Erp, J.B.F, and Spape, M.M.A., (2003). Distilling the Underlying Dimension of Tactile Melodies. *In Proc. Of Eurohaptics 2003.* pp 111-120.

Appendix A: Dissimilarity Matrices

Proceeding pages display the averaged dissimilarity matrix for the pilot study described in Section 4.4.

	1	2	3	4	5	6	7	8	9
1	0								
2	993	0							
3	1000	924	0						
4	1000	978	989	0					
5	872	966	972	908	0				
6	978	1000	1000	983	898	0			
7	1000	672	1000	1000	938	971	0		
8	1000	1000	1000	950	956	889	893	0	
9	973	907	966	871	933	956	950	994	0
10	1000	813	983	622	983	1000	700	951	891
11	978	770	862	674	1000	889	1000	963	848
12	973	938	800	987	989	907	1000	1000	800
13	989	938	926	806	989	1000	815	933	830
14	1000	978	1000	1000	989	1000	956	989	991
15	956	978	947	933	920	1000	1000	1000	920
16	1000	978	1000	972	1000	1000	978	989	965
17	900	933	944	939	858	1000	733	850	858
18	973	844	907	906	917	907	767	1000	1000
19	984	929	956	822	1000	1000	900	969	924
20	1000	973	944	840	833	983	989	947	811
21	971	939	844	973	983	960	956	922	822
22	965	991	830	991	956	898	972	922	960
23	937	989	785	711	956	889	878	1000	968
24	983	948	915	1000	1000	987	666	1000	911
25	870	830	867	956	922	938	585	839	982
26	967	967	889	967	739	839	889	916	933
27	969	924	972	972	807	938	806	911	961
28	989	983	1000	1000	871	933	1000	978	1000
29	839	1000	806	1000	960	1000	1000	972	963
30	900	883	844	978	822	939	883	772	878
31	948	939	630	969	920	939	989	989	944
32	898	987	778	1000	822	928	978	933	880
33	889	889	711	1000	956	828	1000	1000	889
34	937	938	978	933	922	791	889	1000	907
35	952	991	917	974	948	978	928	951	886
36	863	917	963	983	960	960	1000	973	970
37	944	870	926	1000	830	871	889	907	898
38	1000	889	978	1000	704	916	1000	1000	684
39	922	1000	885	1000	839	809	1000	719	917
40	929	983	856	1000	833	889	1000	1000	896
41	858	924	722	1000	807	844	983	1000	861
42	991	989	983	956	978	983	956	933	1000
43	911	978	794	987	972	862	878	811	982
44	889	961	1000	978	972	956	1000	939	972
45	893	989	793	830	1000	900	889	883	937
46	978	970	789	983	960	950	844	987	857
47	1000	938	922	929	836	972	939	951	906
48	985	941	839	950	702	920	917	878	928
49	943	950	1000	1000	967	950	964	982	993
50	904	983	834	956	956	896	889	1000	936
51	978	1000	896	750	973	785	867	989	972

	1	2	3	4	5	6	7	8	9
52	852	1000	858	1000	770	916	978	874	941
53	941	756	822	983	955	950	1000	960	990
54	858	938	817	991	916	978	985	987	920
55	858	911	978	989	1000	987	926	922	1000
56	908	960	1000	991	858	839	907	804	859
57	926	867	983	1000	867	793	815	893	864
58	858	950	822	987	966	966	818	900	889
59	833	989	836	850	917	739	700	961	992
60	987	983	933	922	836	916	1000	906	972
61	853	858	933	911	939	956	911	920	1000
62	983	966	947	896	963	853	1000	834	828
63	800	978	941	972	956	884	1000	1000	902
64	926	874	1000	989	1000	1000	985	985	991
65	782	933	1000	911	858	839	878	933	900
66	978	941	1000	1000	871	933	722	859	844
67	987	1000	872	816	900	956	696	939	852
68	889	924	983	1000	939	911	920	878	881
69	933	748	964	987	987	1000	1000	867	871
70	960	950	822	983	1000	867	766	839	809
71	963	972	956	963	983	1000	889	956	938
72	1000	1000	972	963	944	972	1000	1000	987
73	1000	880	963	1000	978	951	1000	933	896
74	989	950	972	947	978	852	900	960	992
75	939	848	978	978	964	939	867	898	1000
76	889	880	966	942	952	983	944	944	978
77	876	950	904	1000	822	645	666	818	854
78	993	906	948	978	1000	987	929	898	924
79	889	920	906	1000	989	889	1000	893	972
80	933	900	928	964	772	896	963	951	939

	10	11	12	13	14	15	16	17	18
1									
2									
3									
4									
5									
6									
7									
8									
9									
10	0								
11	978	0							
12	973	958	0						
13	807	816	827	0					
14	1000	867	964	943	0				
15	845	933	983	944	962	0			
16	1000	1000	1000	1000	1000	986	0		
17	889	933	901	1000	1000	970	963	0	
18	972	1000	950	978	1000	1000	828	848	0
19	984	1000	989	933	880	1000	858	922	948
20	948	989	825	917	978	938	959	983	918
21	987	959	946	916	983	965	981	1000	989
22	883	959	924	928	963	892	908	818	884
23	922	916	1000	906	896	959	982	917	943
24	1000	889	900	889	967	872	805	911	1000
25	989	989	1000	989	1000	885	948	867	937
26	937	956	933	950	951	835	989	805	889
27	963	856	867	989	1000	795	985	885	800
28	1000	1000	952	956	960	978	943	947	889
29	1000	938	1000	789	973	682	889	911	896
30	1000	920	967	867	1000	874	882	833	966
31	1000	944	1000	862	983	911	963	889	982
32	983	768	922	933	941	866	1000	933	853
33	938	990	893	1000	964	870	982	841	922
34	941	917	966	831	983	947	880	920	907
35	907	955	989	972	983	873	974	784	936
36	769	863	956	964	987	837	1000	951	966
37	856	949	961	952	983	973	952	1000	819
38	987	785	889	1000	1000	966	956	742	928
39	950	896	859	874	918	837	984	849	879
40	978	987	889	941	1000	756	1000	904	889
41	1000	939	815	1000	1000	856	785	911	1000
42	956	978	922	928	811	813	1000	1000	858
43	978	937	905	978	884	905	907	871	1000
44	1000	882	989	1000	1000	917	929	1000	966
45	822	733	863	987	815	983	989	983	922
46	711	901	924	987	983	871	989	845	968
47	811	987	983	1000	822	922	911	972	972
48	983	1000	1000	933	1000	966	947	950	983
49	1000	978	970	858	1000	908	938	956	989
50	971	1000	1000	987	982	870	958	661	992
51	1000	920	816	987	983	878	751	789	972

	10	11	12	13	14	15	16	17	18
52	930	837	796	1000	972	967	941	871	1000
53	987	973	830	933	889	900	989	707	981
54	983	933	876	939	983	756	991	773	867
55	1000	1000	883	938	950	916	978	917	991
56	889	949	898	892	900	944	918	933	830
57	852	989	949	950	1000	841	968	828	911
58	978	1000	939	947	894	839	896	889	983
59	967	835	896	961	889	904	911	922	961
60	1000	987	950	1000	960	966	800	960	983
61	1000	1000	849	898	966	917	933	1000	839
62	862	956	867	966	978	800	806	809	928
63	944	883	828	918	952	966	978	844	933
64	1000	848	1000	985	1000	924	991	867	862
65	987	956	989	1000	1000	853	918	834	944
66	983	947	772	767	904	844	889	956	833
67	944	856	738	867	1000	806	792	805	978
68	792	1000	840	989	978	978	1000	817	919
69	987	884	966	1000	981	694	960	938	924
70	956	972	911	850	987	898	1000	871	1000
71	978	889	844	939	956	793	983	815	924
72	982	1000	875	1000	889	989	924	950	978
73	1000	938	889	906	963	1000	922	831	978
74	990	974	952	973	896	900	975	834	897
75	1000	904	856	789	950	978	933	1000	989
76	973	892	975	970	983	875	968	889	878
77	889	947	946	944	966	841	989	728	733
78	955	870	965	876	861	884	1000	956	893
79	794	1000	804	920	818	811	972	827	983
80	1000	882	874	960	872	993	889	917	994

	19	20	21	22	23	24	25	26	27
1									
2									
3									
4									
5									
6									
7									
8									
9									
10									
11									
12									
13									
14									
15									
16									
17									
18									
19	0								
20	980	0							
21	878	928	0						
22	917	939	978	0					
23	882	964	813	958	0				
24	1000	922	904	933	825	0			
25	937	970	915	962	987	951	0		
26	933	967	944	944	880	933	938	0	
27	813	989	960	858	967	933	950	911	0
28	1000	1000	943	907	871	856	833	928	909
29	972	1000	1000	933	876	933	883	1000	978
30	939	933	970	878	934	961	867	861	928
31	1000	993	900	822	883	886	926	917	929
32	893	881	921	991	1000	908	950	1000	900
33	893	918	848	918	917	861	967	844	889
34	1000	978	827	964	911	960	933	907	1000
35	981	965	848	1000	883	1000	978	951	885
36	769	915	989	933	794	943	971	907	949
37	896	856	966	937	1000	924	943	900	924
38	1000	884	1000	956	793	876	717	767	898
39	915	978	858	870	933	933	990	956	952
40	1000	889	933	867	856	839	836	907	973
41	973	939	907	951	889	924	983	965	906
42	1000	942	939	889	876	872	907	964	872
43	918	903	895	987	973	930	924	1000	922
44	971	978	975	867	1000	915	960	806	787
45	774	835	966	1000	978	1000	900	1000	956
46	787	987	966	1000	856	933	981	809	852
47	993	1000	906	1000	906	769	1000	991	704
48	1000	989	987	911	972	711	911	893	911
49	946	989	896	922	806	832	841	849	738
50	911	1000	911	933	884	901	863	963	911
51	989	978	906	1000	889	922	926	882	1000

	19	20	21	22	23	24	25	26	27
52	867	879	899	948	960	879	973	893	914
53	1000	975	873	956	883	851	900	1000	989
54	1000	972	1000	1000	784	1000	987	1000	1000
55	874	982	951	966	872	1000	844	951	892
56	940	940	914	1000	1000	1000	959	783	978
57	908	902	911	978	839	978	882	989	944
58	925	917	845	972	811	862	907	942	852
59	765	737	871	943	941	927	830	889	911
60	985	920	938	972	983	800	983	961	782
61	772	991	876	989	966	726	811	1000	950
62	1000	844	1000	983	989	831	1000	972	858
63	1000	844	963	767	981	836	875	944	1000
64	993	898	989	922	678	942	858	893	951
65	937	981	959	939	1000	919	902	989	876
66	1000	756	852	844	840	872	961	1000	933
67	920	947	1000	1000	742	893	987	956	787
68	908	978	944	933	822	917	1000	956	944
69	815	1000	1000	978	918	987	987	956	929
70	852	1000	966	966	938	900	853	871	841
71	972	1000	966	966	973	867	744	883	983
72	1000	896	973	929	898	907	833	963	956
73	1000	922	907	1000	978	674	726	983	973
74	941	982	956	982	987	822	944	874	989
75	922	844	827	933	889	1000	933	1000	883
76	990	854	841	805	978	982	952	1000	941
77	1000	733	987	941	917	867	911	778	956
78	930	914	841	987	950	952	956	1000	848
79	938	763	817	1000	956	889	1000	889	983
80	959	1000	915	1000	983	867	818	885	870

	28	29	30	31	32	33	34	35	36
1									
2									
3									
4									
5									
6									
7									
8									
9									
10									
11									
12									
13									
14									
15									
16									
17									
18									
19									
20									
21									
22									
23									
24									
25									
26									
27									
28	0								
29	917	0							
30	856	926	0						
31	836	811	906	0					
32	1000	898	881	931	0				
33	960	920	815	956	891	0			
34	900	900	978	939	902	870	0		
35	933	933	867	911	744	839	995	0	
36	885	933	978	1000	978	987	944	783	0
37	983	778	978	1000	898	960	978	943	921
38	898	834	830	844	884	983	778	933	963
39	818	944	878	916	926	922	863	933	978
40	951	938	939	933	1000	794	867	867	833
41	960	872	822	800	925	951	844	920	822
42	756	911	849	778	763	972	926	983	828
43	778	1000	989	933	1000	916	825	981	896
44	983	987	933	941	918	1000	915	933	938
45	978	955	987	900	969	933	1000	944	992
46	983	955	933	978	933	966	952	1000	925
47	911	989	1000	1000	911	1000	944	900	966
48	956	939	907	978	961	989	950	1000	960
49	862	922	861	902	882	950	816	902	874
50	906	896	898	859	844	944	885	955	885
51	852	956	900	987	920	922	933	1000	783

	28	29	30	31	32	33	34	35	36
52	893	861	882	930	946	950	778	833	973
53	900	950	874	898	737	889	848	895	878
54	778	906	933	939	662	1000	853	785	844
55	789	956	896	839	867	922	929	791	818
56	1000	951	943	896	870	965	908	943	943
57	978	833	937	956	893	896	894	964	989
58	725	889	852	800	756	1000	853	964	966
59	906	978	902	978	989	883	896	987	896
60	987	893	972	920	1000	885	874	987	839
61	1000	956	989	939	963	867	817	983	987
62	920	900	978	1000	939	1000	978	933	1000
63	702	900	966	944	917	817	828	983	983
64	893	989	944	978	911	917	867	852	944
65	1000	862	893	956	943	867	959	989	937
66	922	963	941	963	872	1000	793	922	983
67	952	884	831	1000	806	917	900	871	911
68	1000	916	978	989	906	806	898	889	844
69	947	933	872	978	947	933	956	738	1000
70	800	867	955	748	933	987	840	944	933
71	839	756	839	966	1000	941	844	983	973
72	800	1000	933	867	911	874	729	916	966
73	856	922	807	898	867	966	815	950	893
74	900	1000	911	874	822	989	800	900	978
75	950	898	883	951	911	871	898	1000	920
76	822	947	905	867	870	872	904	948	965
77	1000	1000	941	878	836	967	944	944	895
78	938	849	883	896	852	794	993	989	989
79	1000	950	960	973	960	963	966	880	840
80	978	904	989	848	937	983	975	911	819

	37	38	39	40	41	42	43	44	45
1									
2									
3									
4									
5									
6									
7									
8									
9									
10									
11									
12									
13									
14									
15									
16									
17									
18									
19									
20									
21									
22									
23									
24									
25									
26									
27									
28									
29									
30									
31									
32									
33									
34									
35									
36									
37	0								
38	896	0							
39	882	933	0						
40	917	956	990	0					
41	784	983	836	832	0				
42	883	747	800	871	863	0			
43	871	946	856	944	876	941	0		
44	900	960	917	900	989	960	983	0	
45	678	956	761	900	978	942	933	983	0
46	741	1000	884	898	917	900	1000	875	965
47	896	950	878	938	933	1000	844	856	911
48	852	983	1000	902	878	1000	871	973	840
49	889	900	844	831	885	982	968	973	970
50	872	838	1000	874	889	848	920	1000	885
51	1000	963	933	1000	966	900	989	1000	1000

	37	38	39	40	41	42	43	44	45
52	871	908	800	759	938	968	963	867	785
53	872	867	973	734	813	870	947	956	922
54	889	1000	902	878	917	883	989	883	898
55	883	956	700	836	773	796	919	828	1000
56	1000	973	900	906	987	989	984	1000	989
57	815	989	938	872	911	1000	911	952	898
58	748	917	928	947	826	933	822	1000	972
59	920	950	1000	811	835	904	893	942	963
60	950	894	1000	1000	963	900	917	719	874
61	711	1000	966	722	789	983	978	911	916
62	900	967	950	950	872	987	1000	966	1000
63	989	959	1000	861	795	862	1000	904	966
64	889	793	889	778	1000	1000	970	983	911
65	815	989	983	917	1000	915	851	989	989
66	794	1000	963	944	978	906	978	1000	978
67	720	987	767	911	885	989	1000	978	689
68	822	1000	920	889	898	900	941	966	800
69	964	867	889	1000	933	938	904	978	822
70	800	773	862	827	805	952	1000	889	911
71	883	947	807	844	720	983	966	889	956
72	867	810	833	600	983	933	917	904	806
73	785	941	862	800	600	928	1000	880	938
74	983	797	963	770	872	956	969	1000	826
75	1000	978	850	991	683	933	991	973	813
76	911	907	950	896	972	959	971	989	974
77	859	989	917	1000	861	989	966	989	867
78	874	930	894	972	947	874	903	1000	1000
79	1000	906	978	867	978	872	939	880	970
80	785	841	950	885	951	978	867	813	936

	46	47	48	49	50	51	52	53	54
1									
2									
3									
4									
5									
6									
7									
8									
9									
10									
11									
12									
13									
14									
15									
16									
17									
18									
19									
20									
21									
22									
23									
24									
25									
26									
27									
28									
29									
30									
31									
32									
33									
34									
35									
36									
37									
38									
39									
40									
41									
42									
43									
44									
45									
46	0								
47	940	0							
48	956	975	0						
49	876	956	958	0					
50	840	850	800	854	0				
51	907	800	963	1000	859	0			

	46	47	48	49	50	51	52	53	54
52	969	973	941	841	985	849	0		
53	933	985	933	826	755	796	891	0	
54	978	991	922	836	920	989	747	937	0
55	926	939	906	794	985	911	973	978	949
56	1000	933	994	982	917	925	889	783	880
57	765	960	956	857	950	1000	926	885	906
58	978	939	994	1000	985	845	893	822	956
59	960	956	982	842	989	889	933	1000	733
60	828	922	1000	871	951	1000	840	956	871
61	933	922	922	726	755	815	871	817	680
62	1000	889	933	900	1000	898	928	933	900
63	850	1000	983	884	978	800	983	983	844
64	902	922	982	915	911	911	970	867	1000
65	698	818	970	876	898	851	773	989	938
66	1000	926	896	867	767	963	785	1000	811
67	972	822	844	852	1000	1000	707	917	917
68	938	959	960	800	911	1000	806	960	956
69	956	963	889	956	956	972	983	972	906
70	1000	918	922	822	800	840	662	889	952
71	956	744	911	933	966	844	955	1000	911
72	944	1000	907	614	1000	830	863	867	555
73	956	1000	911	785	917	667	933	850	866
74	902	939	944	674	973	819	870	911	872
75	951	965	933	883	716	839	989	628	1000
76	822	791	851	908	978	907	896	845	856
77	911	844	840	941	792	911	908	849	1000
78	987	849	970	918	972	963	937	834	933
79	970	973	939	902	853	972	1000	880	955
80	836	858	848	943	972	946	944	827	924

	55	56	57	58	59	60	61	62	63
1									
2									
3									
4									
5									
6									
7									
8									
9									
10									
11									
12									
13									
14									
15									
16									
17									
18									
19									
20									
21									
22									
23									
24									
25									
26									
27									
28									
29									
30									
31									
32									
33									
34									
35									
36									
37									
38									
39									
40									
41									
42									
43									
44									
45									
46									
47									
48									
49									
50									
51									

	55	56	57	58	59	60	61	62	63
52									
53									
54									
55	0								
56	953	0							
57	900	938	0						
58	911	783	898	0					
59	987	876	972	909	0				
60	845	1000	983	889	952	0			
61	951	900	939	978	850	870	0		
62	1000	1000	1000	920	983	966	967	0	
63	1000	1000	933	915	866	966	1000	975	0
64	969	978	772	922	951	1000	950	922	892
65	924	863	956	882	916	711	785	922	915
66	933	889	1000	783	862	966	1000	964	1000
67	983	811	853	1000	967	966	960	987	778
68	956	951	867	844	928	839	963	861	1000
69	878	972	861	898	867	966	947	900	922
70	933	889	967	816	916	800	978	738	852
71	955	1000	939	947	844	978	983	884	1000
72	966	987	756	796	1000	689	907	640	978
73	844	983	911	756	950	828	978	756	963
74	878	962	900	847	884	874	1000	978	830
75	956	922	1000	948	858	989	1000	1000	1000
76	936	844	796	882	787	964	1000	763	867
77	900	989	956	870	933	966	963	889	916
78	956	982	915	896	956	989	978	816	896
79	822	818	983	889	950	933	1000	822	922
80	956	969	944	946	896	850	911	983	991

	64	65	66	67	68	69	70	71	72
1									
2									
3									
4									
5									
6									
7									
8									
9									
10									
11									
12									
13									
14									
15									
16									
17									
18									
19									
20									
21									
22									
23									
24									
25									
26									
27									
28									
29									
30									
31									
32									
33									
34									
35									
36									
37									
38									
39									
40									
41									
42									
43									
44									
45									
46									
47									
48									
49									
50									
51									

	64	65	66	67	68	69	70	71	72
52									
53									
54									
55									
56									
57									
58									
59									
60									
61									
62									
63									
64	0								
65	884	0							
66	867	918	0						
67	889	1000	879	0					
68	1000	1000	867	994	0				
69	960	978	944	852	886	0			
70	933	933	960	807	973	854	0		
71	978	896	884	904	983	835	959	0	
72	806	941	898	978	956	634	674	921	0
73	689	963	839	883	1000	706	918	722	692
74	810	782	1000	807	952	900	939	729	852
75	1000	789	1000	991	972	872	941	333	983
76	940	944	939	947	966	844	785	739	839
77	1000	978	933	889	983	983	983	973	978
78	971	983	1000	1000	966	867	755	911	793
79	880	1000	889	878	978	1000	983	944	1000
80	787	941	911	844	883	813	987	811	872

	73	74	75	76	77	78	79	80
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								
16								
17								
18								
19								
20								
21								
22								
23								
24								
25								
26								
27								
28								
29								
30								
31								
32								
33								
34								
35								
36								
37								
38								
39								
40								
41								
42								
43								
44								
45								
46								
47								
48								
49								
50								
51								

	73	74	75	76	77	78	79	80
52								
53								
54								
55								
56								
57								
58								
59								
60								
61								
62								
63								
64								
65								
66								
67								
68								
69								
70								
71								
72								
73	0							
74	899	0						
75	906	994	0					
76	889	920	973	0				
77	963	1000	963	875	0			
78	911	987	886	987	943	0		
79	920	907	885	983	956	936	0	
80	940	1000	978	856	960	920	817	0

Proceeding pages display the averaged dissimilarity matrix for the full-set (gold standard) study described in Chapter 6.

	1	2	3	4	5	6	7	8
1	0							
2	745	0						
3	896	591.57143	0					
4	957.28571	985.85714	849	0				
5	957.28571	843.28571	805.71429	554.42857	0			
6	905.57143	905.57143	743.42857	791	838.57143	0		
7	905.57143	905.57143	971.71429	791	881.85714	449.57143	0	
8	905.57143	648.71429	943.14286	743.57143	725.42857	597.42857	530.57143	0
9	891.14286	738.85714	805.14286	853.57143	957.28571	597.14286	763	834.85714
10	877.28571	792.14286	725.14286	952.57143	1000	706.42857	834	773
11	649	773.28571	929	1000	819.28571	919.71429	919.71429	891.14286
12	863.14286	435.71429	734.14286	985.85714	843.28571	905.57143	905.57143	734.42857
13	863.14286	497	672.85714	938.42857	843.28571	834.85714	819.85714	616
14	839.42857	985.85714	905.57143	791.28571	696.14286	985.85714	985.85714	985.85714
15	914.85714	772.85714	768	877	696.14286	985.85714	900.14286	985.85714
16	829.42857	886.57143	820.42857	650.14286	796.28571	971.71429	971.71429	943.14286
17	896	985.85714	849	654.57143	473.71429	971.71429	971.71429	971.71429
18	668.14286	806.28571	828.85714	1000	957.57143	777.14286	877.28571	848.71429
19	877.28571	558.57143	363.57143	952.57143	914.28571	834	919.71429	843.71429
20	877.28571	611.28571	662.57143	938.71429	914.28571	834	919.71429	891.14286
21	649	601.85714	607	1000	857.42857	919.71429	834	748.57143
22	588.28571	834.85714	943.42857	914.85714	729.14286	858.14286	715.28571	905.57143
23	872.71429	687.71429	354.42857	891.42857	577.42857	695.71429	924.28571	943.14286
24	985.85714	848	678.14286	792.42857	654	985.85714	985.85714	957.28571
25	909.85714	852.71429	801.57143	621.57143	586.85714	971.71429	971.71429	886
26	957.28571	900.14286	849	654.57143	365	924.28571	924.28571	824.71429
27	919.71429	610.71429	805.14286	985.85714	938.42857	583.28571	549.42857	682.28571
28	919.71429	691.42857	985.85714	757.71429	843.28571	407.42857	345.28571	311.14286
29	909.85714	938.42857	715.85714	568.85714	706.42857	459	510.85714	653.42857
30	738.85714	919.71429	985.85714	882.14286	985.85714	734.42857	482.85714	810.42857
31	829.85714	744.71429	820.28571	1000	1000	849	834	820.42857
32	696.42857	649.57143	867.71429	952.57143	857.42857	919.71429	834	700.85714
33	863.14286	553.85714	734.14286	877.14286	985.85714	863.14286	692.28571	743.85714
34	863.14286	635.42857	767.85714	985.85714	757.57143	749.14286	905.57143	663.71429
35	696.85714	985.85714	985.85714	957.28571	957.28571	985.85714	985.85714	985.85714
36	872.71429	673.57143	697	863.14286	724.71429	985.85714	985.85714	957.28571
37	744	901	863.14286	550	553.57143	938.42857	938.42857	985.85714
38	896	985.85714	763.28571	654.57143	388	886	971.71429	971.71429
39	553.85714	834.57143	867.71429	1000	1000	919.71429	919.71429	891.14286
40	857.71429	421	496.42857	985.85714	843.28571	985.85714	985.85714	814.71429
41	682.28571	549.71429	853.57143	938.42857	985.85714	905.57143	905.57143	743.85714
42	696.42857	748.85714	825.28571	886.85714	929.28571	919.71429	777.14286	805.42857
43	734	1000	1000	971.42857	971.42857	1000	1000	1000
44	1000	957.28571	828.85714	952.57143	1000	857.42857	1000	924
45	952.57143	724.85714	924	1000	1000	1000	1000	971.42857
46	886.57143	734.42857	791	800.57143	753.14286	924.28571	924.28571	971.71429
47	957.28571	985.85714	815.28571	800.57143	667.42857	838.57143	924.28571	971.71429
48	795.57143	952.57143	952.57143	1000	1000	833.85714	910.14286	857.42857
49	1000	1000	985.85714	985.85714	985.85714	819.71429	957.28571	985.85714
50	971.42857	733.57143	591.85714	909.85714	957.28571	705.71429	985.85714	710.14286
51	943.14286	985.85714	985.85714	957.28571	957.28571	848.28571	900.14286	843.28571

	1	2	3	4	5	6	7	8
52	938.42857	795.85714	781.14286	1000	1000	857.42857	929.28571	885.71429
53	985.85714	943.14286	957.28571	985.85714	985.85714	985.85714	985.85714	957.28571
54	1000	957.28571	900.71429	952.57143	1000	957.57143	1000	924
55	1000	957.28571	971.42857	1000	952.57143	839.42857	952.57143	900.71429
56	914.85714	1000	1000	971.42857	971.42857	1000	1000	1000
57	938.42857	924	924	1000	1000	1000	1000	971.42857
58	924	952.57143	795.57143	719.57143	719.57143	843.28571	985.85714	985.85714
59	971.42857	957.57143	843.28571	696.57143	744	843.28571	985.85714	938.42857
60	900.14286	971.42857	971.42857	952.57143	1000	957.57143	1000	924
61	985.85714	828.85714	971.42857	952.57143	857.42857	1000	914.28571	781.14286
62	985.85714	885.71429	971.42857	1000	1000	957.57143	1000	885.71429
63	1000	772.28571	885.71429	1000	866.85714	796.14286	809.71429	900.71429
64	862.14286	1000	1000	971.42857	971.42857	1000	1000	1000
65	914.28571	914.85714	971.42857	1000	1000	1000	1000	971.42857
66	985.85714	800.57143	957.28571	938.42857	985.85714	985.85714	915.14286	909.85714
67	971.42857	1000	985.85714	672.14286	719.57143	985.85714	985.85714	938.42857
68	971.42857	914.28571	843.28571	648.85714	648.85714	843.28571	985.85714	900.14286
69	1000	957.57143	985.85714	985.85714	985.85714	862.14286	896	985.85714
70	1000	871.85714	985.85714	985.85714	985.85714	862.14286	957.28571	900.14286
71	919.71429	919.71429	900.14286	985.85714	900.14286	653.71429	877	763
72	871.57143	1000	1000	971.42857	971.42857	862.42857	1000	857.42857
73	952.57143	909.85714	924	1000	1000	1000	1000	971.42857
74	1000	871.57143	971.42857	857.42857	1000	1000	1000	885.71429
75	1000	814.71429	971.42857	1000	857.42857	957.57143	1000	828.85714
76	1000	914.85714	971.42857	1000	1000	1000	914.28571	971.42857
77	829.14286	1000	1000	971.42857	971.42857	1000	1000	1000
78	904.85714	957.28571	885.71429	1000	914.28571	914.28571	1000	971.42857
79	900.71429	929.28571	985.85714	871.57143	957.28571	943.42857	985.85714	985.85714
80	985.85714	900.71429	791.28571	644.85714	606.57143	900.14286	843.28571	909.85714
81	938.42857	924	924	1000	1000	957.57143	938.71429	971.42857
82	1000	957.28571	971.42857	1000	1000	1000	857.42857	971.42857
83	1000	957.28571	828.85714	1000	1000	815	914.28571	971.42857
84	1000	957.28571	971.42857	1000	1000	886.85714	914.28571	900.71429

	9	10	11	12	13	14	15	16
1								
2								
3								
4								
5								
6								
7								
8								
9	0							
10	834.85714	0						
11	815.71429	582.71429	0					
12	563.28571	596.85714	375	0				
13	625.71429	474.28571	697	359.42857	0			
14	924	1000	771.85714	938.42857	985.85714	0		
15	881.57143	857.71429	611.57143	778.14286	801.14286	648.71429	0	
16	882.14286	971.42857	885.71429	957.28571	957.28571	905.57143	905.57143	0
17	871.57143	1000	819.28571	900.14286	985.85714	634.85714	696.14286	563.85714
18	772.42857	625.14286	720.28571	648.71429	639.28571	952.57143	938.42857	900.71429
19	682.28571	711.71429	707.28571	540.57143	502.57143	1000	663.28571	971.42857
20	520.85714	682.85714	725.28571	558.57143	416	1000	943.42857	862.71429
21	905.57143	535.85714	725.28571	596.85714	449.85714	1000	857.71429	971.42857
22	891.14286	919.71429	738.85714	905.57143	905.57143	563.57143	776.57143	872.71429
23	943.42857	786.42857	644.57143	811.14286	872.42857	724.71429	483.42857	792.14286
24	857.42857	896	776.57143	943.14286	881.85714	724.71429	615.42857	644.57143
25	853.57143	1000	819.28571	985.85714	985.85714	696.14286	696.14286	650.14286
26	957.28571	1000	819.28571	985.85714	985.85714	553.57143	696.14286	753.85714
27	568	863.14286	905.57143	738.85714	738.85714	1000	1000	843.28571
28	748.85714	858.14286	905.57143	777.14286	729.42857	1000	1000	985.85714
29	814.71429	1000	1000	985.85714	985.85714	734.42857	877	849
30	597.71429	819.85714	858.14286	872.28571	834	952.57143	866.85714	882.14286
31	692.28571	141.71429	582.71429	596.85714	521.71429	1000	857.71429	971.42857
32	905.57143	345.85714	512	383.57143	402.14286	1000	857.71429	971.42857
33	677.57143	549.42857	596.85714	401.85714	497	985.85714	929.28571	896
34	801.57143	241.57143	549.42857	392.42857	450.71429	938.42857	881.85714	957.28571
35	838.28571	1000	952.57143	852.71429	985.85714	511.57143	839.14286	985.85714
36	809.71429	914.85714	625.42857	792	900.71429	677.28571	321.71429	763.85714
37	867.71429	985.85714	805.14286	971.71429	971.71429	467.85714	682	593.57143
38	814.71429	1000	819.28571	985.85714	985.85714	634.85714	696.14286	563.85714
39	730	664	492.57143	563.85714	493.14286	952.57143	853.57143	971.42857
40	771.85714	914.85714	720.42857	468.42857	577.42857	938.42857	654.71429	871.57143
41	919.71429	588.28571	739.42857	725.28571	616.57143	985.85714	786.71429	957.28571
42	905.57143	521.42857	582.71429	596.85714	678.14286	1000	943.42857	881.57143
43	924	1000	866.85714	952.57143	1000	682.42857	924	733.57143
44	1000	767	886.57143	886.57143	909.85714	1000	985.85714	790.71429
45	771.71429	914.85714	957.28571	871.57143	957.28571	1000	985.85714	790.71429
46	734.14286	1000	957.57143	762.71429	762.71429	957.28571	914.85714	901
47	772.28571	1000	957.57143	943.42857	943.42857	957.28571	914.85714	971.71429
48	819.14286	1000	857.42857	914.28571	1000	943.42857	1000	1000
49	757.57143	1000	952.57143	866.85714	1000	952.57143	952.57143	985.85714
50	667.28571	795.57143	985.85714	819.28571	771.85714	971.42857	971.42857	985.85714
51	828.85714	914.28571	952.57143	938.42857	900.14286	895.71429	824.14286	985.85714

	9	10	11	12	13	14	15	16
52	985.85714	772.28571	957.28571	971.42857	828.85714	985.85714	1000	971.42857
53	871.85714	957.28571	914.85714	815	900.71429	800.85714	929.28571	909.85714
54	914.28571	909.85714	957.28571	871.57143	909.85714	1000	843.28571	971.42857
55	701	886.57143	957.28571	871.57143	886.57143	1000	985.85714	971.42857
56	909.85714	985.85714	938.42857	952.57143	1000	516.57143	924	1000
57	1000	971.42857	971.42857	971.42857	971.42857	985.85714	1000	828.85714
58	943.14286	843.28571	915.14286	929.28571	1000	971.42857	971.42857	890.71429
59	857.42857	753.14286	985.85714	914.28571	952.57143	971.42857	971.42857	985.85714
60	1000	924	885.71429	971.42857	924	943.42857	1000	743.14286
61	957.57143	838.28571	929	786.42857	653	943.42857	871.85714	971.42857
62	809.71429	971.42857	924	924	971.42857	938.42857	952.57143	971.42857
63	929.28571	844.14286	957.28571	957.28571	886.57143	1000	985.85714	971.42857
64	924	1000	952.57143	952.57143	1000	724.85714	924	1000
65	952.57143	914.85714	824.14286	909.85714	957.28571	809.71429	938.42857	705
66	914.28571	909.85714	957.28571	857.42857	895.71429	985.85714	971.71429	814.71429
67	914.85714	952.57143	957.57143	957.57143	910.14286	971.42857	929	890.71429
68	895.71429	843.28571	938.42857	952.57143	1000	924	924	890.71429
69	757.57143	957.57143	952.57143	866.85714	1000	952.57143	952.57143	985.85714
70	700.42857	957.57143	952.57143	952.57143	1000	952.57143	952.57143	985.85714
71	763	919.71429	872.28571	872.28571	919.71429	952.57143	809.71429	985.85714
72	828.85714	1000	866.85714	952.57143	1000	867.42857	924	771.71429
73	914.28571	957.28571	957.28571	871.57143	814.71429	957.57143	985.85714	828.85714
74	938.42857	848	800.57143	814.71429	719.57143	910.14286	938.42857	790.71429
75	866.85714	957.28571	909.85714	681.28571	672.14286	952.57143	938.42857	971.42857
76	1000	829.14286	957.28571	957.28571	871.57143	1000	757.57143	924
77	924	1000	866.85714	952.57143	1000	516.57143	924	914.28571
78	795.57143	943.14286	895.71429	909.85714	957.28571	952.57143	795.85714	971.42857
79	909.85714	1000	952.57143	952.57143	1000	695.71429	924	915.14286
80	1000	867.42857	957.28571	943.14286	895.71429	905.57143	891.42857	877
81	771.71429	971.42857	971.42857	885.71429	828.85714	943.42857	1000	971.42857
82	952.57143	957.28571	909.85714	909.85714	957.28571	910.14286	938.42857	790.71429
83	952.57143	729	909.85714	909.85714	729	952.57143	852.71429	971.42857
84	929.28571	800.85714	957.28571	957.28571	658.28571	1000	757.57143	924

	17	18	19	20	21	22	23	24
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								
16								
17	0							
18	914.28571	0						
19	1000	862.85714	0					
20	952.57143	625.14286	553.85714	0				
21	1000	586.85714	616.57143	682.85714	0			
22	672.85714	849	919.71429	919.71429	919.71429	0		
23	710.57143	758.14286	563.14286	843.28571	786.42857	687	0	
24	421.57143	914.85714	800.85714	767	896	762.71429	601	0
25	544.42857	1000	1000	938.71429	952.57143	734.14286	710.57143	682.28571
26	431.28571	1000	1000	1000	1000	686.71429	663.14286	611.57143
27	985.85714	905.57143	724.71429	682.28571	863.14286	872.28571	938.42857	1000
28	985.85714	905.57143	858.14286	905.57143	763	919.71429	985.85714	1000
29	749.71429	1000	914.28571	914.28571	952.57143	914.85714	805.71429	792.42857
30	843.28571	677.28571	905.57143	844.28571	639	919.71429	985.85714	1000
31	1000	767.71429	759.14286	540.28571	488.42857	919.71429	929	753.42857
32	1000	586.85714	641	682.85714	212.42857	919.71429	929	896
33	985.85714	781.85714	606.28571	497.28571	739.42857	763	914.85714	943.14286
34	985.85714	734.42857	687.57143	611.28571	493.14286	905.57143	829.14286	881.85714
35	810.28571	866.85714	1000	1000	1000	601.71429	985.85714	985.85714
36	682.28571	839.14286	819.71429	772.28571	914.85714	606.28571	426.57143	401.85714
37	558.57143	915.14286	985.85714	924.57143	985.85714	473.42857	606.57143	668.14286
38	180	1000	914.28571	724	1000	672.85714	624.85714	278.71429
39	914.28571	311.14286	716.71429	512	483.14286	919.71429	886.57143	896
40	985.85714	909.85714	496.42857	734.14286	629.71429	985.85714	615.85714	848
41	985.85714	601	583	626.28571	454.85714	905.57143	914.85714	881.85714
42	839.42857	586.85714	759.14286	677.85714	483.14286	734.71429	929	735.42857
43	828.85714	952.57143	1000	1000	1000	914.85714	1000	1000
44	1000	711	839.14286	957.28571	957.28571	914.28571	828.85714	914.85714
45	914.28571	871.57143	957.28571	772.28571	867.42857	1000	971.42857	814.71429
46	943.14286	929.28571	776.85714	819.28571	1000	839.14286	811.14286	985.85714
47	943.14286	1000	871.85714	771.71429	929.28571	909.85714	796.14286	843.28571
48	914.28571	914.28571	1000	1000	952.57143	943.42857	1000	1000
49	900.14286	866.85714	1000	1000	1000	1000	985.85714	1000
50	957.28571	843.28571	757.71429	805.14286	915.14286	971.42857	843.28571	1000
51	957.28571	952.57143	1000	1000	914.28571	943.14286	985.85714	985.85714

	17	18	19	20	21	22	23	24
52	1000	629.71429	957.28571	772.28571	867.42857	985.85714	828.85714	929
53	852.71429	871.57143	914.85714	909.85714	957.28571	943.42857	914.85714	895.71429
54	914.28571	871.57143	767	957.28571	886.57143	1000	971.42857	957.28571
55	914.28571	767.85714	957.28571	814.71429	957.28571	952.57143	924	772.28571
56	971.42857	938.42857	985.85714	985.85714	985.85714	819.71429	1000	914.28571
57	1000	971.42857	971.42857	971.42857	924	985.85714	971.42857	885.71429
58	862.14286	843.28571	915.14286	985.85714	938.42857	971.42857	843.28571	914.28571
59	800.85714	757.57143	938.42857	943.42857	943.42857	971.42857	843.28571	929.28571
60	1000	971.42857	924	971.42857	971.42857	943.42857	971.42857	971.42857
61	1000	971.42857	881.57143	971.42857	743.14286	943.42857	929	971.42857
62	1000	881.57143	971.42857	828.85714	971.42857	985.85714	971.42857	786.42857
63	1000	853.57143	871.57143	829.14286	914.85714	809.71429	838.28571	914.85714
64	971.42857	952.57143	1000	1000	1000	914.85714	1000	1000
65	1000	806.14286	957.28571	914.85714	914.85714	1000	971.42857	914.85714
66	900.14286	767.85714	909.85714	957.28571	957.28571	985.85714	957.28571	900.71429
67	719.57143	1000	910.14286	1000	1000	971.42857	943.42857	1000
68	791.42857	795.57143	985.85714	985.85714	985.85714	971.42857	843.28571	929.28571
69	900.14286	866.85714	1000	957.57143	957.57143	1000	985.85714	1000
70	985.85714	952.57143	1000	815	957.57143	1000	985.85714	857.42857
71	985.85714	872.28571	691.42857	834	919.71429	919.71429	900.14286	1000
72	971.42857	952.57143	1000	1000	1000	914.85714	1000	1000
73	914.28571	667.71429	957.28571	814.71429	909.85714	957.57143	971.42857	957.28571
74	1000	596.71429	943.14286	705.42857	848	957.57143	971.42857	957.28571
75	914.28571	577.85714	957.28571	814.71429	814.71429	1000	971.42857	914.85714
76	952.57143	853.57143	814.71429	867.42857	829.14286	1000	971.42857	867.42857
77	971.42857	952.57143	1000	1000	1000	819.71429	1000	1000
78	1000	895.71429	857.42857	714.85714	943.14286	1000	885.71429	814.71429
79	957.28571	839.42857	1000	1000	1000	900.71429	915.14286	957.57143
80	834.85714	957.28571	824.14286	829.14286	914.85714	843.28571	791.28571	792.14286
81	914.28571	743.14286	971.42857	686.28571	924	943.42857	971.42857	828.85714
82	1000	848.57143	957.28571	957.28571	957.28571	815	971.42857	871.57143
83	1000	520.71429	957.28571	814.71429	871.57143	1000	828.85714	914.85714
84	952.57143	711	814.71429	767.28571	871.57143	1000	971.42857	867.42857

	25	26	27	28	29	30	31	32
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								
16								
17								
18								
19								
20								
21								
22								
23								
24								
25	0							
26	316.14286	0						
27	900.14286	852.71429	0					
28	900.14286	900.14286	435.71429	0				
29	773	607.14286	644.42857	463.71429	0			
30	882.14286	985.85714	625.14286	554.42857	748.14286	0		
31	952.57143	1000	863.14286	905.57143	952.57143	819.85714	0	
32	1000	1000	863.14286	715.28571	1000	639	393.28571	0
33	838.85714	900.14286	653.14286	673.42857	985.85714	787.71429	596.85714	549.42857
34	985.85714	985.85714	877.28571	777.14286	900.14286	872.28571	241.57143	350.57143
35	957.28571	814.71429	1000	1000	814.71429	952.57143	1000	1000
36	682.28571	682.28571	1000	1000	863.14286	952.57143	772.28571	914.85714
37	454.85714	368.28571	952.57143	1000	692	896.28571	985.85714	985.85714
38	544.42857	431.28571	985.85714	985.85714	664	985.85714	857.42857	1000
39	1000	1000	905.57143	905.57143	1000	677.28571	664	483.14286
40	985.85714	985.85714	819.28571	857.42857	985.85714	910.14286	914.85714	772.28571
41	900.14286	900.14286	791.57143	786.57143	985.85714	738.85714	635.71429	407.42857
42	871.85714	801.14286	819.85714	819.85714	886.85714	724.71429	521.42857	340.57143
43	971.42857	971.42857	1000	1000	971.42857	809.71429	1000	1000
44	1000	1000	1000	952.57143	1000	957.57143	957.28571	839.14286
45	952.57143	1000	957.57143	1000	952.57143	1000	724.57143	914.85714
46	943.14286	895.71429	757.71429	985.85714	943.14286	985.85714	1000	1000
47	943.14286	895.71429	938.42857	985.85714	857.42857	985.85714	857.42857	1000
48	952.57143	1000	900.71429	929	952.57143	971.42857	952.57143	1000
49	985.85714	985.85714	957.28571	914.85714	985.85714	909.85714	1000	1000
50	871.57143	871.57143	705.28571	796.14286	957.28571	971.71429	985.85714	938.42857
51	957.28571	957.28571	929.28571	957.57143	957.28571	866.85714	914.28571	914.28571

	25	26	27	28	29	30	31	32
52	866.85714	914.28571	857.71429	829.42857	952.57143	915.14286	867.42857	914.85714
53	985.85714	843.28571	1000	1000	843.28571	1000	957.28571	957.28571
54	1000	1000	1000	910.14286	1000	1000	957.28571	909.85714
55	1000	952.57143	952.57143	957.57143	1000	1000	744	957.28571
56	971.42857	971.42857	985.85714	985.85714	971.42857	938.42857	985.85714	985.85714
57	952.57143	1000	857.42857	1000	952.57143	957.57143	924	971.42857
58	814.71429	862.14286	971.71429	971.71429	909.85714	929.28571	938.42857	915.14286
59	957.28571	886.57143	929.28571	924.28571	886.57143	971.71429	943.42857	896
60	1000	1000	786.71429	910.14286	1000	1000	971.42857	924
61	1000	1000	929.28571	809.71429	1000	914.28571	885.71429	695.42857
62	914.28571	914.28571	843.57143	871.85714	1000	952.57143	828.85714	971.42857
63	1000	952.57143	910.14286	1000	914.28571	1000	844.14286	914.85714
64	971.42857	971.42857	1000	1000	971.42857	952.57143	1000	1000
65	1000	857.42857	957.57143	1000	857.42857	952.57143	914.85714	914.85714
66	985.85714	985.85714	857.42857	881.85714	985.85714	929.28571	957.28571	909.85714
67	791.42857	862.14286	985.85714	938.42857	957.28571	843.28571	1000	952.57143
68	776.42857	705.71429	886	886	886.57143	924.28571	985.85714	985.85714
69	985.85714	985.85714	844.14286	957.28571	985.85714	909.85714	957.57143	957.57143
70	900.14286	900.14286	758.42857	871.57143	985.85714	909.85714	815	957.57143
71	985.85714	985.85714	806.28571	834.57143	900.14286	829.57143	919.71429	919.71429
72	971.42857	971.42857	786.71429	957.57143	971.42857	952.57143	1000	1000
73	952.57143	1000	786.71429	1000	952.57143	1000	909.85714	957.28571
74	914.28571	914.28571	829.42857	900.14286	1000	896	848	848
75	1000	1000	929.28571	815	1000	952.57143	957.28571	814.71429
76	1000	1000	886.85714	1000	1000	914.28571	829.14286	829.14286
77	971.42857	971.42857	1000	1000	971.42857	952.57143	1000	1000
78	1000	1000	985.85714	985.85714	914.28571	896	800.57143	943.14286
79	957.28571	957.28571	985.85714	943.42857	957.28571	938.42857	1000	1000
80	905.57143	834.85714	957.57143	952.57143	749.14286	1000	914.85714	867.42857
81	952.57143	1000	929.28571	957.57143	952.57143	1000	781.14286	971.42857
82	1000	1000	929.28571	1000	1000	952.57143	957.28571	957.28571
83	1000	1000	929.28571	957.57143	1000	866.85714	871.57143	871.57143
84	1000	1000	1000	957.57143	1000	914.28571	800.85714	871.57143

	33	34	35	36	37	38	39	40
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								
16								
17								
18								
19								
20								
21								
22								
23								
24								
25								
26								
27								
28								
29								
30								
31								
32								
33	0							
34	582.71429	0						
35	985.85714	938.42857	0					
36	900.71429	853.28571	938.42857	0				
37	910.42857	971.71429	672.14286	597.42857	0			
38	985.85714	900.14286	896	539.71429	558.57143	0		
39	739.42857	630.71429	866.85714	867.42857	985.85714	1000	0	
40	720	710.42857	867.71429	696.85714	971.71429	985.85714	867.42857	0
41	592.14286	621.57143	985.85714	900.71429	971.71429	985.85714	426.57143	900.71429
42	368.57143	535.57143	1000	872.42857	943.42857	839.42857	483.14286	914.85714
43	1000	952.57143	825	952.57143	971.42857	971.42857	952.57143	786.57143
44	909.85714	957.28571	1000	871.57143	985.85714	1000	957.28571	834.57143
45	957.28571	914.85714	834	814.71429	985.85714	857.42857	871.57143	957.28571
46	805.14286	985.85714	957.28571	915.14286	839.14286	943.14286	957.57143	724.71429
47	985.85714	900.14286	957.28571	843.28571	909.85714	714.85714	957.57143	905.57143
48	957.57143	1000	857.71429	1000	1000	1000	771.71429	919.71429
49	957.57143	952.57143	866.85714	952.57143	1000	985.85714	866.85714	872.28571
50	643.71429	1000	971.42857	1000	971.42857	957.28571	985.85714	738.85714
51	943.42857	938.42857	715	938.42857	957.28571	957.28571	952.57143	858.14286

	33	34	35	36	37	38	39	40
52	815	929	985.85714	971.42857	1000	1000	814.71429	891.14286
53	943.14286	943.14286	715.14286	943.14286	829.14286	938.42857	829.14286	862.85714
54	867.42857	957.28571	914.28571	957.28571	985.85714	1000	871.57143	877
55	914.85714	886.57143	914.28571	814.71429	938.42857	857.42857	871.57143	877
56	1000	952.57143	549.14286	952.57143	971.42857	971.42857	938.42857	872.28571
57	971.42857	971.42857	724.71429	971.42857	1000	1000	971.42857	929
58	1000	1000	710.28571	1000	876.28571	862.14286	985.85714	957.57143
59	952.57143	957.57143	624.57143	1000	971.42857	886.57143	900.14286	1000
60	881.57143	971.42857	943.42857	971.42857	1000	1000	971.42857	805.42857
61	924	828.85714	943.42857	971.42857	1000	1000	929	748.57143
62	843.28571	924	938.42857	781.14286	1000	857.42857	924	843.71429
63	814.71429	758.42857	1000	957.28571	938.42857	914.28571	957.28571	877
64	1000	952.57143	606.28571	952.57143	971.42857	971.42857	952.57143	952.57143
65	957.28571	867.42857	729.42857	909.85714	843.28571	1000	909.85714	824.14286
66	825	943.14286	639	943.14286	971.71429	985.85714	871.57143	943.14286
67	952.57143	1000	891.14286	1000	876.28571	862.14286	957.57143	1000
68	914.28571	952.57143	662.85714	952.57143	876.28571	791.42857	938.42857	952.57143
69	1000	910.14286	866.85714	952.57143	1000	985.85714	866.85714	872.28571
70	914.28571	910.14286	952.57143	809.71429	1000	843.28571	952.57143	872.28571
71	877.28571	786.57143	952.57143	952.57143	1000	900.14286	872.28571	952.57143
72	957.57143	952.57143	867.42857	952.57143	971.42857	971.42857	952.57143	786.57143
73	957.28571	957.28571	871.85714	957.28571	985.85714	1000	729	877
74	776.42857	814.71429	910.14286	909.85714	985.85714	1000	658	787.14286
75	914.85714	767	866.85714	909.85714	985.85714	1000	681.57143	686.71429
76	957.28571	914.85714	1000	957.28571	985.85714	952.57143	957.28571	877
77	1000	952.57143	468.71429	952.57143	971.42857	971.42857	952.57143	866.85714
78	957.28571	824.14286	691.42857	624.42857	985.85714	771.71429	895.71429	867.42857
79	957.57143	952.57143	662.85714	881.85714	815	957.28571	952.57143	952.57143
80	752.85714	815	805.14286	862.85714	891.42857	749.14286	957.28571	943.14286
81	929	971.42857	857.71429	828.85714	1000	857.42857	743.14286	891.14286
82	814.71429	909.85714	910.14286	909.85714	985.85714	1000	909.85714	829.57143
83	914.85714	909.85714	952.57143	909.85714	985.85714	1000	767.28571	829.57143
84	914.85714	886.57143	1000	957.28571	985.85714	952.57143	814.71429	877

	41	42	43	44	45	46	47	48
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								
16								
17								
18								
19								
20								
21								
22								
23								
24								
25								
26								
27								
28								
29								
30								
31								
32								
33								
34								
35								
36								
37								
38								
39								
40								
41	0							
42	411.57143	0						
43	1000	1000	0					
44	909.85714	957.28571	724.71429	0				
45	772.28571	957.28571	805.14286	667.28571	0			
46	985.85714	1000	891.14286	919.71429	1000	0		
47	985.85714	1000	891.14286	919.71429	753.71429	535.28571	0	
48	1000	1000	849	905.57143	852.71429	919.71429	919.71429	0
49	1000	1000	858.14286	905.57143	838.85714	905.57143	844.28571	472.85714
50	866.85714	900.14286	891.14286	729.42857	938.71429	696.14286	745	639.57143
51	985.85714	1000	829.57143	919.71429	1000	834.57143	877	554.71429

	41	42	43	44	45	46	47	48
52	843.28571	871.57143	905.57143	706.14286	881.57143	919.71429	919.71429	677.28571
53	943.14286	909.85714	877.28571	877	871.57143	863.14286	682.28571	791.57143
54	767	957.28571	905.57143	815.42857	796.14286	919.71429	606.85714	777.42857
55	957.28571	957.28571	905.57143	759.14286	714.85714	872.28571	548.57143	777.42857
56	1000	985.85714	602.14286	919.71429	1000	891.14286	891.14286	863.14286
57	971.42857	971.42857	971.71429	677.14286	734.42857	1000	1000	924.28571
58	1000	985.85714	971.42857	744.28571	872.28571	711	814.71429	952.57143
59	910.14286	915.14286	971.42857	809.71429	687.85714	719.57143	615.85714	914.28571
60	924	971.42857	763.28571	829.57143	957.28571	919.71429	738.85714	735.85714
61	924	971.42857	849	591.85714	862.14286	877.28571	696.42857	778.28571
62	885.71429	885.71429	844	834.57143	814.71429	919.71429	596.28571	778.28571
63	772.28571	814.71429	905.57143	664	663	872.28571	605.71429	905.57143
64	1000	1000	544.85714	985.85714	905.57143	885.71429	971.42857	929.28571
65	914.85714	957.28571	672	421	544.42857	1000	1000	985.85714
66	753.14286	957.28571	1000	711	553.57143	985.85714	985.85714	914.28571
67	952.57143	1000	814.71429	938.42857	905.57143	582.85714	591.57143	985.85714
68	914.28571	829.42857	924	857.42857	816	814.71429	711	1000
69	957.57143	1000	858.14286	905.57143	857.71429	905.57143	905.57143	383.28571
70	871.85714	914.28571	858.14286	905.57143	739.57143	905.57143	701.71429	530.28571
71	777.14286	919.71429	938.42857	985.85714	924.57143	985.85714	658.14286	606.42857
72	1000	1000	687.28571	905.57143	985.85714	891.14286	710.28571	498.14286
73	957.28571	957.28571	863.14286	801.57143	810	919.71429	919.71429	478.42857
74	776.42857	762.28571	649	592.42857	776.57143	919.71429	919.71429	806.57143
75	957.28571	957.28571	858.14286	759.14286	857.42857	919.71429	738.85714	706.71429
76	772.28571	909.85714	905.57143	759.14286	900.71429	919.71429	919.71429	654
77	1000	1000	521.28571	985.85714	905.57143	971.42857	971.42857	929.28571
78	957.28571	943.14286	857.42857	914.85714	734.42857	1000	771.71429	1000
79	1000	1000	767.28571	943.42857	905.57143	730.14286	862.14286	943.42857
80	853.28571	744	1000	909.85714	811.14286	748.14286	558.71429	1000
81	971.42857	971.42857	849	877	681.28571	919.71429	777.14286	360.57143
82	957.28571	814.71429	634.85714	383	667.28571	919.71429	919.71429	792.42857
83	957.28571	957.28571	858.14286	616.57143	943.14286	919.71429	919.71429	611.57143
84	814.71429	909.85714	905.57143	664	848	919.71429	738.85714	863.14286

	49	50	51	52	53	54	55	56
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								
16								
17								
18								
19								
20								
21								
22								
23								
24								
25								
26								
27								
28								
29								
30								
31								
32								
33								
34								
35								
36								
37								
38								
39								
40								
41								
42								
43								
44								
45								
46								
47								
48								
49	0							
50	664.28571	0						
51	734.71429	611	0					

	49	50	51	52	53	54	55	56
52	738.85714	677.28571	905.57143	0				
53	763.28571	919.71429	844.28571	796	0			
54	673.71429	655.42857	877.28571	796	372.71429	0		
55	777.42857	877.28571	877.28571	753.57143	372.71429	316.14286	0	
56	872.28571	877	587.42857	891.42857	816	919.71429	919.71429	0
57	943.42857	957.57143	805.14286	848.57143	971.42857	914.85714	957.28571	719.42857
58	985.85714	800.57143	748.28571	795.57143	1000	1000	1000	690.85714
59	838.85714	691.57143	790.71429	800.85714	914.28571	805.57143	914.28571	776.57143
60	863.14286	829.85714	731.14286	781.85714	368.85714	368.57143	416	801.85714
61	905.57143	872.28571	687.85714	877	564.14286	648.71429	696.14286	801.85714
62	815.71429	791.57143	745	592.42857	472.57143	416	231	858.14286
63	905.57143	919.71429	919.71429	806.28571	696.14286	682	460.14286	919.71429
64	938.42857	971.42857	525	985.85714	896.28571	985.85714	985.85714	482.85714
65	938.42857	1000	952.57143	886.57143	814.71429	943.14286	839.42857	952.57143
66	914.28571	952.57143	805.14286	858.28571	857.42857	824.14286	767.85714	819.28571
67	971.71429	909.85714	929	1000	776.85714	757.71429	805.14286	971.42857
68	877.14286	653.57143	743.28571	757.57143	1000	938.71429	1000	729.14286
69	125.85714	768	706.42857	696.42857	834	777.42857	819.85714	872.28571
70	293.14286	621	706.42857	610.71429	919.71429	801.85714	763	872.28571
71	654.42857	602	601.71429	1000	819.28571	516.42857	762.71429	952.57143
72	720.57143	611	417.42857	905.57143	635.14286	682.28571	682.28571	725.85714
73	639	919.71429	849	425.14286	653.71429	682	620.71429	877.28571
74	872.28571	819.85714	801.57143	648.71429	834.57143	877	815.71429	815.71429
75	730	877.28571	759.14286	611	515.28571	458.71429	355	872.28571
76	724.71429	919.71429	763.28571	530.28571	591.85714	482.57143	521.42857	919.71429
77	795.57143	971.42857	729.14286	985.85714	957.57143	985.85714	985.85714	335.85714
78	910.14286	943.42857	771.85714	896	957.28571	914.85714	814.71429	757.71429
79	881.85714	914.85714	558	957.57143	1000	943.42857	901	600.71429
80	938.71429	891.28571	805.14286	929	943.14286	848.57143	957.28571	819.28571
81	596.57143	877.28571	792.42857	411	667.85714	653.71429	511.14286	863.14286
82	858.14286	919.71429	801.57143	891.14286	834.57143	862.85714	801.57143	744.14286
83	634.85714	734.71429	673.42857	287.57143	781.85714	725.28571	621.57143	872.28571
84	863.14286	877.28571	791.57143	706.14286	648.71429	497	465.14286	919.71429

	57	58	59	60	61	62	63	64
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								
16								
17								
18								
19								
20								
21								
22								
23								
24								
25								
26								
27								
28								
29								
30								
31								
32								
33								
34								
35								
36								
37								
38								
39								
40								
41								
42								
43								
44								
45								
46								
47								
48								
49								
50								
51								

	57	58	59	60	61	62	63	64
52								
53								
54								
55								
56								
57	0							
58	563.28571	0						
59	738.85714	396.85714	0					
60	800.57143	1000	952.57143	0				
61	705.42857	1000	952.57143	460.14286	0			
62	881.85714	1000	1000	331.14286	611.28571	0		
63	862.14286	1000	957.57143	696.14286	601	653.71429	0	
64	710.57143	710.28571	710.28571	868	868	924.28571	985.85714	0
65	568.57143	919.71429	877.28571	871.57143	719.57143	867.42857	701.85714	858.14286
66	472.57143	738.85714	605.71429	781.14286	828.85714	929	615.85714	738.85714
67	905.57143	535.57143	687	757.71429	715.28571	805.14286	805.14286	791.28571
68	738.85714	301.71429	222.42857	1000	1000	866.85714	1000	662.85714
69	943.42857	985.85714	857.71429	834.85714	834.85714	787.42857	863.14286	938.42857
70	943.42857	985.85714	882.14286	834.85714	834.85714	558.85714	863.14286	938.42857
71	943.42857	985.85714	924.57143	692	734.42857	644.57143	719.42857	938.42857
72	829.14286	971.42857	971.42857	265.42857	536.14286	550	724.71429	792
73	767	952.57143	914.28571	526.14286	763.85714	711.14286	801.57143	901
74	929	943.42857	985.85714	778	778	687.28571	815.71429	910.14286
75	957.28571	1000	914.28571	487.85714	482.85714	398	578.28571	938.42857
76	957.28571	1000	957.57143	568.57143	720.57143	526.14286	716.71429	985.85714
77	710.57143	710.28571	710.28571	843.57143	929.28571	924.28571	985.85714	449.57143
78	493.42857	682.28571	724.71429	971.42857	971.42857	719.85714	871.57143	596.28571
79	724.71429	634.85714	601	943.42857	985.85714	853.57143	943.42857	363.28571
80	790.71429	676.71429	317.28571	924	924	971.42857	686.57143	819.28571
81	895.71429	952.57143	914.28571	612.14286	749.71429	512	877	929.28571
82	633.85714	914.28571	1000	763.85714	526.14286	758.85714	563.85714	853.57143
83	957.28571	857.42857	857.42857	668.71429	720.57143	578.85714	759.14286	938.42857
84	862.14286	1000	1000	653.71429	515.28571	611.28571	412.42857	985.85714

	65	66	67	68	69	70	71	72
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								
16								
17								
18								
19								
20								
21								
22								
23								
24								
25								
26								
27								
28								
29								
30								
31								
32								
33								
34								
35								
36								
37								
38								
39								
40								
41								
42								
43								
44								
45								
46								
47								
48								
49								
50								
51								

	65	66	67	68	69	70	71	72
52								
53								
54								
55								
56								
57								
58								
59								
60								
61								
62								
63								
64								
65	0							
66	678.14286	0						
67	905.57143	872.28571	0					
68	872.28571	696.42857	639.28571	0				
69	896	914.28571	971.71429	938.42857	0			
70	896	1000	971.71429	791.42857	241.28571	0		
71	938.42857	1000	791	877.14286	687.42857	626.14286	0	
72	852.71429	857.42857	776.57143	924	692.28571	692.28571	406.85714	0
73	881.85714	667.71429	985.85714	1000	568.28571	654	915.14286	649.85714
74	667.85714	896	1000	852.71429	801.57143	715.85714	881.85714	759.14286
75	792	767.85714	805.14286	952.57143	701.71429	787.42857	644.57143	564.14286
76	797	853.57143	985.85714	1000	611.57143	611.57143	772.57143	834.85714
77	772.42857	738.85714	877	662.85714	795.57143	938.42857	938.42857	767.57143
78	758.85714	696.14286	919.71429	677.28571	910.14286	767.28571	824.42857	952.57143
79	815.71429	696.42857	801.57143	648.71429	924.28571	924.28571	881.85714	867.42857
80	914.85714	715	810	502.28571	957.57143	896.28571	853	1000
81	957.28571	885.71429	985.85714	1000	507	511.42857	872.71429	735.85714
82	416	800.85714	985.85714	952.57143	787.42857	787.42857	867.71429	745
83	792	853.57143	985.85714	809.71429	606.57143	606.57143	825.28571	745
84	744.28571	758.42857	805.14286	1000	905.57143	905.57143	620.14286	682.28571

	73	74	75	76	77	78	79	80
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								
16								
17								
18								
19								
20								
21								
22								
23								
24								
25								
26								
27								
28								
29								
30								
31								
32								
33								
34								
35								
36								
37								
38								
39								
40								
41								
42								
43								
44								
45								
46								
47								
48								
49								
50								
51								

	73	74	75	76	77	78	79	80
52								
53								
54								
55								
56								
57								
58								
59								
60								
61								
62								
63								
64								
65								
66								
67								
68								
69								
70								
71								
72								
73	0							
74	560	0						
75	407.42857	555	0					
76	454.85714	745	593.28571	0				
77	943.42857	910.14286	938.42857	985.85714	0			
78	957.28571	853.28571	909.85714	957.28571	691.42857	0		
79	985.85714	952.57143	853.57143	943.42857	506.14286	691.42857	0	
80	957.28571	957.28571	957.28571	914.85714	819.28571	690.85714	724.14286	0
81	212.14286	635.42857	440.42857	530.28571	929.28571	828.85714	943.42857	971.42857
82	646	474.28571	683.42857	730.85714	896	909.85714	938.42857	814.71429
83	312.28571	555	360.85714	326.71429	938.42857	909.85714	853.57143	957.28571
84	659	673.14286	393.28571	483.14286	985.85714	957.28571	901	957.28571

	81	82	83	84
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				
16				
17				
18				
19				
20				
21				
22				
23				
24				
25				
26				
27				
28				
29				
30				
31				
32				
33				
34				
35				
36				
37				
38				
39				
40				
41				
42				
43				
44				
45				
46				
47				
48				
49				
50				
51				

	81	82	83	84
52				
53				
54				
55				
56				
57				
58				
59				
60				
61				
62				
63				
64				
65				
66				
67				
68				
69				
70				
71				
72				
73				
74				
75				
76				
77				
78				
79				
80				
81	0			
82	763.85714	0		
83	345.28571	683.42857	0	
84	692	706.42857	488.42857	0

Proceeding pages display the averaged dissimilarity matrix for part one of the subset study described in Section 7.2.

	1	2	3	4	5	6	7	8
1	0							
2	687.3333	0						
3	906.7778	494	0					
4	829.7778	788.6667	886.0833	0				
5	967	934	967	769	0			
6	967	967	901	967	895.5	0		
7	967	934	934	950.5	813	587.5	0	
8	810.6667	755.6667	893.1667	827.1667	802	703	868	0
9	884.5	956	908.3333	703	868	835	785.5	835
10	965.44446	929.3333	870.1111	936.1111	967	835	967	656.6667
11	703	791	967	868	983.5	835	884.5	692
12	928.7778	687.3333	873.7778	917.7778	1000	868	1000	783.1667
13	983.5	967	879	945	1000	967	950.5	835
14	466.5	791	886.3333	952.3333	967	967	983.5	1000
15	945	802	809.3333	948.6667	967	857	934	1000
16	549	857	835	802	769	1000	1000	895.5
17	909.6667	876.6667	761.1667	772.1667	835	769	934	725
18	909.6667	794.6667	798.3333	928.7778	967	934	945	689.6667
19	819.3333	848.1111	876.6667	909.6667	868	967	983.5	805.1667
20	884.7778	761.1667	905.3333	822.8333	934	868	884.5	744.6667
21	950.5	967	923	956	857	868	631.5	780
22	578.3333	840.5	912	854.25	967	967	967	862.5
23	857	675.5	829.5	912	901	967	967	827.6667
24	827.1667	591.44446	518.1111	793.1111	730.5	983.5	897.3333	593
25	654.3333	848.1111	871.1667	491.66666	901	637	967	711.6667
26	906.5	824	787.3333	673.6667	307	967	851.5	934
27	967	857	912	945	967	703	538	780
28	967	956	879	952.3333	901	406	406	912
29	967	934	967	934	747	631.5	703	505
30	717.1667	761.1667	876.6667	601.6667	967	868	917.5	794.1667
31	965.44446	896.3333	826.1111	954.44446	934	868	934	882.1667
32	761.1667	574.1667	865.6667	794.1667	967	967	868	761.6667
33	1000	1000	930.3333	919.3333	967	835	884.5	582
34	687.3333	826.1111	827.1667	761.1667	934	835	835	755.6667
35	624.44446	731.3333	965.44446	866.44446	967	824	967	865.6667
36	945	824	849.6667	901	967	857	983.5	967
37	860.6667	527	912	527	769	967	934	796.5
38	852.3333	881.1111	739.1667	689.6667	604	967	983.5	747
39	1000	967	912	1000	967	967	802	983.5
40	835	967	919.3333	937.6667	1000	868	868	967
41	967	967	802	934	950.5	950.5	983.5	934
42	1000	901	901	967	1000	934	670	956
43	670	978	980.75	980.75	967	868	983.5	950.5
44	983.5	703	945	1000	1000	967	978	879
45	930.3333	983.5	835	1000	1000	967	978	923
46	912	961.5	972.5	759.1667	868	934	917.5	950.5
47	967	989	901	934	802	835	785.5	771.3333
48	948.1667	915.1667	931.6667	948.1667	967	703	703	877.44446
49	967	857	1000	967	950.5	736	802	725
50	1000	978	917.5	967	967	934	818.5	769
51	879	950.5	950.5	851.5	967	835	967	967

	1	2	3	4	5	6	7	8
53	835	897.3333	983.5	917.5	967	1000	1000	813
54	928.5	956	945	956	1000	901	989	884.5
55	989	1000	969.75	945	1000	967	983.5	873.5
56	952.3333	945	947.75	947.75	1000	868	851.5	884.5
57	362	983.5	1000	934	983.5	934	974.3333	1000
58	1000	902.55554	804.3333	1000	967	967	829.5	901
59	852.3333	925.1111	887.6667	772.1667	1000	934	818.5	893.1667
60	890	952.3333	879	912	868	934	945	798.8333
61	945	707.6667	829.2222	926.6667	967	835	967	917.5
62	928.5	967	970.6667	948.6667	1000	967	978	1000
63	945	961.5	945	961.5	967	1000	1000	875.3333
64	882.6667	663.6667	876.8889	963.3333	1000	967	1000	725
65	904.1667	948.1667	948.1667	964.6667	1000	868	967	976.44446
66	983.5	703	934	989	967	1000	961.5	824
67	1000	862.5	835	983.5	967	895.5	967	857
68	941.3333	923	952.3333	715.55554	967	791	868	879
69	852.3333	870.1111	805.1667	854.6667	736	901	835	730.5
70	967	891.55554	787.8333	983.5	967	670	851.5	785.5
71	929.3333	917.7778	810.6667	926.1667	934	736	653.5	766.6667
72	934	983.5	983.5	950.5	917.5	703	714	736
73	736	868	1000	1000	1000	967	703	1000
74	1000	967	945	945	1000	1000	835	1000
75	758	967	1000	901	1000	934	1000	1000
76	890	765.8333	815.3333	961.5	934	934	950.5	862.5
77	1000	1000	945	945	967	802	967	857
78	764.3333	932.44446	865.6667	948.1667	1000	1000	901	964.6667
79	882.1667	730.5	758	937.1667	1000	967	967	888.44446
80	983.5	967	978	824	934	967	967	846
81	912	952.3333	765.3333	930.3333	934	967	983.5	799.8889
82	901	956	934	1000	1000	934	785.5	967
83	917.5	923	974.3333	952.3333	1000	967	1000	835
84	967	985.3333	879	862.5	1000	934	950.5	934

	9	10	11	12	13	14	15	16
1								
2								
3								
4								
5								
6								
7								
8								
9	0							
10	917.5	0						
11	791	967	0					
12	934	706.6667	868	0				
13	956	593	934	505	0			
14	941.3333	934	802	1000	934	0		
15	978	956	934	1000	934	842.3333	0	
16	802	879	857	967	967	802	813	0
17	967	898.6667	802	882.1667	868	868	846	945
18	851.5	530.1667	681	695.1667	780	983.5	967	824
19	1000	670	626	296	923	868	868	857
20	857	717.6667	901	475.66666	813	1000	923	1000
21	923	824	604	708.5	743.3333	989	912	967
22	820.3333	1000	659	952.3333	978	607.6667	897.3333	648
23	835	862.5	857	796.5	1000	945	1000	851.5
24	961.5	931.6667	862.5	893.1667	950.5	961.5	950.5	659
25	824	670	802	560	1000	956	901	604
26	919.3333	983.5	967	967	989	945	915.6667	890
27	923	945	890	967	1000	1000	912	917.5
28	882.6667	912	901	956	1000	974.3333	893.6667	1000
29	868	967	851.5	1000	1000	967	967	934
30	472	931.6667	780	931.6667	934	934	967	824
31	884.5	541.6667	1000	703	719.5	1000	840.5	1000
32	879	909.6667	461	794.1667	934	934	879	912
33	725	626	868	538	772.6667	956	978	890
34	1000	929.3333	593	786.3333	637	967	956	824
35	884.5	976.44446	1000	943.44446	983.5	411.5	807.5	835
36	974.3333	972.5	967	835	923	915.6667	417	945
37	934	970.6667	1000	919.3333	1000	945	917.5	912
38	956	929.3333	967	786.3333	967	956	736	670
39	835	1000	736	736	802	967	912	983.5
40	868	917.5	538	967	952.3333	945	853.3333	780
41	868	461	785.5	340	670	967	934	934
42	967	703	736	582	395	868	934	983.5
43	838.6667	978	1000	978	1000	732.3333	989	835
44	989	901	934	741.5	820.3333	967	934	1000
45	989	825.55554	967	840.2222	919.3333	919.3333	978	1000
46	681	941.3333	967	945	923	941.3333	967	802
47	967	967	934	1000	857	1000	1000	967
48	945	816.1667	967	931.6667	1000	1000	967	1000
49	934	967	862.5	934	967	868	1000	857
50	967	703	1000	967	1000	1000	956	1000
51	706.6667	978	967	934	978	945	985.3333	835

	9	10	11	12	13	14	15	16
53	1000	934	593	1000	967	802	923	857
54	917.5	950.5	752.5	917.5	983.5	879	961.5	1000
55	934	708.2222	835	631.2222	794.6667	923	945	945
56	952.3333	923	835	923	875.3333	956	967	1000
57	983.5	1000	917.5	967	950.5	466.5	1000	967
58	769	835	934	1000	967	1000	505	934
59	956	863.3333	967	852.3333	670	956	967	736
60	1000	967	1000	923	967	1000	967	1000
61	950.5	893.6667	703	923	846	967	972.5	945
62	919.3333	906.5	769	967	879	875.3333	959.6667	945
63	923	754.8333	703	732.8333	945	1000	967	983.5
64	967	791	637	794.6667	791	912	950.5	1000
65	967	796.5	868	835	967	912	1000	983.5
66	945	956	967	813	893.6667	956	934	1000
67	1000	1000	879	868	983.5	1000	857	890
68	846	967	967	941.3333	934	983.5	879	802
69	791	830.3333	703	786.3333	1000	956	901	1000
70	967	868	802	1000	1000	1000	956	1000
71	923	698.3333	967	929.3333	1000	956	1000	967
72	884.5	967	917.5	824	967	967	961.5	1000
73	912	1000	868	1000	967	967	1000	934
74	967	945	736	1000	967	967	945	972.5
75	967	868	703	835	868	857	967	1000
76	967	884.5	802	681	835	758	1000	1000
77	1000	857	505	747	747	1000	912	956
78	967	929.3333	1000	929.3333	1000	538	956	967
79	1000	810.6667	890	788.6667	648	901	912	901
80	846	1000	1000	956	864.3333	945	978	835
81	917.5	895.5	967	945	752.5	950.5	983.5	967
82	901	934	736	1000	967	1000	923	1000
83	835	983.5	769	967	934	941.3333	978	835
84	824	791	835	835	901	956	1000	1000

	17	18	19	20	21	22	23	24
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								
16								
17	0							
18	926.1667	0						
19	893.1667	591.44446	0					
20	893.1667	848.1111	626	0				
21	879	582	758	864.3333	0			
22	961.5	952.3333	846	914.75	1000	0		
23	952.3333	807.5	824	862.5	736	637	0	
24	780	877.8333	892.1111	807.7778	895.5	882.6667	791	0
25	794.1667	928.7778	798.3333	631.5	1000	708.5	895.5	848.1111
26	505	983.5	901	912	1000	908.3333	879	747
27	917.5	725	857	670	967	967	912	857
28	824	928.5	1000	912	761.6667	952.3333	967	961.5
29	703	967	967	934	901	967	901	884.5
30	948.1667	884.7778	909.6667	865.6667	868	818.5	912	807.7778
31	904.1667	612.6667	505	593	752.5	959.6667	917.5	898.6667
32	802	706.1667	673.1667	761.1667	703	818.5	758	576.5
33	857	846	956	395	857	989	835	983.5
34	860.1667	793.1111	804.1111	744.6667	571	730.5	840.5	771.1111
35	964.6667	948.1667	929.3333	899.44446	983.5	765.3333	939.5	865.6667
36	912	983.5	967	923	890	937.6667	868	813
37	862.5	945	890	875.3333	983.5	805.6667	895.5	846
38	461	906.7778	884.7778	810.6667	967	873.5	879	596.6667
39	851.5	703	505	835	901	912	934	967
40	879	664.5	329	787.3333	688.3333	879	890	967
41	901	549	538	307	967	967	934	917.5
42	846	703	835	824	516	1000	879	901
43	983.5	956	1000	983.5	945	765.0833	920.6667	974.3333
44	967	868	967	930.3333	926.6667	989	890	983.5
45	917.5	890	837.3333	849.9167	915.6667	974.0833	909.6667	703
46	1000	923	950.5	950.5	978	858.5833	876.6667	963.3333
47	705.3333	963.3333	974.3333	895.5	824	948.1667	931.6667	825.55554
48	884.7778	915.1667	948.1667	865.6667	967	956	967	948.1667
49	967	912	890	1000	835	967	824	912
50	879	890	989	967	1000	967	967	967
51	1000	989	851.5	909.25	978	903.75	934	1000

	17	18	19	20	21	22	23	24
53	835	842.3333	798.3333	917.5	967	818.5	928.5	875.3333
54	818.5	851.5	879	923	917.5	989	950.5	991.75
55	983.5	842.3333	870.3333	772.9167	937.6667	983.5	1000	967
56	917.5	904.6667	840.5	945	791	964.25	945	875.3333
57	967	1000	917.5	1000	983.5	703	1000	1000
58	983.5	748.55554	989	1000	967	964.6667	948.1667	967
59	909.6667	873.7778	928.7778	860.1667	835	939.5	879	914.1111
60	716.3333	952.3333	941.3333	961.5	967	926.1667	909.6667	821.8889
61	917.5	639.3333	1000	912	835	956	967	967
62	1000	802	802	967	857	989	1000	983.5
63	871.6667	796.5	699.8333	721.8333	967	895.5	952.3333	928.5
64	752.5	518.3333	923	868	813	963.3333	923	928.5
65	888.44446	915.1667	769	835	1000	967	926.6667	964.6667
66	945	917.5	967	948.6667	930.3333	978	967	967
67	967	886.3333	912	934	967	1000	758	677.3333
68	945	928.5	890	974.3333	967	921.44446	909.6667	945
69	681	862.7778	895.7778	876.6667	1000	857	912	739.6667
70	703	847.55554	945	983.5	901	967	983.5	989
71	948.1667	844.44446	976.44446	882.1667	868	961.5	967	862.7778
72	692	923	983.5	912	934	950.5	967	978
73	967	835	1000	1000	1000	912	1000	901
74	983.5	571	736	1000	1000	967	1000	1000
75	967	670	736	901	571	1000	1000	1000
76	873.5	744.8889	730.5	796.5	901	961.5	895.5	941.3333
77	884.5	703	736	879	879	1000	983.5	967
78	931.6667	921.44446	965.44446	948.1667	1000	835	917.5	943.44446
79	932.44446	763.5	843.6667	854.6667	879	1000	875.3333	893.1667
80	945	895.5	758	974.3333	945	978	1000	967
81	799.8889	871.6667	937.6667	941.3333	961.5	939.7778	837.1111	766.8889
82	901	846	901	1000	967	1000	1000	945
83	1000	620.5	505	967	824	901	835	961.5
84	912	846	879	851.5	1000	961.5	1000	974.3333

	25	26	27	28	29	30	31	32
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								
16								
17								
18								
19								
20								
21								
22								
23								
24								
25	0							
26	626	0						
27	967	967	0					
28	923	879	692	0				
29	736	593	769	637	0			
30	783.1667	934	890	967	967	0		
31	670	923	692	697.5	934	931.6667	0	
32	810.6667	912	945	912	967	645.6667	827.1667	0
33	868	963.3333	868	912	967	967	686.5	967
34	752.7778	1000	791	802	835	662.1667	896.3333	607.1667
35	764.3333	939.5	967	967	967	717.1667	954.44446	849.1667
36	956	879	912	879	967	1000	791	780
37	725	769	912	950.5	934	785.5	967	846
38	760.1111	494	967	956	901	931.6667	896.3333	747
39	967	912	895.5	835	868	1000	516	648
40	1000	948.6667	615	827.6667	901	835	923	835
41	934	934	901	824	901	884.5	428	703
42	1000	1000	884.5	967	1000	868	967	730.5
43	978	945	769	893.6667	967	884.5	886.3333	917.5
44	1000	956	934	978	1000	967	967	923
45	853.8333	989	835	978	1000	967	869.55554	879
46	730.5	875.3333	967	919.3333	868	884.5	948.6667	917.5
47	956	703	868	868	670	950.5	967	837.3333
48	931.6667	967	659	769	868	865.6667	854.6667	936.1111
49	934	967	593	769	851.5	895.5	1000	857
50	978	868	901	703	835	967	835	983.5
51	934	967	879	901	967	705.3333	970.6667	906.5

	25	26	27	28	29	30	31	32
53	934	1000	857	1000	967	780	967	714
54	901	983.5	967	983.5	917.5	978	983.5	818.5
55	804.3333	945	912	875.3333	1000	950.5	766.8889	1000
56	961.5	945	769	780	901	950.5	897.3333	961.5
57	1000	983.5	967	934	983.5	967	1000	1000
58	989	1000	835	1000	967	934	868	967
59	804.1111	956	802	956	1000	915.1667	731.3333	909.6667
60	952.3333	1000	901	1000	967	983.5	802	831.8333
61	1000	967	560	835	967	884.5	901	967
62	901	1000	780	941.3333	1000	967	912	802
63	815.3333	967	818.5	692	967	835	782.3333	875.3333
64	923	901	967	873.5	1000	950.5	912	923
65	818.5	780	956	901	1000	849.1667	796.5	921.44446
66	1000	945	967	959.6667	967	1000	967	967
67	983.5	983.5	791	983.5	967	945	967	890
68	626	967	1000	983.5	967	851.5	970.6667	934
69	870.1111	560	934	824	967	882.1667	797.3333	697.5
70	923	868	868	868	703	983.5	901	983.5
71	928.7778	824	703	659	835	849.1667	896.3333	948.1667
72	967	851.5	802	703	670	950.5	1000	1000
73	901	1000	873.5	1000	1000	703	945	890
74	1000	901	829.5	1000	1000	1000	1000	851.5
75	1000	1000	967	1000	1000	934	901	736
76	945	1000	934	1000	983.5	967	917.5	780
77	802	901	956	967	802	934	967	752.5
78	954.44446	967	802	967	1000	948.1667	929.3333	964.6667
79	964.6667	1000	868	967	1000	876.6667	931.6667	906.7778
80	1000	978	1000	959.6667	934	604	983.5	967
81	941.3333	983.5	967	1000	934	934	1000	851.2222
82	1000	1000	967	1000	1000	835	802	868
83	956	985.3333	813	985.3333	1000	967	972.5	912
84	930.3333	923	967	923	1000	967	791	917.5

	33	34	35	36	37	38	39	40
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								
16								
17								
18								
19								
20								
21								
22								
23								
24								
25								
26								
27								
28								
29								
30								
31								
32								
33	0							
34	1000	0						
35	1000	731.3333	0					
36	1000	1000	791	0				
37	1000	692	842.3333	884.5	0			
38	1000	906.7778	896.3333	626	857	0		
39	835	604	967	780	1000	857	0	
40	875.3333	1000	983.5	919.3333	956	967	912	0
41	340	571	1000	967	967	934	604	967
42	791	868	1000	1000	1000	890	780	890
43	989	983.5	783.6667	930.3333	967	978	923	923
44	930.3333	1000	917.5	835	967	967	835	919.3333
45	886.3333	1000	945	934	978	802	1000	897.3333
46	945	983.5	923	930.3333	882.6667	912	945	897.3333
47	923	912	1000	1000	1000	759.55554	1000	956
48	868	948.1667	948.1667	967	967	876.6667	917.5	835
49	967	857	835	868	934	934	1000	868
50	967	887.8889	967	1000	896.3333	853.3333	857	1000
51	945	978	879	934	910.44446	1000	912	776.3333

	33	34	35	36	37	38	39	40
53	901	761.6667	835	967	764.3333	978	736	769
54	917.5	897.3333	934	917.5	948.1667	1000	736	835
55	739.6667	978	967	893.6667	958.1111	1000	802	948.6667
56	952.3333	813	956	945	906.7778	945	923	897.3333
57	1000	934	538	934	934	1000	1000	1000
58	1000	989	1000	505	1000	978	1000	967
59	1000	873.7778	929.3333	923	923	925.1111	967	967
60	1000	974.3333	1000	967	824	777.8889	967	967
61	884.5	967	945	923	914.1111	967	967	758
62	923	1000	917.5	904.6667	956	967	692	783.6667
63	813	763.5	983.5	967	909.6667	890	774.5	835
64	824	923	967	901	950.7778	758	967	813
65	1000	915.1667	904.1667	967	1000	964.6667	917.5	1000
66	970.6667	1000	901	868	950.5	967	802	967
67	950.5	923	824	862.5	934	917.5	967	967
68	983.5	923	897.3333	912	842.3333	923	912	983.5
69	967	928.7778	896.3333	956	857	684.6667	967	1000
70	967	985.3333	967	1000	929.3333	989	1000	1000
71	967	921.44446	929.3333	857	1000	939.7778	1000	1000
72	895.5	978	967	1000	967	846	857	917.5
73	1000	1000	868	967	1000	1000	939.5	1000
74	1000	736	967	912	945	1000	835	945
75	868	736	967	934	967	1000	736	571
76	736	846	934	868	909.6667	890	626	967
77	879	703	1000	780	945	967	769	912
78	967	961.7778	296	901	1000	976.44446	1000	1000
79	879	909.6667	964.6667	615	972.5	882.1667	967	912
80	985.3333	901	851.5	923	758	967	934	974.3333
81	868	875.3333	983.5	983.5	945	733.8889	983.5	923
82	1000	941.3333	901	967	929.3333	989	670	868
83	901	901	917.5	908.3333	983.5	923	879	534.3333
84	703	923	1000	956	929.3333	948.6667	725	1000

	41	42	43	44	45	46	47	48
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								
16								
17								
18								
19								
20								
21								
22								
23								
24								
25								
26								
27								
28								
29								
30								
31								
32								
33								
34								
35								
36								
37								
38								
39								
40								
41	0							
42	868	0						
43	1000	967	0					
44	1000	582	934	0				
45	967	747	888.8333	659	0			
46	1000	923	872.3333	860.6667	844.8333	0		
47	857	967	948.1667	1000	761.1667	893.1667	0	
48	835	912	738.3333	934	917.5	956	868	0
49	983.5	1000	571	692	835	659	571	472
50	1000	857	945	769	917.5	950.5	934	758
51	1000	1000	727.75	890	917.5	788.25	983.5	653.5

	41	42	43	44	45	46	47	48
53	703	868	666.8333	835	901	983.5	945	765.8333
54	868	868	730.2222	851.5	886.3333	890	978	798.8333
55	593	714	931.25	728.6667	822.4167	925.75	967	840.5
56	934	967	836.1667	798.3333	826.75	914.75	961.5	694.3333
57	1000	1000	516	912	840.5	917.5	950.5	967
58	967	802	931.6667	923	788.6667	843.6667	884.7778	868
59	1000	769	683.3333	967	967	780	868	752.5
60	890	967	865.6667	791	728.1667	865.6667	754.3333	983.5
61	901	1000	868	950.5	923	879	967	598.5
62	1000	1000	809.3333	908.3333	871.6667	879	1000	912
63	538	835	851.5	879	743.8333	967	967	758
64	769	802	886.3333	785.5	864.3333	952.3333	967	917.5
65	736	934	879	1000	853.8333	950.5	967	844.44446
66	967	747	945	424.33334	802	853.3333	835	967
67	934	802	917.5	851.5	472	901	983.5	967
68	1000	1000	943.44446	967	877.44446	530.6667	863.3333	972.5
69	890	835	978	1000	917.5	829.5	601.8889	865.6667
70	1000	835	895.5	868	934	983.5	868	747
71	802	1000	776.8333	1000	983.5	945	846	483
72	983.5	857	835	983.5	934	901	868	758
73	1000	851.5	813	1000	1000	945	967	774.5
74	736	851.5	912	901	1000	1000	901	769
75	818.5	868	824	901	791	934	1000	967
76	769	725	829.5	835	813	901	950.5	813
77	593	774.5	967	615	879	1000	824	967
78	1000	835	884.5	1000	983.5	983.5	978	948.1667
79	1000	873.5	950.5	714	923	983.5	934	921.44446
80	967	1000	945	897.3333	945	615	857	1000
81	967	884.5	976.44446	670	624.44446	884.7778	721.3333	989
82	703	571	650.3333	967	967	983.5	978	716.3333
83	1000	1000	813	923	901	857	901	681
84	868	560	831.8333	670	802	895.5	923	666.8333

	50	51	52	53	54	55	56	57
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								
16								
17								
18								
19								
20								
21								
22								
23								
24								
25								
26								
27								
28								
29								
30								
31								
32								
33								
34								
35								
36								
37								
38								
39								
40								
41								
42								
43								
44								
45								
46								
47								
48								
49	0							
50	769	0						
51	538	739.1667	0					

	50	51	52	53	54	55	56	57
53	725	870.1111	722.6667	0				
54	901	782.1111	738.1111	618.6667	0			
55	868	656.6667	784.3333	755.6667	826.1111	0		
56	626	882.1667	836.5833	719.5	824	767.8333	0	
57	851.5	983.5	868	769	750.6667	950.5	868	0
58	868	956	983.5	923	941.3333	917.5	950.5	824
59	901	978	901	847.55554	891.55554	884.5	782.3333	934
60	934	912	934	868	978	917.5	763.5	912
61	802	522.3333	697.7778	654.3333	711.6667	818.7778	859.1111	835
62	835	1000	871.6667	868	483	860.6667	857	774.5
63	934	827.1667	887.6667	717.1667	733.6667	538	612.6667	967
64	967	632.3333	921.44446	863.3333	827.1667	672.1111	767.44446	857
65	1000	1000	884.5	950.5	934	815.3333	956	802
66	725	1000	934	967	818.5	849.6667	758	912
67	895.5	879	961.5	890	875.3333	928.5	851.5	934
68	901	967	915.6667	967	950.5	967	952.3333	934
69	967	890	983.5	1000	989	851.5	879	1000
70	769	683.1111	706.1667	683.1111	646.44446	656.6667	799.6667	763.5
71	505	644.3333	846	843.8889	634.8889	912	837.3333	983.5
72	752.5	576.5	846	879	853.3333	928.5	868	934
73	1000	1000	483	1000	857	890	912	824
74	868	967	637	604	560	736	483	824
75	950.5	967	934	703	582	868	802	879
76	818.5	777.6667	816.1667	816.1667	741.7778	502.66666	810.6667	862.5
77	967	901	1000	703	472	527	703	1000
78	967	908.3333	862.5	941.3333	816.6667	961.5	1000	516
79	758	868	934	846	901	780	950.5	934
80	692	1000	846	967	934	956	820.3333	785.5
81	901	901	945	923	901	945	904.6667	934
82	1000	914.1111	887.6667	662.6667	472	871.1667	785.5	796.5
83	835	1000	692	769	571	919.3333	915.6667	818.5
84	835	686.7778	750.1667	791	703	601.6667	670	868

	58	59	60	61	62	63	64	65
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								
16								
17								
18								
19								
20								
21								
22								
23								
24								
25								
26								
27								
28								
29								
30								
31								
32								
33								
34								
35								
36								
37								
38								
39								
40								
41								
42								
43								
44								
45								
46								
47								
48								
49								
50								
51								

	66	67	68	69	70	71	72	73
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								
16								
17								
18								
19								
20								
21								
22								
23								
24								
25								
26								
27								
28								
29								
30								
31								
32								
33								
34								
35								
36								
37								
38								
39								
40								
41								
42								
43								
44								
45								
46								
47								
48								
49								
50								
51								

	66	67	68	69	70	71	72	73
53								
54								
55								
56								
57								
58								
59								
60								
61								
62								
63								
64								
65								
66	0							
67	884.5	0						
68	945	659	0					
69	703	884.5	923	0				
70	1000	961.5	1000	901	0			
71	802	917.5	967	906.7778	820.3333	0		
72	950.5	842.3333	934	901	763.5	686.5	0	
73	967	1000	912	868	824	1000	1000	0
74	703	967	967	1000	758	967	967	565.5
75	934	950.5	934	1000	967	967	917.5	967
76	868	901	912	912	785.5	868	895.5	967
77	736	736	890	857	802	868	1000	967
78	1000	813	967	965.44446	860.6667	866.44446	879	659
79	835	824	928.5	948.1667	804.3333	915.1667	769	884.5
80	787.3333	950.5	516	868	835	1000	983.5	934
81	983.5	752.5	794.1667	821.8889	934	934	967	983.5
82	769	879	967	945	723.44446	689.8889	813	758
83	923	884.5	923	956	1000	791	835	714
84	1000	835	934	919.3333	723.44446	733.8889	912	967

	74	75	76	77	78	79	80	81
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								
16								
17								
18								
19								
20								
21								
22								
23								
24								
25								
26								
27								
28								
29								
30								
31								
32								
33								
34								
35								
36								
37								
38								
39								
40								
41								
42								
43								
44								
45								
46								
47								
48								
49								
50								
51								

	74	75	76	77	78	79	80	81
53								
54								
55								
56								
57								
58								
59								
60								
61								
62								
63								
64								
65								
66								
67								
68								
69								
70								
71								
72								
73								
74	0							
75	670	0						
76	769	648	0					
77	692	736	736	0				
78	659	934	1000	1000	0			
79	890	967	688.8333	774.5	849.1667	0		
80	934	967	868	890	1000	890	0	
81	967	527	862.5	917.5	967	945	813	0
82	758	868	849.1667	835	871.6667	950.5	967	989
83	769	637	835	967	868	967	890	835
84	868	703	563.1667	736	923	967	736	758

	82	83	84
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			
21			
22			
23			
24			
25			
26			
27			
28			
29			
30			
31			
32			
33			
34			
35			
36			
37			
38			
39			
40			
41			
42			
43			
44			
45			
46			
47			
48			
49			
50			
51			

	82	83	84
53			
54			
55			
56			
57			
58			
59			
60			
61			
62			
63			
64			
65			
66			
67			
68			
69			
70			
71			
72			
73			
74			
75			
76			
77			
78			
79			
80			
81			
82	0		
83	571	0	
84	769	857	0

Proceeding pages display the averaged dissimilarity matrix for part one of the subset study described in Section 7.2.

	1	2	3	4	5	6	7	8
1	0							
2	792.6	0						
3	905.3333	523.8571	0					
4	814.5833	818.8571	847.26666	0				
5	950.5	960.4	884.5	725	0			
6	960.4	983.5	854.8	894.4	887.8	0		
7	960.4	943.4286	938.125	950.5	864.3333	571	0	
8	833.25	804.375	847.1429	790.5714	861.4	615	670	0
9	868	961.5	917.5	732.7	917.5	785.5	802	884.5
10	920.2308	937.8	869.5833	943.8333	917.5	821.8	960.4	717.75
11	736	829.5	884.5	934	991	838	901	846
12	928.7778	733.2	873.7778	917.7778	1000	851.5	802	713.625
13	975.25	960.4	881.2	950.5	983.5	901	925.75	821.8
14	457.85715	874.6	887.8	947.2	950.5	943.4286	978	983.5
15	946.375	752.5	828.4	953.8	967	912	943.4286	980.2
16	659	912	846	653.5	820.8571	924.5714	983.5	868
17	906.7778	880.1429	775.5	804.375	582	841.6	950.5	711.25
18	895.125	832	781	941.7273	978	835	909.25	730.125
19	860.1667	869.5833	870.375	932.25	943.4286	983.5	987.625	841.5
20	905.3333	767	889.06665	829.6667	950.5	835	901	719.8571
21	943.4286	967	923	956	894.4	769	649.375	824
22	610.6	806.7143	911.1539	857.8461	960.4	950.5	967	864.3333
23	868	645.25	839.7143	910.4286	920.8	884.5	829.5	850.2308
24	858	584.7273	480.5	813.8	769	985.8571	916	645.25
25	667.6667	826.3077	903.375	457.875	917.5	802	975.25	734.25
26	917.5	763.5	808.6	706.3	318	917.5	872.7143	782.2
27	967	912	947.2	967	967	631.5	543.5	842.3333
28	901	961.5	891.1	907.6	884.5	367.5	349.42856	729.4
29	960.4	917.5	894.4	874.6	755.8	580	670	417
30	763.125	808.5	880.1429	644.4286	962.875	824	875.3333	807.2
31	960.8461	918	853.0833	957.5833	901	802	940.6	886.875
32	796.125	620.8571	800	738.7143	861.4	920.8	861.4	788.25
33	985.8571	1000	894.4	884.5	950.5	655.8571	791	587.5
34	753	841.53845	781.1429	795.2857	940.6	835	858.5714	767.25
35	638.7273	838.8	959	869.9	960.4	879	967	909.5
36	934	815.2	838	871	943.4286	863.2857	980.2	912
37	870.75	478.6	844	451	642.5	912	957.5714	825.1
38	893.1667	877.8333	705.375	721.875	509.7143	967	987.625	698.875
39	1000	967	802	945	985.8571	910.4286	884.5	934
40	839.125	967	907.6	943.9	934	868	886.8571	960.4
41	967	980.2	785.5	950.5	955	910.9	919.3333	901
42	983.5	901	730.5	912	971.7143	811.4286	802	901
43	652	981.1429	983.5	976.4286	960.4	901	989	962.875
44	987.625	736	940.6	1000	1000	980.2	983.5	927.4
45	947.75	957.5714	861.4	1000	1000	980.2	983.5	934
46	884.5	967	978	736.93335	873.5	960.4	938.125	957.5714
47	980.2	984.7692	915.1429	943.4286	637	901	759.5714	779
48	961.125	936.375	941.4286	955.5714	980.2	835	851.5	915.1539
49	983.5	914.2	1000	983.5	946	759.1	827.6667	835
50	1000	954.3077	929.2857	971.7143	894.4	835	844.4286	826.75
51	901	957.5714	957.5714	872.7143	881.2	881.2	937.6667	975.25

	1	2	3	4	5	6	7	8
53	868	914.75	975.25	938.125	985.8571	967	1000	859.75
54	946.375	928	940.6	960.4	1000	915.1429	991	913.375
55	991.75	1000	969.2	956	1000	940.6	987.625	891.5714
56	964.25	952.8571	951.6	958.2	1000	881.2	888.625	901
57	664.5	987.625	1000	945	930.7	950.5	982.2308	1000
58	983.5	926.9167	853.25	950.5	901	983.5	872.125	925.75
59	792.6	940.53845	903.7143	804.7143	1000	967	844.4286	919.875
60	928.5	939.5	909.25	888.625	863.2857	967	958.75	849.125
61	950.5	804.8	836.4	934	983.5	835	983.5	938.125
62	938.7143	983.5	970.6667	948.6667	1000	967	983.5	1000
63	963.3333	967	884.5	909.25	901	1000	1000	906.5
64	918.7692	798.2	899.4167	956	1000	980.2	1000	793.75
65	928.125	948.75	955.5714	969.7143	1000	868	983.5	983.6923
66	985.8571	656.8	940.6	990.1	983.5	985.8571	974.3333	912
67	901	825.5714	863.875	987.625	978	930.3333	946.6923	862.5
68	925.75	934	961	758.2727	960.4	874.6	868	875.3333
69	911.4	902.46155	833	875.4286	722.8	934	858.5714	797.875
70	983.5	918.6667	840.875	987.625	943.4286	681	843.25	839.125
71	964.6667	938.3333	858	944.625	929.2857	714	694.75	825
72	960.4	987.625	987.625	962.875	912	683.2	760.75	868
73	851.5	934	1000	980.2	983.5	967	807.5	1000
74	1000	967	956	972.5	1000	915.1429	868	1000
75	815.2	934	894.4	874.6	1000	925	1000	945
76	926.6667	799.2857	849.125	971.125	956	950.5	967	908.3333
77	835	934	923	890	985.8571	901	917.5	843.25
78	858.6	953.2308	884.8571	955.5714	1000	983.5	915.1429	973.5
79	911.625	769	792.5714	946.1429	1000	980.2	980.2	916.3333
80	987.625	983.5	980.2	821.8	967	983.5	971.7143	907.6
81	877.9	951.7692	771.75	881.75	818.5	872.125	980.2	805.61536
82	841.6	961.9231	915.1429	1000	1000	917.5	759.5714	962.875
83	929.2857	961.5	974.3333	952.3333	1000	868	975.25	884.5
84	980.2	982.2308	896.2857	882.1429	1000	917.5	957.5714	950.5

	9	10	11	12	13	14	15	16
1								
2								
3								
4								
5								
6								
7								
8								
9	0							
10	925.75	0						
11	783.1429	807.5	0					
12	945	706.6667	868	0				
13	931.46155	513.25	741.5	591.625	0			
14	931.25	929.2857	802	1000	937	0		
15	942.25	954.625	967	1000	917.5	862	0	
16	763.5	923	884.5	980.2	960.4	719.5	808.6	0
17	967	910.44446	901	899	917.5	901	887.8	934
18	813	540.375	574.3	734.25	665.875	987.625	975.25	912
19	985.8571	593	561.5714	557.8	802	861.4	787.8571	928.5
20	871.3	667.25	807.5	475.66666	745.9	990.1	901	846
21	870.53845	721.8571	554.5	772.6667	683.75	983.5	917.5	960.4
22	835	990.1	730.5	961	980.2	631.5	907.6	769
23	851.5	872.125	879	760.75	980.2	956	861.4	863.875
24	963.3333	924	884.5	866	938.125	967	943.4286	775.6
25	858.5714	818.5	915.1429	736	967	953.8	917.5	646.4286
26	923	938.125	983.5	975.25	975.25	946	926.38464	914.2
27	868	906.5	882.1429	980.2	967	967	907.6	940.6
28	895.5	847.375	950.5	905.125	958.75	970	911.1539	980.2
29	835	901	883	901	920.8	957.5714	967	896.2857
30	472	924	798.3333	862.125	881.2	934	960.4	874.6
31	901	517.6923	857	703	608.125	985.8571	868	901
32	808.6	870.375	617.2	795.2857	940.6	874.6	907.6	910.4286
33	694.75	594.5714	844.4286	611.3333	805	954.3077	973	857
34	901	799.2	648	871.8	498.4	960.4	945	879
35	901	971.7273	813	953.7273	987.625	387.66666	855.625	782.2
36	936.53845	967	962.875	857	934	880.6923	376	773.7143
37	950.5	961.5	1000	907	967	952.3333	826.75	725
38	952.8571	948.1667	943.4286	871.8	957.5714	953.8	830.2857	758
39	901	857	769	716.2	709.6	983.5	887.8	961
40	868	826.75	653.5	975.25	947.75	901	845.1539	848.2
41	868	571	766	465.4	626	950.5	901	943.4286
42	884.5	692	707.125	729.4	445.6	851.5	940.6	943
43	844	982	967	980.2	1000	772	989	901
44	992.38464	913.375	950.5	806.125	868	973	950.5	1000
45	990.1	819.6667	967	840.2222	917.5	927.4	980.2	1000
46	712.9	914.75	983.5	945	930.7	947.2	970.3	807.5
47	967	980.2	967	920.8	835	980.2	983.5	950.5
48	972.5	800.25	967	948.75	1000	983.5	980.2	1000
49	934	983.5	925	960.4	967	917.5	983.5	882.1429
50	983.5	821.8	967	960.4	1000	901	978	934
51	751	982	967	940.6	980.2	910	985.3333	851.5

	9	10	11	12	13	14	15	16
53	1000	950.5	797.2857	894.4	971.7143	881.2	967	846
54	945	950.5	802	886.8571	975.25	839.7143	967	901
55	940.6	772.9167	884.5	631.2222	805.3	930.7	950.5	972.5
56	957.1	934	901	923	877.9	960.4	970.3	868
57	989	1000	946	980.2	962.875	600.3333	975.25	971.7143
58	901	917.5	971.7143	1000	943.4286	1000	698.2857	950.5
59	906.5	918	983.5	911.4	802	894.4	950.5	664.5
60	1000	901	985.8571	934	985.8571	1000	985.8571	939.5
61	930.7	894.4	773.7143	934	886.3333	967	979.375	972.5
62	944.1539	919.8571	868	978	909.25	865.25	969.75	887.8
63	961.5	825.55554	818.5	771	956	1000	980.2	987.625
64	975.25	847.6923	802	794.6667	830.875	924.5714	962.875	884.5
65	917.5	847.375	868	876.25	940.6	956	960.4	975.25
66	958.75	962.2857	943.4286	831.3333	895	931.46155	928	934
67	1000	980.2	858.1	782.2	975.25	945	877.4286	846
68	868	975.25	967	907.6	950.5	946.375	896.2857	868
69	879	898.2	851.5	871.8	980.2	953.8	934	967
70	971.7143	884.5	915.1429	1000	1000	1000	981.1429	1000
71	952.8571	717.1667	985.8571	957.6	1000	973.6	1000	983.5
72	814.375	920.8	901	894.4	971.7143	967	971.125	1000
73	858.5714	1000	915.1429	1000	934	901	1000	868
74	934	956	814.375	980.2	960.4	901	967	904
75	983.5	881.2	802	796.5	920.8	825.5714	983.5	853.8571
76	983.5	912	857	726.5714	901	854.8	1000	1000
77	1000	879	686.5	689.8	828.4	983.5	947.2	976
78	983.5	957.6	983.5	957.6	1000	557.8	862.5	983.5
79	1000	858	848.2	818.8571	749.2	940.6	907.6	915.1429
80	848.75	1000	1000	967	881.75	955	969.53845	716.2
81	955	907.6	956	952.3333	851.5	960.4	990.1	901
82	884.5	920.8	769	960.4	980.2	1000	961.5	1000
83	855.3077	985.8571	802	956	950.5	947.75	983.5	901
84	912	874.6	884.5	795.4	940.6	874.6	1000	884.5

	17	18	19	20	21	22	23	24
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								
16								
17	0							
18	944.625	0						
19	917.7778	597.3333	0					
20	907.5	797.7273	575.125	0				
21	907.6	463.75	763.5	864.3333	0			
22	896.875	934	825.5714	913.6923	973	0		
23	906.5	793.75	855.625	868	769	648	0	
24	752.5	888.2143	893.7273	817.1	909.25	854.8	707.125	0
25	820.875	946.5833	852.7692	723.625	967	651.1429	847.375	836.5833
26	326.8	987.625	943.4286	920.8	983.5	917.5	907.6	726.5714
27	934	755.8	862.5	802	835	960.4	930.3333	912
28	874.6	896.875	943.4286	920.8	763.5	957.1	920.8	952.8571
29	802	879	983.5	874.6	818.5	967	901	901
30	943.44446	851.7273	919.875	870.7143	818.5	824	825.1	833.7273
31	914.1111	577.5	543.5	573.75	660.5714	920.8	913.375	899.25
32	835	677.4286	606.375	781.1429	643.6	814.375	785.5	622.8571
33	894.4	760.75	973.6	363.1	802	983.5	818.5	985.8571
34	823.5714	794.7273	828.3333	781.1429	554.5	769	855.625	767.7273
35	965.44446	961.125	957.6	899.6	989	749.75	953.8	899.25
36	840.5	989	901	898	909.25	887.25	769	810.25
37	838.3	963.3333	895.5	859	978	813	808.6	798.3333
38	365.66666	888.8333	918.8571	858	950.5	891.5714	896.875	634
39	888.625	576.5	543.5	670	920.8	947.2	950.5	960.4
40	907.6	711.25	340	808.6	700.25	841.6	914.2	957.5714
41	934	505	637	444.5	861.4	960.4	736	915.1429
42	872.125	510.5	725	697.5	505	980.2	896.875	920.8
43	975.25	964	1000	985.8571	950.5	768	940.5	979
44	983.5	888.625	971.7143	927.4	945	990.1	934	975.25
45	938.125	901	865.625	860.13336	915.6667	950.6923	922.5714	722.8
46	987.625	937	962.875	927.4	978	869.46155	894.2857	967
47	747.4286	970	980.75	910.4286	846	955.5714	936.375	848.2727
48	913.5833	936.375	961.125	814.1429	983.5	959.6667	977.1539	961.125
49	983.5	941.3333	952.8571	1000	881.2	914.2	894.4	924.5714
50	896.2857	892	983.5	971.7143	1000	957.5714	975.25	973
51	938.125	991	872.7143	922.2143	980.2	910.4286	950.5	1000

	17	18	19	20	21	22	23	24
53	890	832.25	856.2143	863.875	983.5	844.4286	946.375	889
54	863.875	865.6429	892	920.8	938.125	990.1	950.5	960.4
55	987.625	844	890.375	811.73334	937.6667	984.7692	1000	960.4
56	938.125	895	868	916.4	791	941.61536	952.8571	877.9
57	967	1000	945	1000	987.625	754.3333	1000	1000
58	978	811.4167	992.9286	1000	934	969.7143	961.125	973
59	922.5714	869.7273	946.5833	880.1429	917.5	948.1429	896.875	920.7273
60	777.8889	964.25	955.2143	946.375	983.5	936.7143	932.25	854.2727
61	938.125	704.55554	895.5	910.9	846	955	975.25	962.875
62	1000	851.5	829.5	967	901	982	1000	987.625
63	911.1539	835	788.8889	779	980.2	921.625	964.25	934
64	835	626.375	945	851.5	839.7143	934	942.25	934
65	916.3333	886.875	802	858.5714	934	941.3333	941.61536	961.125
66	967	938.125	920.8	953.8	947.75	947.75	983.5	929.2857
67	983.5	906.5	921.625	938.125	975.25	1000	862.5	691
68	952.3333	946.375	934	979	975.25	935.7273	928.7778	952.3333
69	726.5714	887.7273	921.8333	894.2857	983.5	877.4286	921.625	778
70	802	869.1667	964.6429	987.625	950.5	971.7143	987.625	991
71	965.44446	866.8333	984.8571	911.625	934	967	975.25	887.7273
72	815.2	901	987.625	934	913.375	919.3333	934	980.2
73	967	901	1000	1000	1000	861.4	1000	950.5
74	987.625	686.5	763.5	901	940.6	960.4	962.875	980.2
75	980.2	780	785.5	874.6	785.5	983.5	1000	957.5714
76	924.1	782.2727	809.3333	835	940.6	929.875	930.3333	943
77	913.375	785.5	802	923	927.4	914.2	987.625	901
78	941.4286	935.7273	949.3333	955.5714	1000	844.4286	938.125	953.7273
79	949.3333	797.2857	858	875.4286	927.4	1000	906.5	908.4286
80	967	921.625	868	976.9	958.75	980.2	1000	971.7143
81	785.3077	888.3077	938.7143	920.25	976.9	946.5833	872	792.1667
82	915.1429	811	865.25	1000	917.5	1000	962.875	955
83	1000	641.125	681	967	832.46155	910	884.5	971.125
84	924.5714	856	901	872.7143	1000	952.8571	1000	979

	25	26	27	28	29	30	31	32
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								
16								
17								
18								
19								
20								
21								
22								
23								
24								
25	0							
26	730.5	0						
27	957.5714	940.6	0					
28	846	878.1539	531.4	0				
29	802	609.5	752.5	554.5	0			
30	787.875	940.6	846	920.8	901	0		
31	818.5	892.75	780	661.75	841.6	924	0	
32	823.5714	927.4	909.25	828.4	920.8	719.7778	808.5	0
33	901	961	851.5	856	740.7143	818.5	717.1429	920.8
34	806	917.5	747	868	785.5	684.75	819	649.1429
35	858.6	954.625	967	975.25	967	810.5	953.7273	888.44446
36	928.5	892	948.1429	892	910.4286	983.5	830.875	848.2
37	582	752.5	948.1429	950.5	895.5	851.5	958.75	820.3333
38	811.0769	500.2857	950.5	952.8571	934	936.375	931.6667	797.875
39	985.8571	947.2	907.6	861.4	811.4286	1000	582	698.2857
40	967	926.38464	670	819.7692	884.5	881.2	855.625	663.4
41	950.5	950.5	934	829.5	881.2	674.125	521.5	762.4
42	985.8571	980.2	851.5	920.8	783.1429	821.8	708.5	754.8571
43	938.125	945	868	893.6667	980.2	901	877	929.2857
44	1000	967	967	958.75	1000	980.2	962.875	953.8
45	890.375	990.1	881.2	980.2	1000	971.7143	866.4167	896.2857
46	698.875	887.8	960.4	877.9	920.8	901	961.5	929.2857
47	954.3077	730.5	917.5	917.5	747	962.875	980.2	860.5714
48	948.75	980.2	750.6667	861.4	934	909.5	891	952.0833
49	901	934	582	796.5	901	921.625	862.5	914.2
50	961.9231	813	934	818.5	763.5	962.875	861.4	985.8571
51	950.5	967	846	901	980.2	766.625	976	919.8571

	25	26	27	28	29	30	31	32
53	954.3077	1000	928.5	929.2857	950.5	835	851.5	785.5
54	917.5	985.8571	983.5	985.8571	929.2857	982	975.25	844.4286
55	853.25	950.5	947.2	868	960.4	957.5714	800.4167	1000
56	934	950.5	841.6	782.2	901	957.5714	829.5	967
57	989	987.625	950.5	950.5	990.1	967	945	1000
58	984.7692	1000	917.5	1000	983.5	950.5	934	975.25
59	818.6923	978	901	978	1000	936.375	838.8	922.5714
60	959.38464	971.7143	950.5	957.5714	983.5	987.625	901	873.875
61	1000	975.25	697.5	876.25	917.5	851.5	901	971.7143
62	950.5	1000	873.5	956	1000	967	924.5714	881.2
63	861.5	980.2	879	815.2	980.2	890	843.8889	865.25
64	928.5	925.75	967	905.125	1000	962.875	868	942.25
65	851.5	868	915.6667	940.6	934	869.9	806.125	941.0833
66	1000	955	967	940	985.8571	983.5	924.5714	980.2
67	975.25	985.8571	895.5	985.8571	978	963.3333	960.4	934
68	604	971.7143	1000	985.8571	980.2	901	936.75	909.25
69	894.8461	730.5	950.5	895.5	884.5	911.625	878.4	740.7143
70	946.6923	901	802	769	769	987.625	868	987.625
71	950.6923	882.1429	719.5	679.4286	835	886.875	865.6667	961.125
72	975.25	851.5	861.4	715.375	752.5	956	835	901
73	943.4286	1000	898	1000	1000	785.5	917.5	917.5
74	1000	940.6	887.8	960.4	971.7143	980.2	868	872.7143
75	950.5	1000	967	1000	970	945	841.6	841.6
76	958.75	1000	950.5	1000	987.625	975.25	842.3333	853.3333
77	844.4286	940.6	963.7	881.2	915.1429	940.6	912	787.8571
78	968.46155	983.5	884.5	983.5	1000	948.75	957.6	969.7143
79	969.7143	1000	901	980.2	1000	917.7778	948.75	930.0833
80	785.5	984.7692	1000	972.0769	967	762.4	987.625	980.2
81	948.1429	910.9	982	1000	921.625	950.5	990.1	880.1667
82	1000	1000	983.5	1000	983.5	876.25	881.2	886.8571
83	978	989	824	972.5	901	868	976.4286	907.6
84	951.7692	961.5	967	928.5	967	962.875	835	929.2857

	33	34	35	36	37	38	39	40
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								
16								
17								
18								
19								
20								
21								
22								
23								
24								
25								
26								
27								
28								
29								
30								
31								
32								
33	0							
34	901	0						
35	989	838.8	0					
36	959.38464	960.4	827.6667	0				
37	952.3333	775.6	883.2308	808.6	0			
38	980.2	864.0833	937.8	780	769	0		
39	725	752.5	980.2	858.5714	945	846	0	
40	889	934	962.875	862	954.625	957.5714	947.2	0
41	554.5	663.4	980.2	957.5714	967	886.8571	566.2857	967
42	725	752.5	980.2	924.5714	928.5	796.5	673	854.8
43	991	985.8571	814	939.5	967	967	945	923
44	943	1000	938.125	876.25	978	985.8571	881.2	939.5
45	897.7	1000	950.5	946	982	851.5	983.5	907.6
46	901	985.8571	930.7	934	871	888.625	972.5	907.6
47	934	901	1000	960.4	980.2	811.4167	1000	961.5
48	851.5	961.125	949.1	983.5	980.2	907.5	938.125	901
49	967	894.4	901	929.2857	967	957.5714	1000	917.5
50	980.2	922.38464	960.4	920.8	937.8	890	879	1000
51	955	981.1429	892	917.5	932.8333	929.2857	956	776.3333

	33	34	35	36	37	38	39	40
53	854.8	821.25	901	983.5	882.1667	985.8571	730.5	901
54	929.2857	907	938.125	839.125	921.44446	1000	821.8	858.5714
55	765.7	910.4286	970.3	913	965.7273	1000	835	953.8
56	957.1	839.7143	960.4	955	923.7273	958.75	912	907.6
57	1000	950.5	703	950.5	957.5714	967	1000	1000
58	1000	967	920.8	538	983.5	964.6429	1000	985.8571
59	940.6	905	957.6	854.8	835	927.3333	934	983.5
60	1000	980.75	1000	967	851.5	786.5	967	985.8571
61	846	901	946	953.8	935.5833	983.5	901	764.875
62	917.5	1000	890	895.5	970.6667	983.5	755.8	802
63	887.8	797.2857	989	983.5	896.3	926.6667	806.125	901
64	849.1429	953.8	973	925.75	963.0833	879	967	859.75
65	934	874.5	932.6	967	970.3	973.5	938.125	1000
66	979.6923	1000	923	855.3077	912	980.2	868	973
67	967	919.8571	829.5	877.9	915.1429	938.125	950.5	971.7143
68	987.625	934	923	919.3333	837.53845	934	939.5	985.8571
69	960.4	935.46155	937.8	934	894.4	694.75	983.5	983.5
70	980.2	989	960.4	1000	964.6667	992.9286	967	957.5714
71	980.2	941.0833	957.6	928.5	1000	961.2857	967	957.5714
72	897.3333	983.5	967	1000	983.5	884.5	879	901
73	950.5	1000	917.5	915.1429	985.8571	983.5	924.1	1000
74	901	868	960.4	905.7143	972.5	1000	838	881.2
75	835	835	967	825.5714	879	1000	764.2857	620.5
76	841.6	868	945	934	945.8	926.6667	755.8	980.2
77	890	818.5	980.2	863.2857	923	983.5	820	907.6
78	980.2	973.53845	478.6	940.6	1000	965.8333	1000	950.5
79	927.4	922.5714	895.7778	769	959.6667	911.625	957.5714	947.2
80	961	950.5	888.625	937	731.875	985.8571	901	982.2308
81	877.9	868	980.2	934	844	694.5714	952	934
82	861.4	906.0769	920.8	980.2	957.6	991.75	752.5	829.5
83	851.5	884.5	923	931.25	989	961.5	868	615
84	676.6	946.6923	980.2	894.4	957.6	961.5	692	1000

	41	42	43	44	45	46	47	48
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								
16								
17								
18								
19								
20								
21								
22								
23								
24								
25								
26								
27								
28								
29								
30								
31								
32								
33								
34								
35								
36								
37								
38								
39								
40								
41	0							
42	693.5714	0						
43	1000	983.5	0					
44	983.5	729.4	940.6	0				
45	967	857	881.1429	607.3	0			
46	1000	961.5	883.5	874.6	869.26666	0		
47	835	967	955.5714	980.2	795.2857	866	0	
48	901	934	791.375	960.4	915.1429	830.2857	863.875	0
49	982	915.1429	676.6	846	846	813	683.2	465.4
50	960.4	895.5	938.7143	841.6	929.2857	957.5714	829.9231	769
51	1000	1000	775.6	901	922.2143	787.8571	985.8571	637

	41	42	43	44	45	46	47	48
53	830.2857	884.5	714.4286	679.4286	880.375	987.625	950.5	725.375
54	872.7143	901	779.2727	744.25	811.9	901	973	774.875
55	747	807.5	934	719.5	833.73334	920.8	929.2857	863.2857
56	917.5	934	836	782.2	808.6	925.2	967	723.8571
57	1000	1000	574.6667	872.125	846	923	901	950.5
58	943.4286	901	927.2857	882.1429	841.5	870.375	847.5833	888.625
59	1000	884.5	728.5714	940.6	971.7143	811.4286	893.38464	814.375
60	952.8571	983.5	870.7143	783.1429	672.375	853.875	741.5	950.5
61	934	917.5	883	849.6667	894.4	891.1	980.2	612.25
62	1000	1000	818.5	881.75	871.6667	879	1000	791
63	752.5	863.875	876.25	697.5	762.5	934	957.5714	766.25
64	868	884.5	877	682.375	807.5	939.5	960.4	925.75
65	841.6	901	868	881.2	813.4286	943.4286	962.875	854.2308
66	983.5	873.5	907	391	739.3	868	901	967
67	945	884.5	934	719.5	496.75	884.5	915.1429	895.5
68	1000	917.5	919.0833	962.875	899.7273	562	799.2	948.6667
69	914.2	901	981.1429	980.2	929.2857	811.4286	579.6923	862.125
70	971.7143	884.5	910.4286	886.8571	950.5	987.625	876.25	637
71	886.8571	967	808.7143	858.5714	925.75	958.75	851.5	513.25
72	967	895.5	810.25	985.8571	909.25	925.75	901	780
73	1000	910.9	802	884.5	934	947.2	967	805.6667
74	844.4286	892	923	868	939.5	895.5	901	773.125
75	891.1	783.1429	854.8	920.8	874.6	960.4	1000	884.5
76	835	716.2	830.875	774.5	760.75	913.375	915.1429	765.3333
77	750.1429	868	895.5	696.4	824	917.5	846	962.875
78	1000	868	844.4286	980.2	985.8571	985.8571	984.7692	936.375
79	1000	891.5714	943.4286	769	934	985.8571	943.4286	932.8333
80	983.5	1000	945	865.25	950.5	633.7	912	1000
81	975.25	898	974.0833	772.3	677.0833	883.3333	730.9231	992.38464
82	821.8	769	643.7143	920.8	971.7143	985.8571	984.7692	713
83	960.4	920.8	821.8	843.25	901	857	950.5	758
84	881.2	747	841.7143	722.8	830.2857	868	916.2308	675.875

	50	51	52	53	54	55	56	57
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								
16								
17								
18								
19								
20								
21								
22								
23								
24								
25								
26								
27								
28								
29								
30								
31								
32								
33								
34								
35								
36								
37								
38								
39								
40								
41								
42								
43								
44								
45								
46								
47								
48								
49	0							
50	861.4	0						
51	630.4	705.7143	0					

	50	51	52	53	54	55	56	57
53	769	869.5833	762.2857	0				
54	915.1429	785.7273	749.7273	628	0			
55	934	677.4286	815.1429	684.75	807.2	0		
56	692	870.7143	852.8571	595.75	805.3	710.86664	0	
57	868	938.125	835	824	796	967	864.3333	0
58	943.4286	967	985.8571	936.3571	943	938.125	962.875	739.6667
59	841.6	946.6923	858.5714	852.6667	866.2727	901	813.4286	925.75
60	971.7143	851.5	943.4286	879.7857	982	938.125	822.625	930.3333
61	851.5	673.8	716.7273	585.1667	738.375	751.1	787.4	868
62	901	917.5	805.3	901	488.5	860.6667	857	756.625
63	917.5	837.7143	874.5	562.1111	717.75	533.875	602.25	950.5
64	912	759.6	926.7273	557.6667	787.875	641.3333	635.8333	857
65	914.2	1000	802	925.75	888.625	841.7143	901	697.5
66	807.5	881.2	901	901	745.4286	854.8	749.2	860.6667
67	908.3333	896.2857	937.6667	872.125	784	888.625	843.25	954.3077
68	841.6	960.4	912	960.4	956	964	961	950.5
69	920.8	832.46155	985.8571	991.75	982	830.2857	896.2857	1000
70	646.4286	721.0833	696.2857	739.7143	656.7273	717.75	825	842.3333
71	533.2857	683.75	816.1429	800.6429	701.2727	909.25	853.25	989
72	743.3333	583.375	872.125	847.375	868	847.375	773.125	921.3077
73	901	901	565.5	901	862.5	815.2	782.2	824
74	943.4286	851.5	681	527	584.2	543.5	400.5	910.4286
75	841.6	901	861.4	730.5	542.7143	901	821.8	917.5
76	721.3333	767	849.75	694.1111	746.7273	519.75	697.125	860.6667
77	938.7143	917.5	983.5	642.5	551.2	604	653.5	938.7143
78	980.2	906.0769	868	956	841	967	1000	562.75
79	854.8	886.8571	943.4286	884.5	901	797.2857	957.5714	901
80	714	983.5	846	971.7143	943.4286	960.4	808.6	839.125
81	962.875	817.2308	958.75	901	917.5	958.75	928.5	937
82	1000	932.9231	903.7143	689.25	487	818.8571	816.1429	797.875
83	901	983.5	683.2	752.5	628.75	919.3333	915.6667	814.375
84	901	714.61536	715.1429	749.75	649	587.8571	688.8571	888.625

	58	59	60	61	62	63	64	65
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								
16								
17								
18								
19								
20								
21								
22								
23								
24								
25								
26								
27								
28								
29								
30								
31								
32								
33								
34								
35								
36								
37								
38								
39								
40								
41								
42								
43								
44								
45								
46								
47								
48								
49								
50								
51								

	58	59	60	61	62	63	64	65
53								
54								
55								
56								
57								
58	0							
59	868	0						
60	918.8571	906.5	0					
61	771.3333	920.8	884.5	0				
62	714	851.5	961.5	717.6667	0			
63	945	924.5714	941.3333	709.5	861.4	0		
64	787.8333	894.4	879	637	825.5714	668.44446	0	
65	925.75	837.375	987.625	839.125	758	781.1667	835	0
66	815.2	901	716.2	890	815.75	960.4	877.4286	846
67	987.625	957.5714	715.375	840.5	913.375	802	788.8	983.5
68	918	696.4	832.2	937	925.75	941.3333	947.75	934
69	934	925.3077	695.9167	920.8	1000	882.1429	874.6	948.75
70	850.1429	950.5	886.8571	532.5	884.5	932.44446	769	950.5
71	971.7143	840.5	893.9286	813	884.5	769	785.5	911.625
72	1000	987.625	863.875	881.2	925.75	835	736	895.5
73	796.5	615	967	703	642.5	838.6667	697.5	820.3333
74	813	752.5	967	494	630.4	632.875	675.5	863.875
75	983.5	917.5	983.5	637	697.5	742.6	617.2	829.5
76	869.55554	910.4286	908.3333	657.625	795.4	569.6	519.6667	787.3333
77	967	884.5	928.5	747	769	719.5	642.5	896.875
78	824	877.0769	958.75	881.2	835	985.8571	1000	800.25
79	593.375	800	962.875	851.1429	808.6	958.75	758.375	792.5833
80	915.1429	384	858.5714	863.875	909.25	980.2	929.875	821.8
81	949.5	913.6923	644.0714	970.3	960.4	908.61536	934	944.1539
82	906.5	818.3077	925.75	627.6	598.5	771.7143	858.6	839.125
83	901	846	983.5	655.3333	619.2308	729.4	773.7143	571
84	958.75	800.53845	925.75	680.4	736	682.1429	799.2	872.125

	66	67	68	69	70	71	72	73
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								
16								
17								
18								
19								
20								
21								
22								
23								
24								
25								
26								
27								
28								
29								
30								
31								
32								
33								
34								
35								
36								
37								
38								
39								
40								
41								
42								
43								
44								
45								
46								
47								
48								
49								
50								
51								

	66	67	68	69	70	71	72	73
53								
54								
55								
56								
57								
58								
59								
60								
61								
62								
63								
64								
65								
66	0							
67	824	0						
68	946.375	730.5	0					
69	821.8	886.8571	914.2	0				
70	940.6	946.375	1000	901	0			
71	821.8	913.375	960.4	897.0833	637	0		
72	908.3333	857	940.6	925.75	698.875	641.125	0	
73	829.5	983.5	877.4286	917.5	851.5	840.5	795.4	0
74	736	851.5	967	950.5	846	950.5	785.5	600.7
75	844.4286	901	841.6	1000	884.5	884.5	881.2	868
76	854.8	871.6667	886.3333	882.1429	758	769	818.5	846
77	714	675.5	862.5	928.5	901	934	912	897.7
78	980.2	839.7143	980.2	976.0769	879	891.5833	896.875	796.5
79	782.2	874.6	934	955.5714	840.875	936.375	841.6	901
80	763	844.4286	457.85715	917.5	915.1429	971.7143	987.625	775.6
81	930.7	811.9	846.8	744.6923	957.5714	950.5	930.7	973
82	841.6	896.2857	980.2	961.9231	743.0833	767.4167	822.625	829.5
83	909.25	913.375	905.125	978	967	796.5	863.875	642.5
84	881.2	858.5714	940.6	913.6923	734.8333	775.6667	896.875	835

	74	75	76	77	78	79	80	81
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								
16								
17								
18								
19								
20								
21								
22								
23								
24								
25								
26								
27								
28								
29								
30								
31								
32								
33								
34								
35								
36								
37								
38								
39								
40								
41								
42								
43								
44								
45								
46								
47								
48								
49								
50								
51								

	74	75	76	77	78	79	80	81
53								
54								
55								
56								
57								
58								
59								
60								
61								
62								
63								
64								
65								
66								
67								
68								
69								
70								
71								
72								
73								
74	0							
75	773.7143	0						
76	689.8	612.25	0					
77	700	787.8571	604	0				
78	813	884.5	985.8571	983.5	0			
79	891.5714	960.4	781.55554	721.8571	743.4286	0		
80	901	917.5	920.8	874.6	983.5	894.4	0	
81	973	756.625	917.5	901	977.1539	958.75	848.2	0
82	692	901	856.5714	769	903.53845	943.4286	934	946.6923
83	696.4	785.5	782.2	901	884.5	802	884.5	901
84	703	697.5	625.5714	785.5	939.0769	971.7143	802	817.2308

	82	83	84
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			
21			
22			
23			
24			
25			
26			
27			
28			
29			
30			
31			
32			
33			
34			
35			
36			
37			
38			
39			
40			
41			
42			
43			
44			
45			
46			
47			
48			
49			
50			
51			

	82	83	84
53			
54			
55			
56			
57			
58			
59			
60			
61			
62			
63			
64			
65			
66			
67			
68			
69			
70			
71			
72			
73			
74			
75			
76			
77			
78			
79			
80			
81			
82	0		
83	598.5	0	
84	748.6923	813	0

Appendix B: Individual MDS Plots

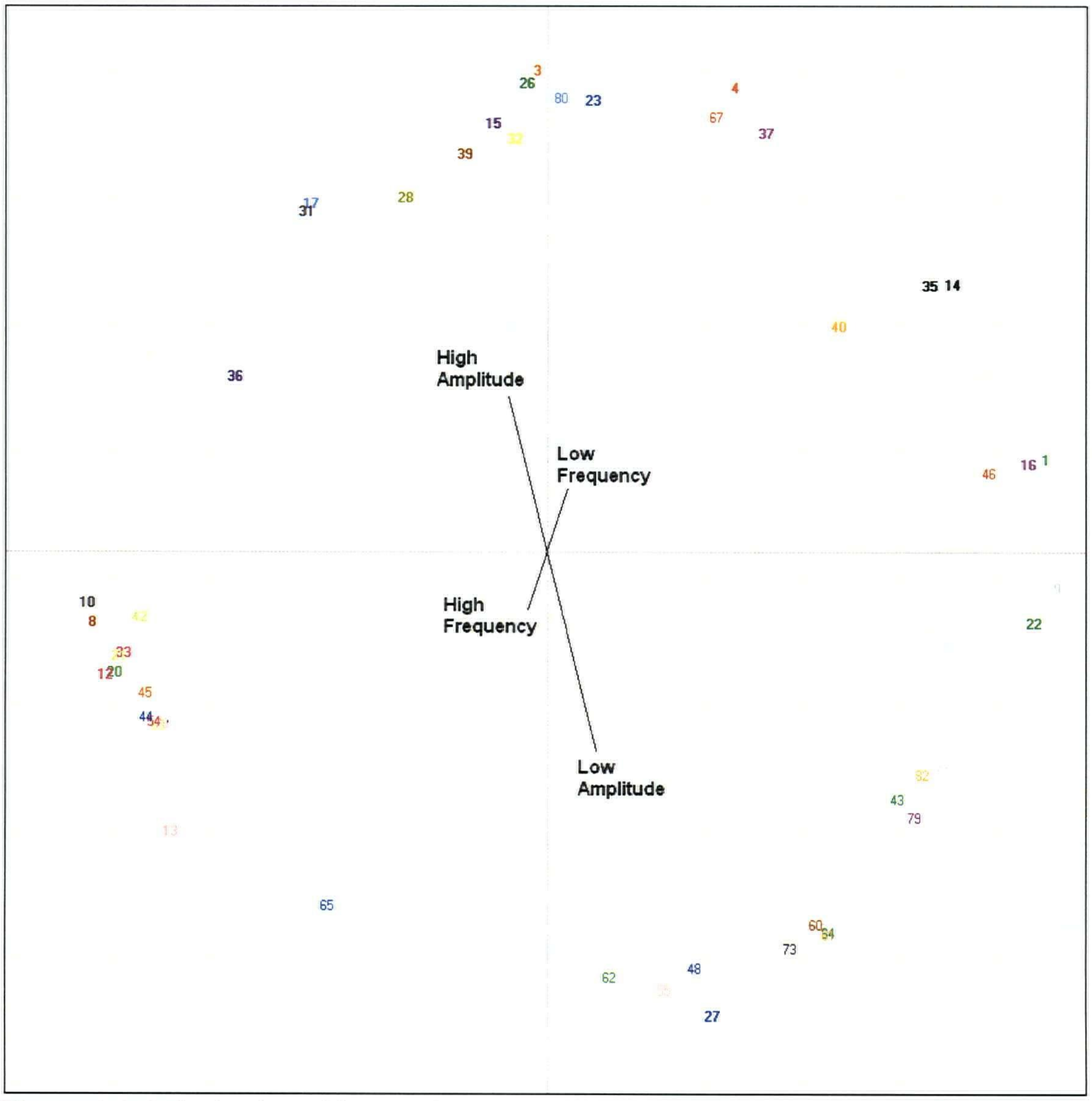

Figure 1. MDS plot for subject 1 of subset study. Axes labeled accordingly. Stimuli numbers as in Table 3.4. Colours are to help distinguish between rhythms. All proceeding figures are presented in the same manner, for the next 14 subjects.

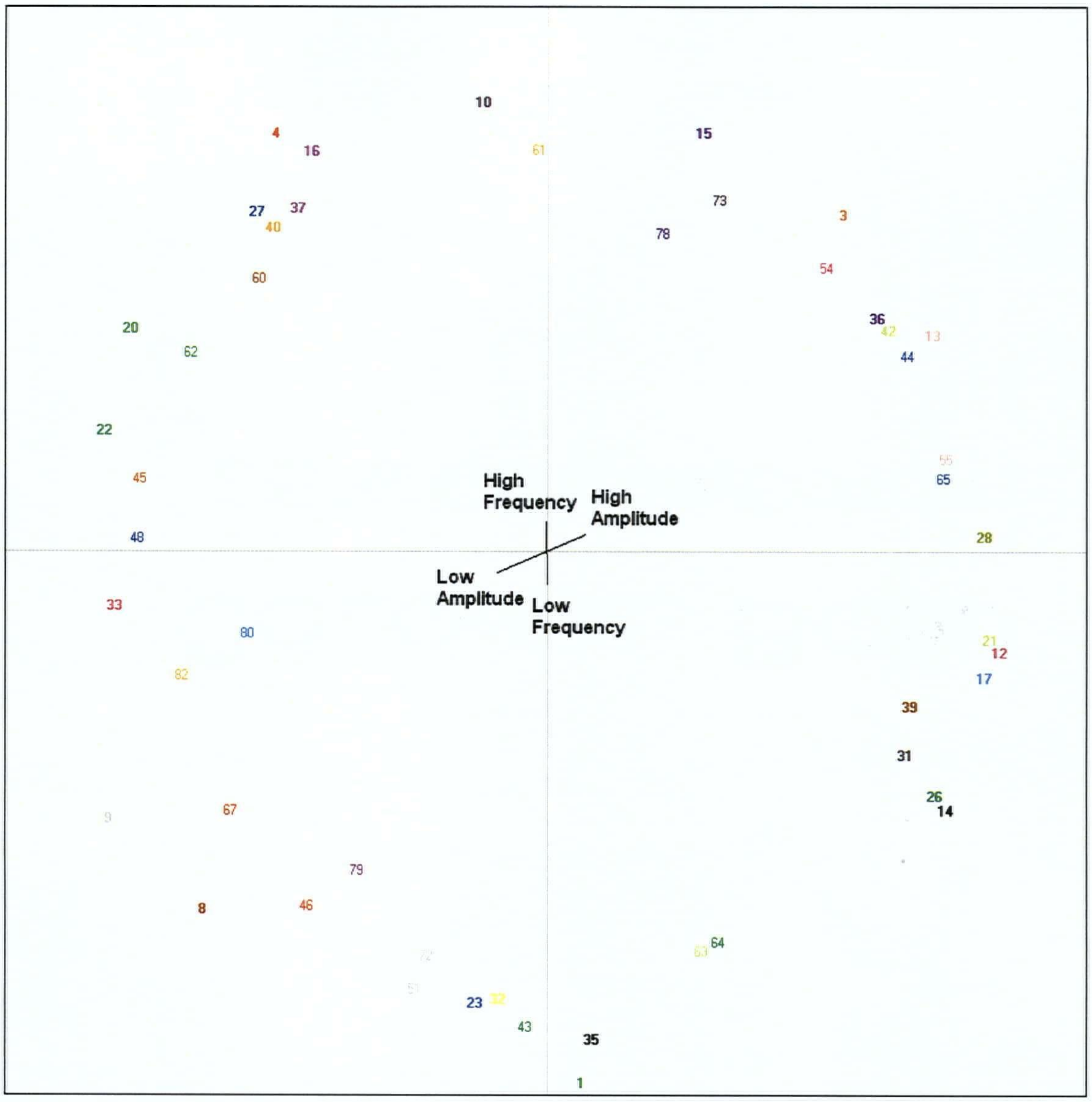

Figure 2. MDS plot for subject 2 of subset study.

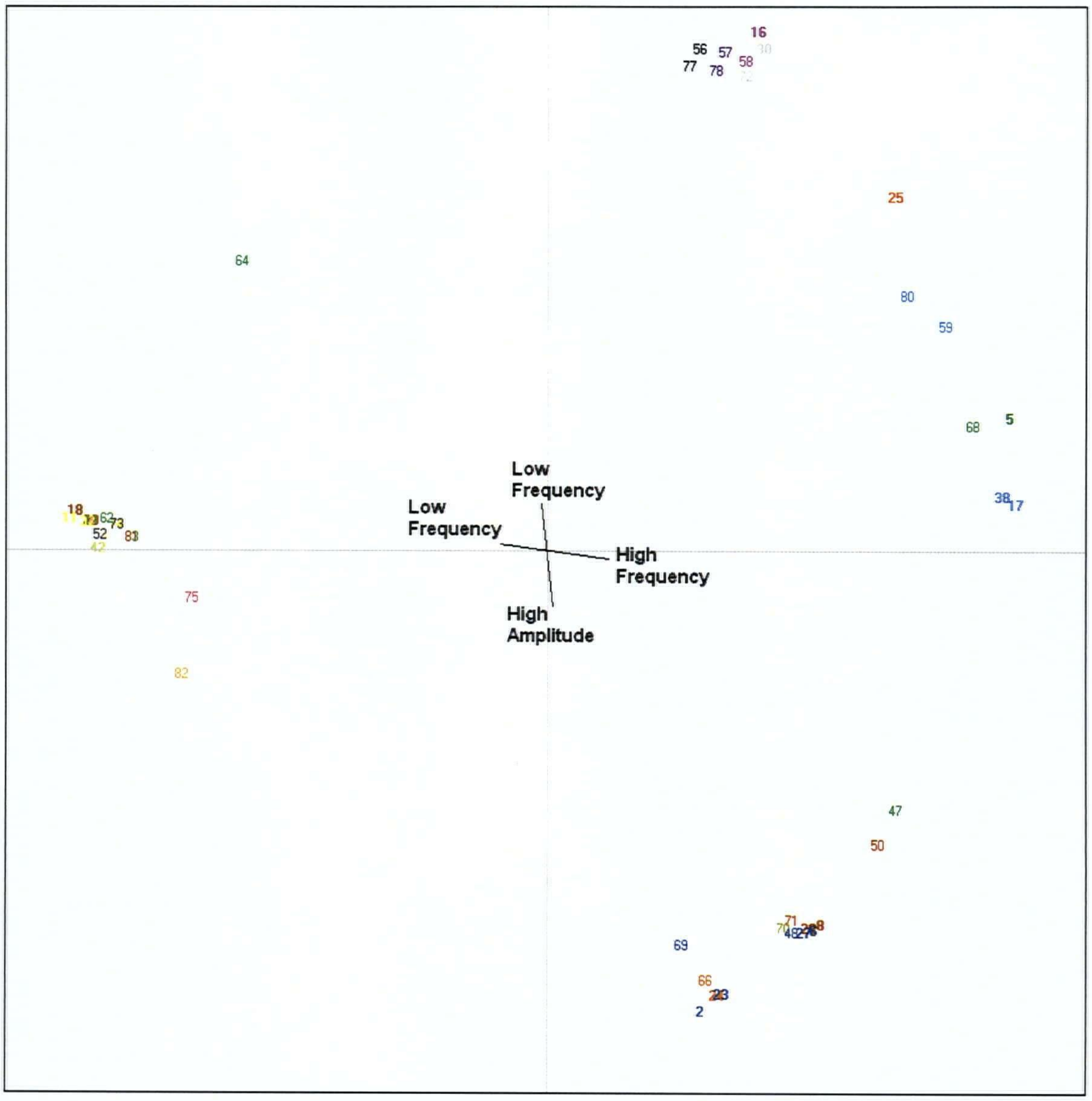

Figure 3. MDS plot for subject 3 of subset study.

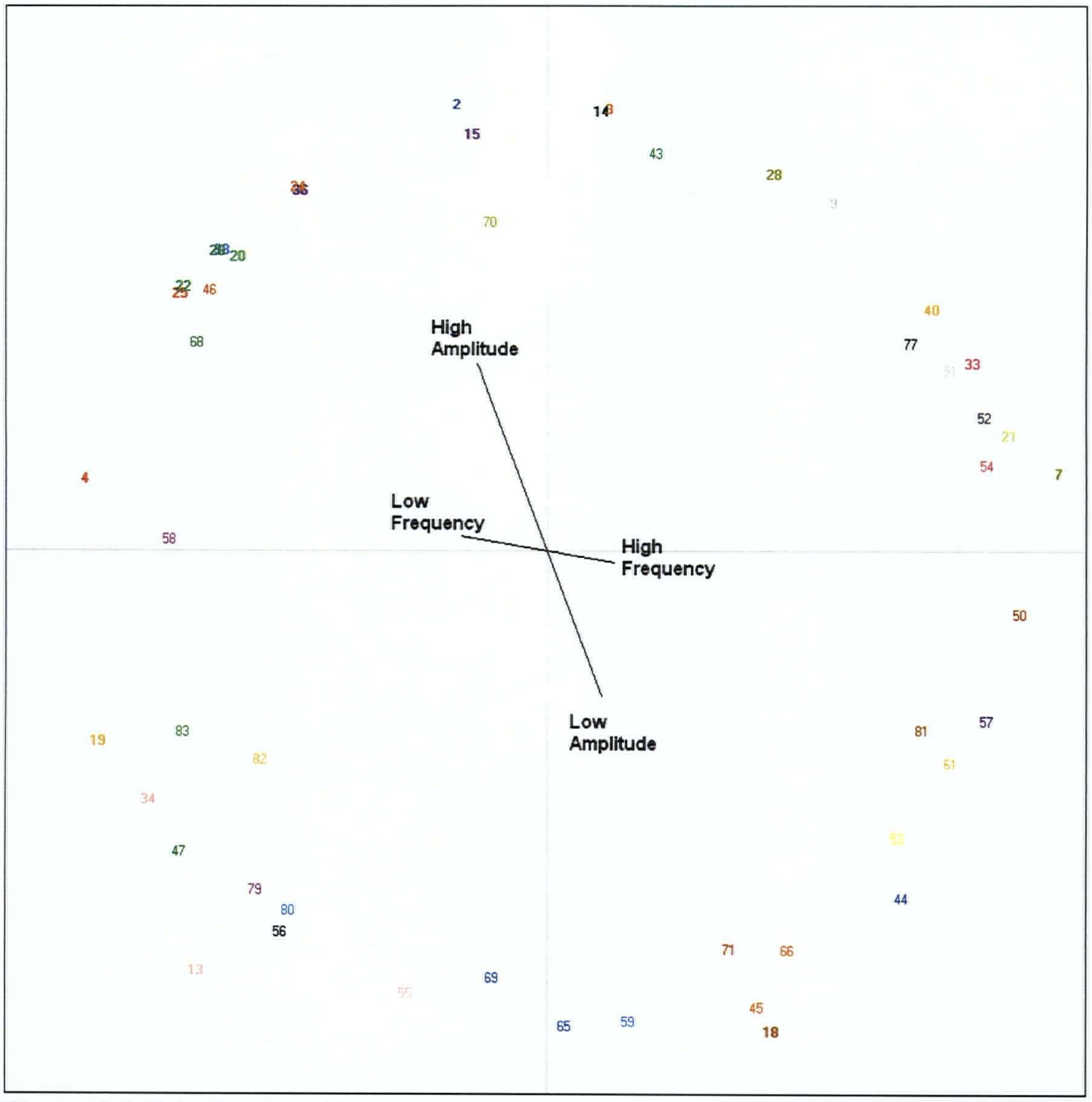

Figure 4. MDS plot for subject 4 of subset study.

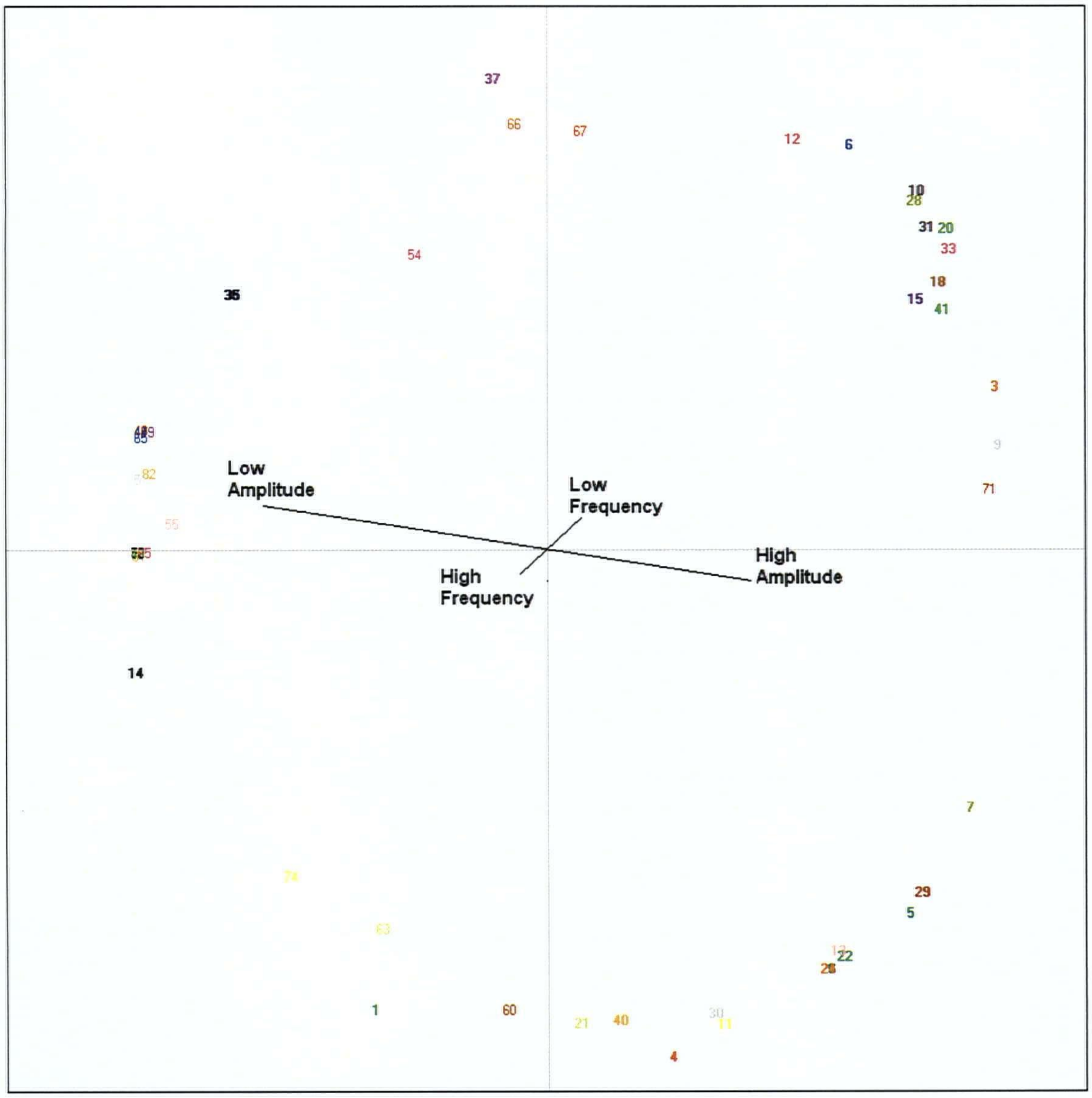

Figure 5. MDS plot for subject 5 of subset study.

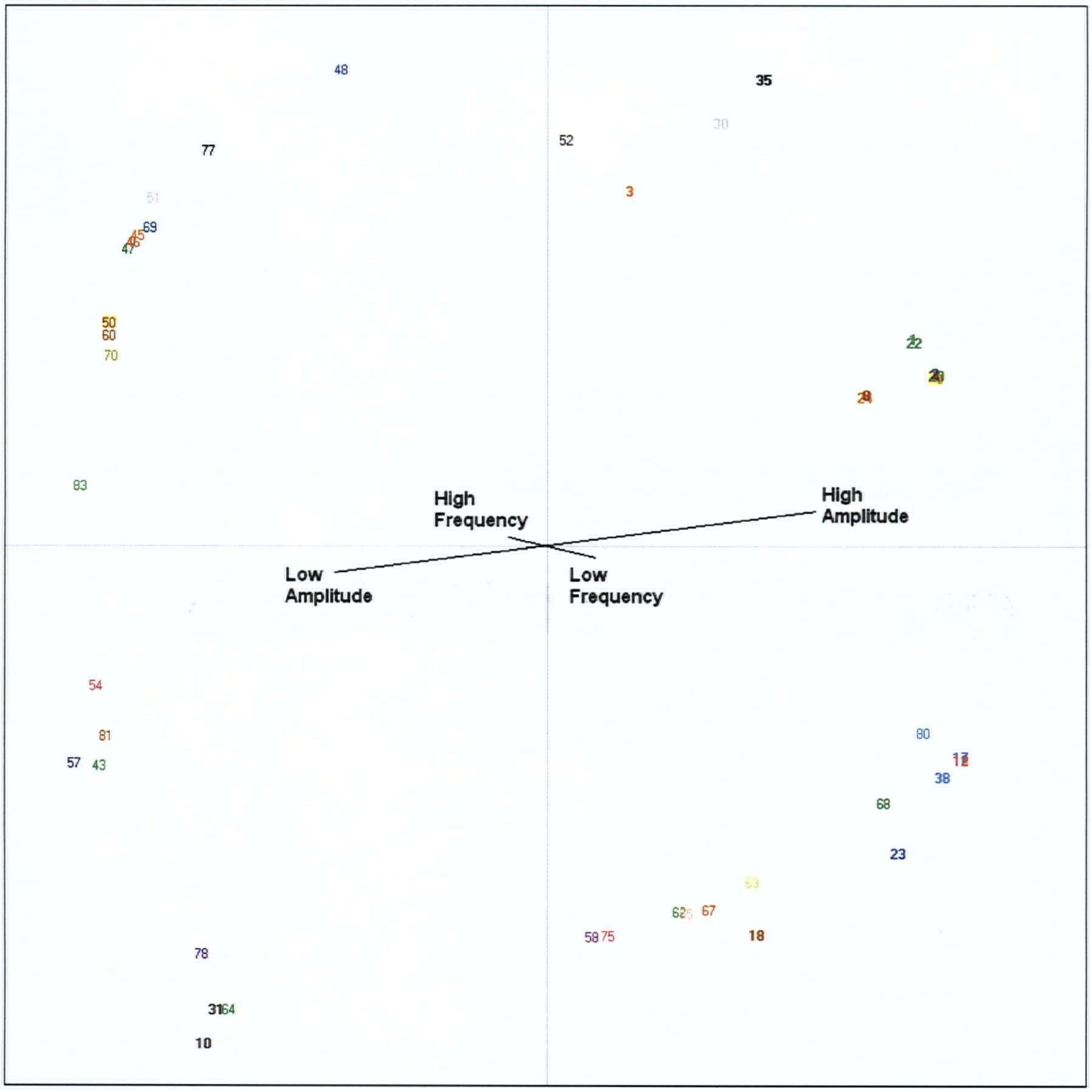

Figure 6. MDS plot for subject 6 of subset study.

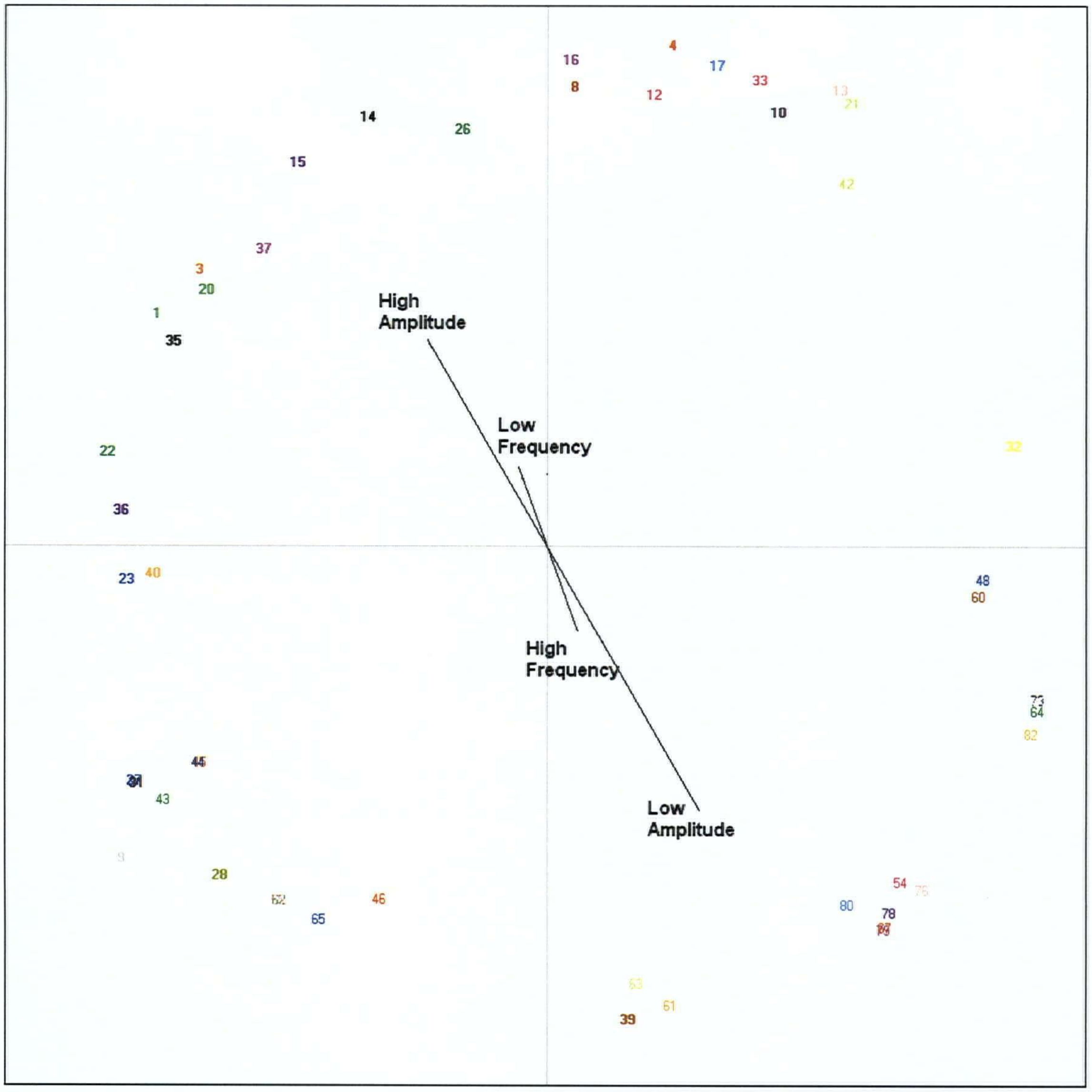

Figure 7. MDS plot for subject 7 of subset study.

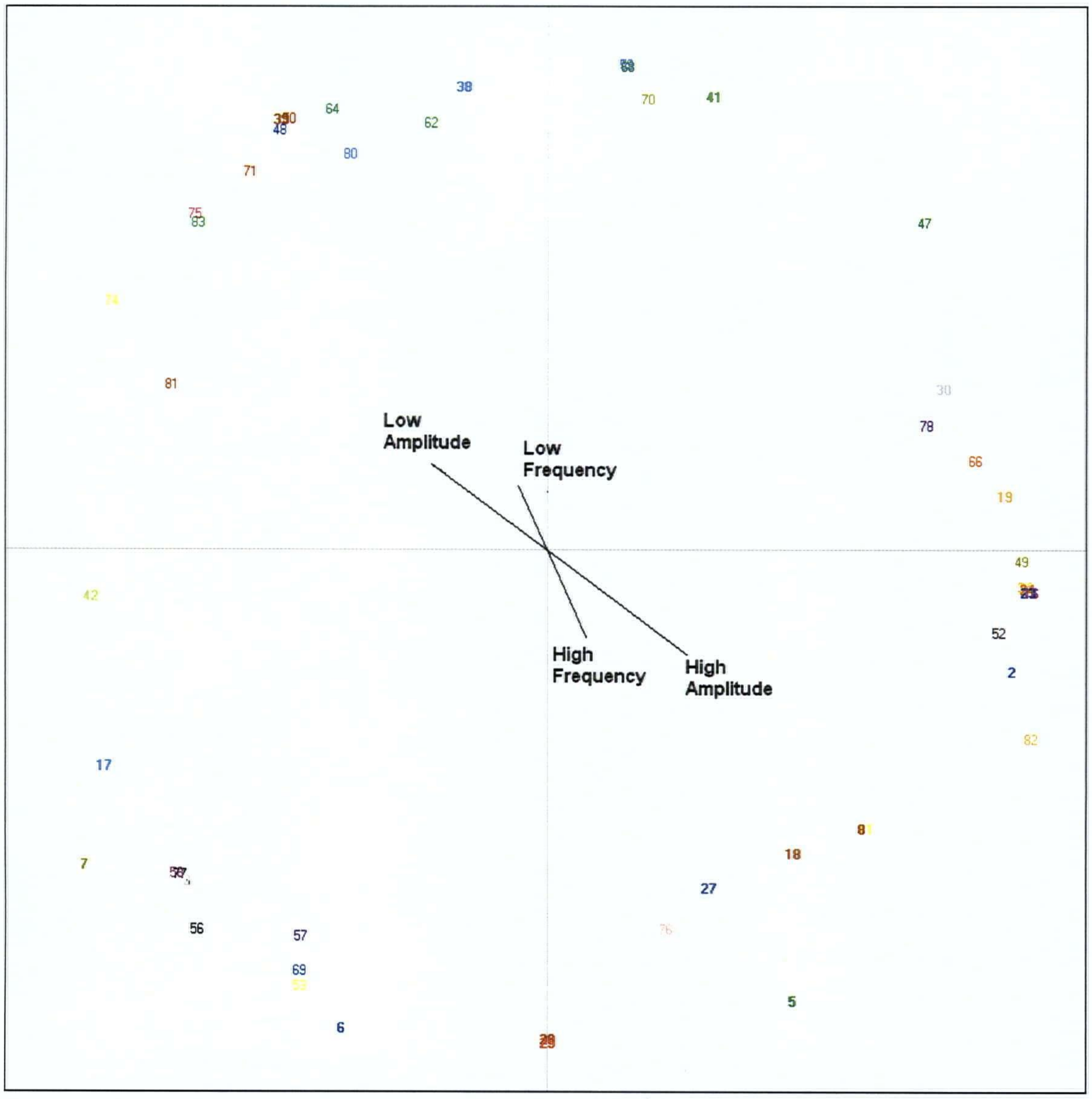

Figure 8. MDS plot for subject 8 of subset study.

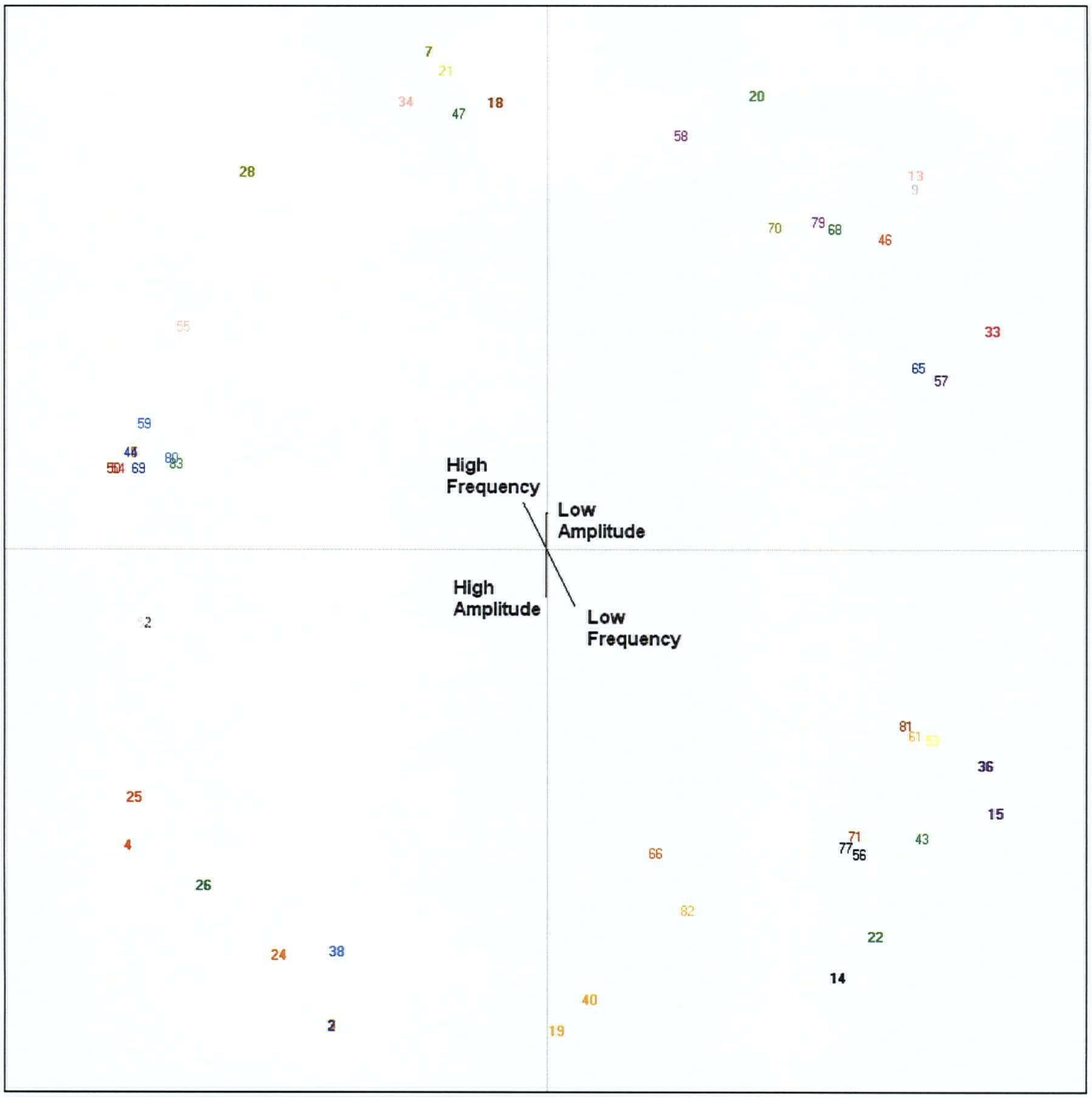

Figure 9. MDS plot for subject 9 of subset study.

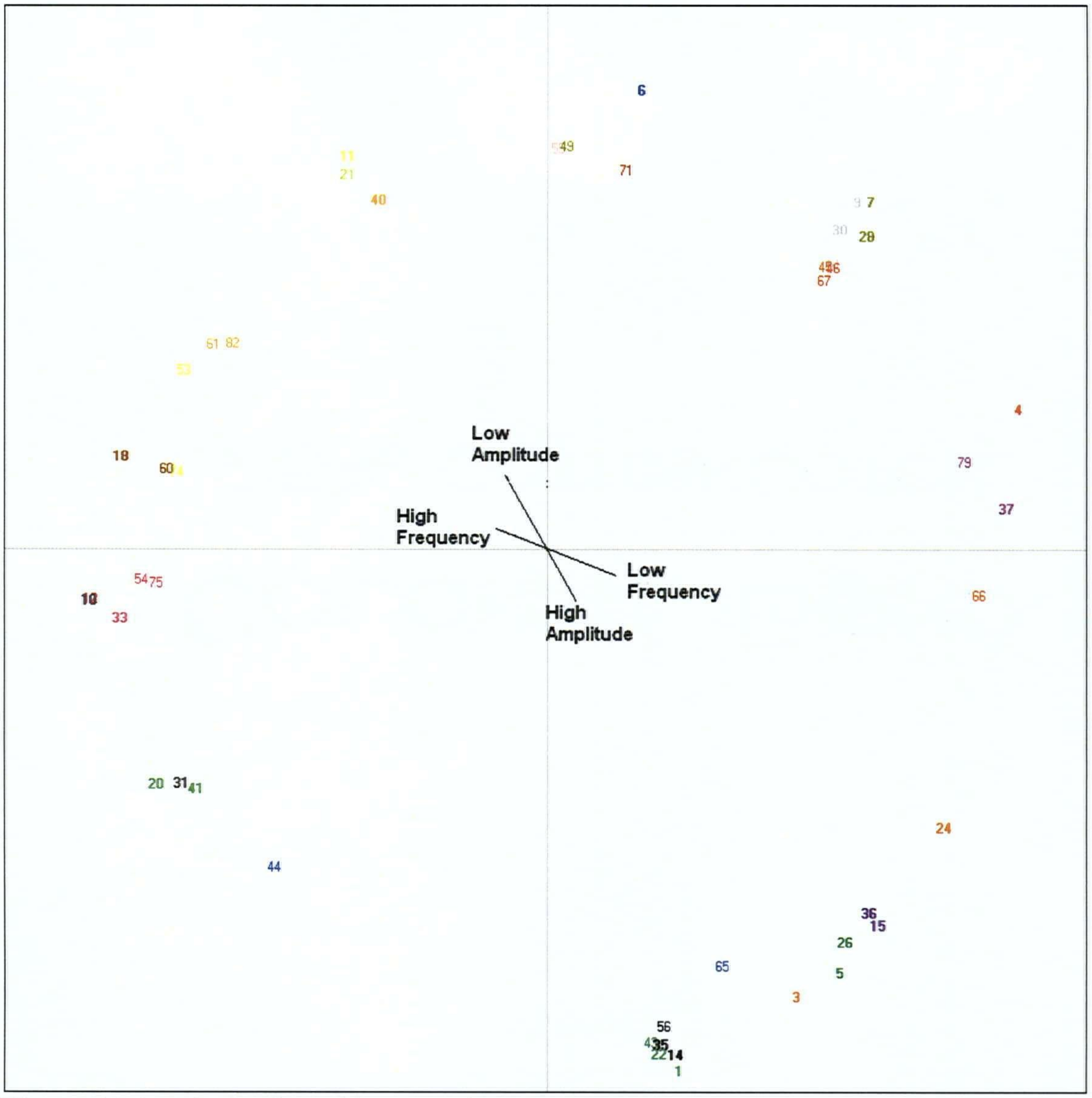

Figure 10. MDS plot for subject 10 of subset study.

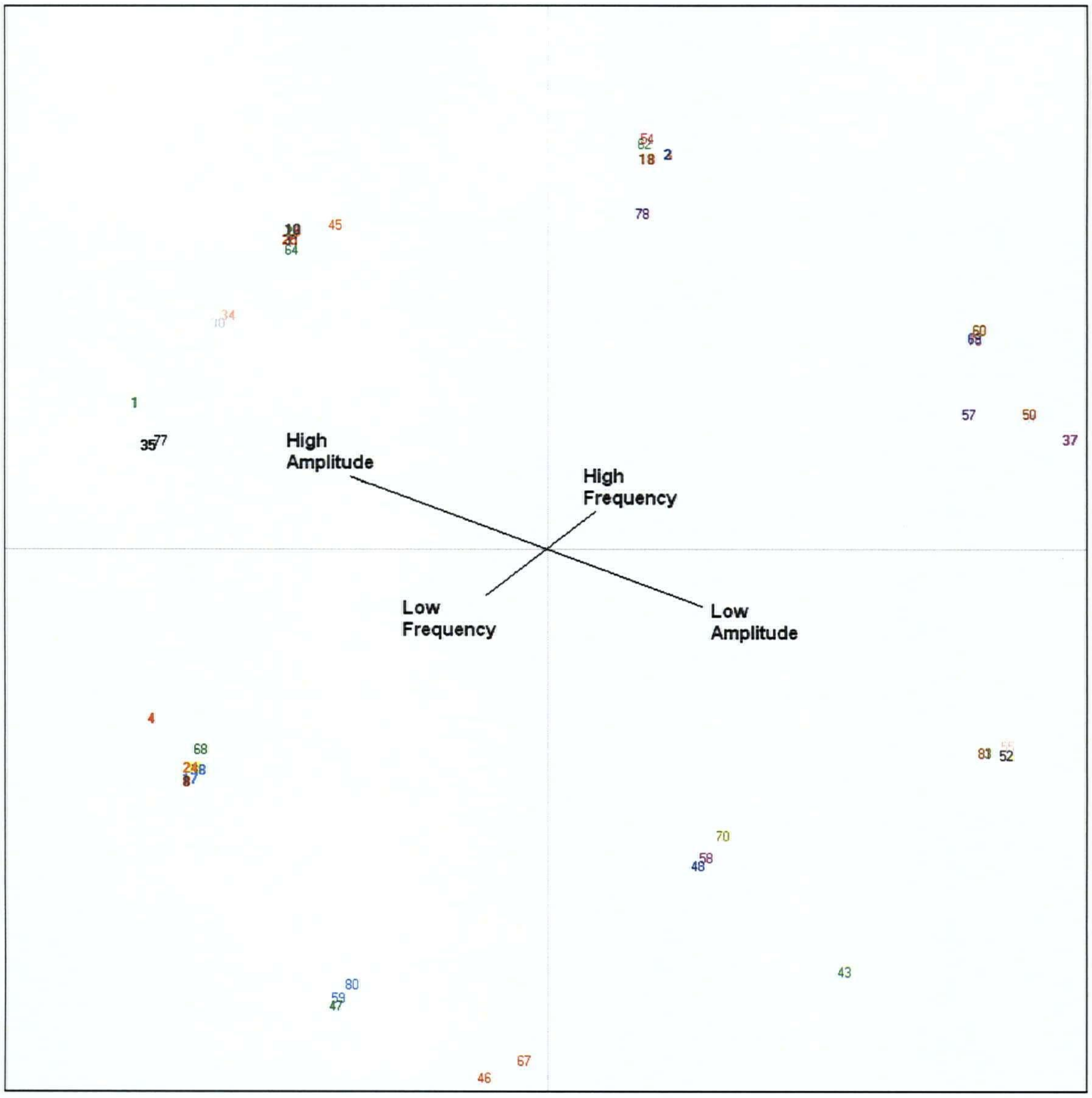

Figure 11. MDS plot for subject 11 of subset study.

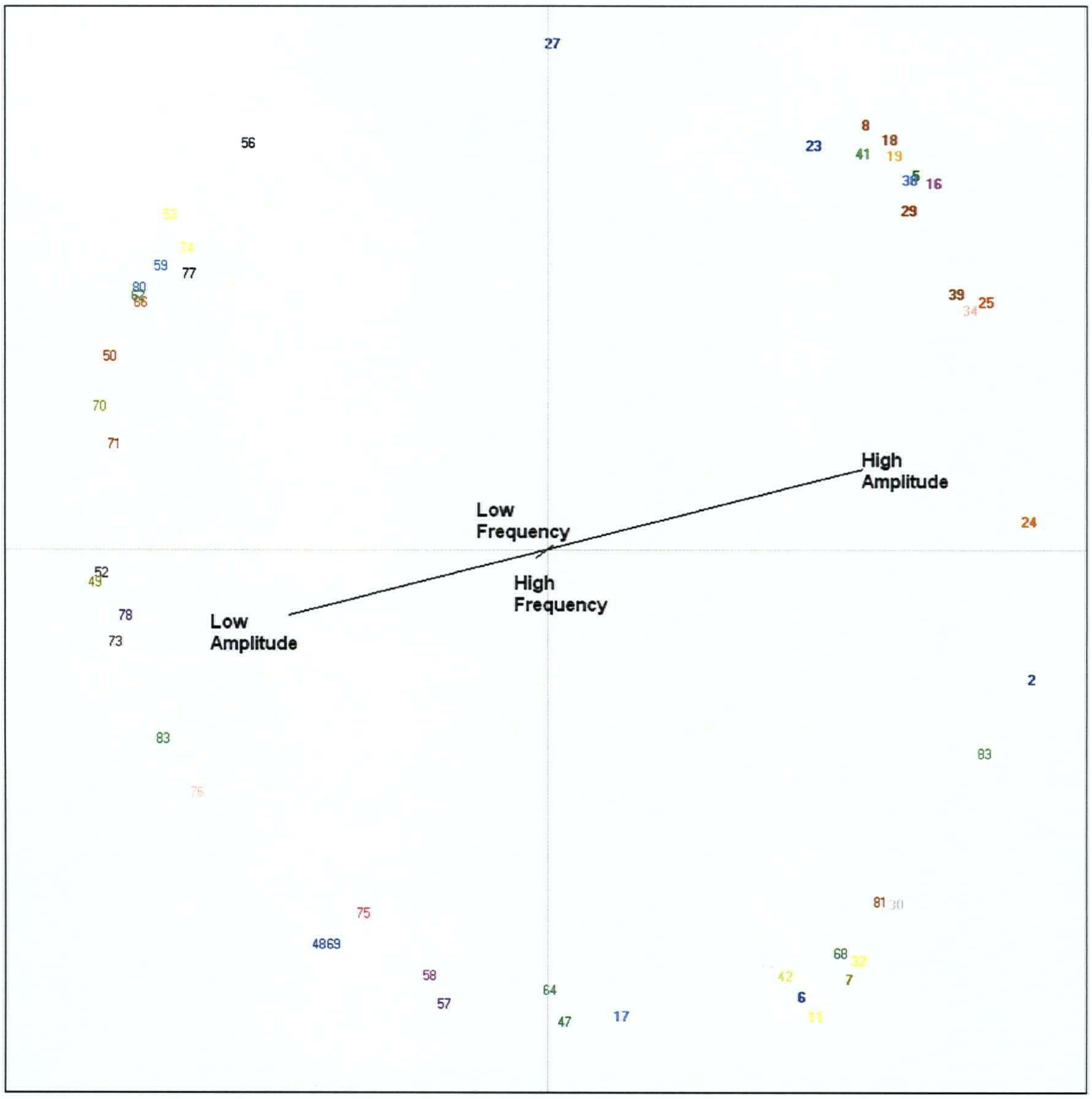

Figure 12. MDS plot for subject 12 of subset study.

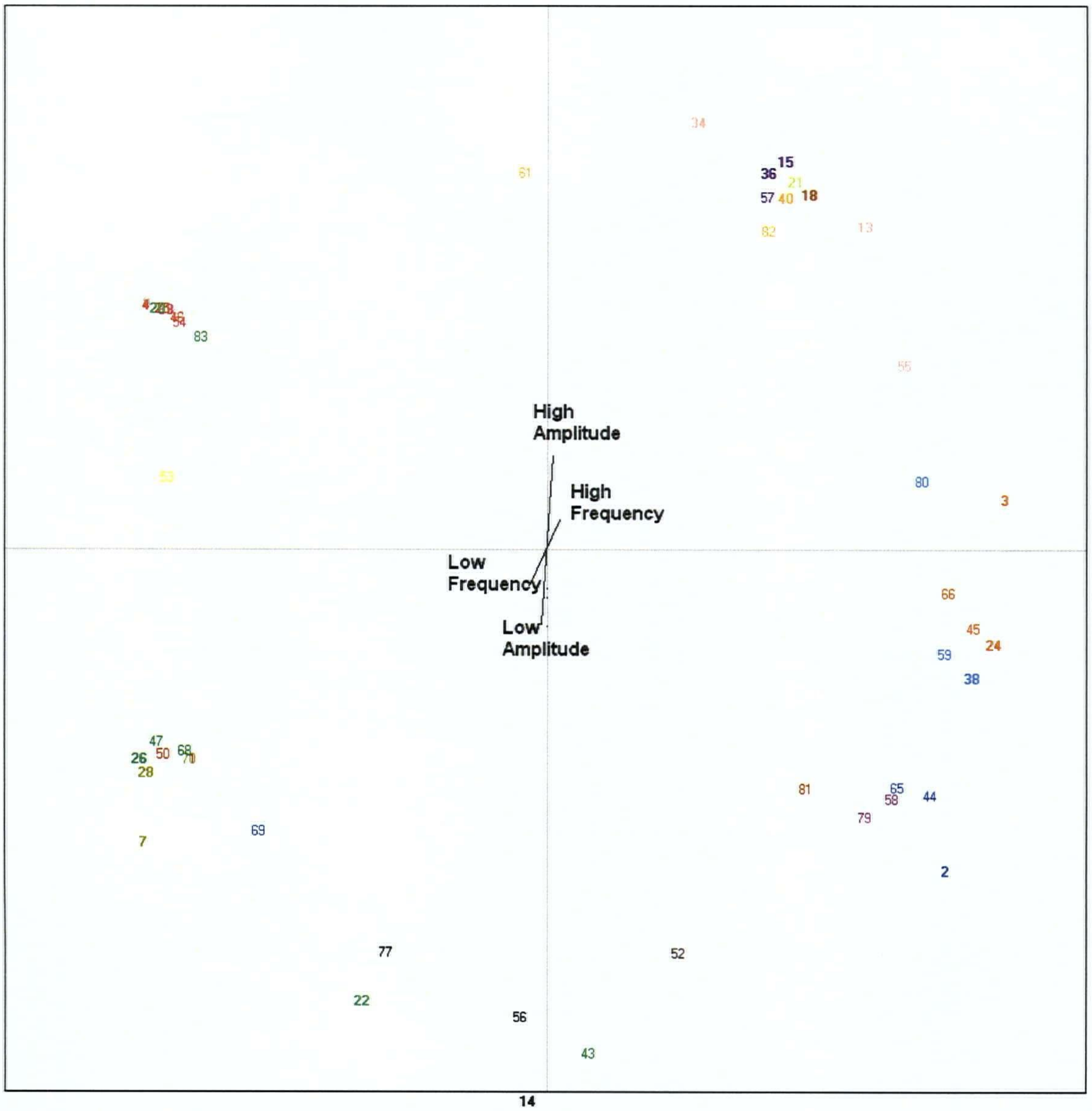

Figure 13. MDS plot for subject 13 of subset study.

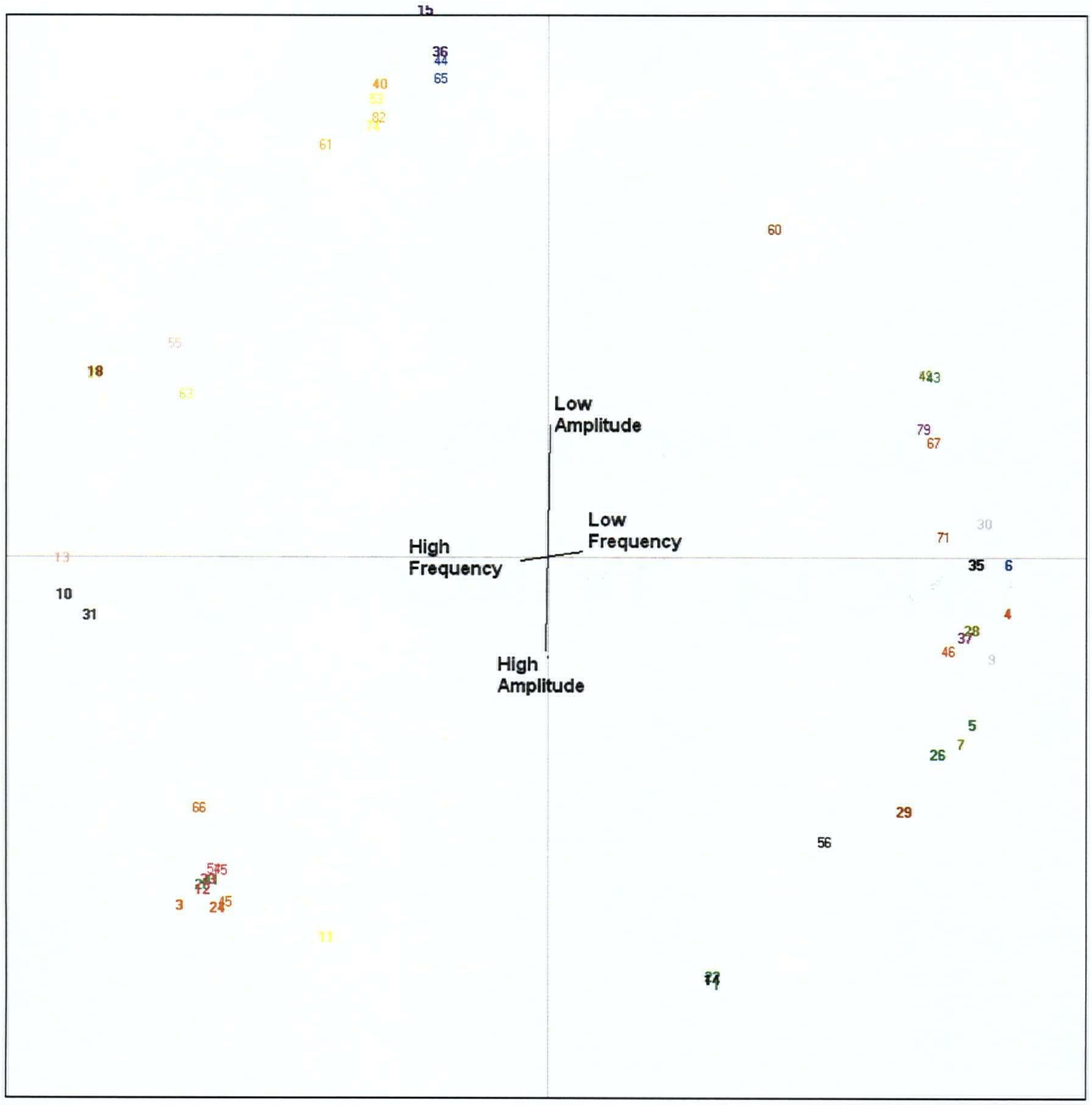

Figure 14. MDS plot for subject 14 of subset study.

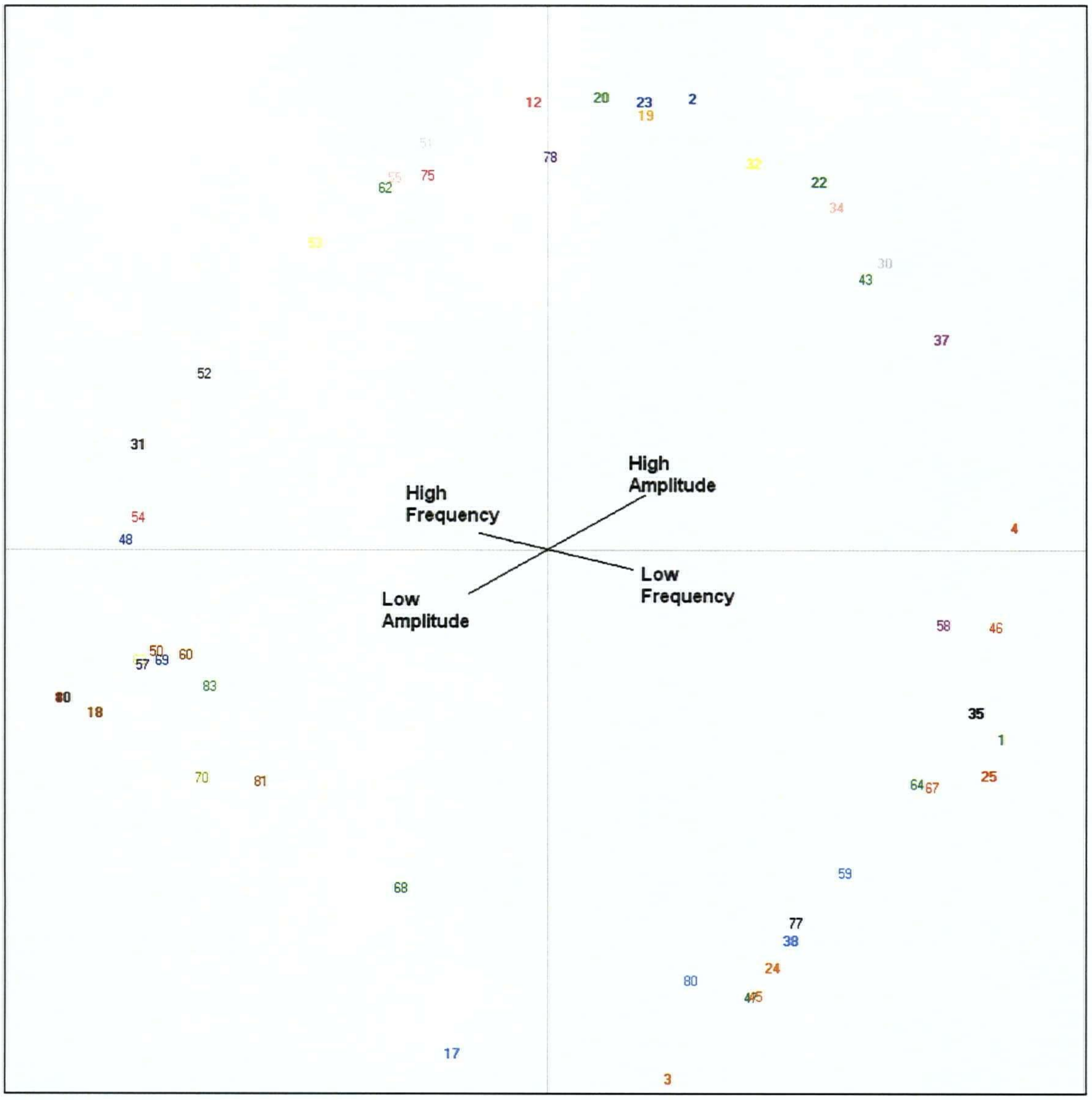

Figure 15. MDS plot for subject 15 of subset study.

Appendix C: Subsets

Table 1. Subsets Used in First Part of Subset Study. Listed are the numbers of all the stimuli used in each of the 5 subsets given to participants in the study described in Section 7.2.1

Subset 1	Subset 2	Subset 3	Subset 4	Subset 5
1	2	2	1	1
3	5	3	3	2
4	6	4	4	3
8	7	7	5	4
9	8	9	6	8
10	11	13	7	10
12	16	14	9	12
13	17	15	10	17
14	18	18	11	18
15	19	19	12	19
16	23	20	13	20
17	24	21	14	22
20	25	22	15	23
21	27	24	18	24
22	29	25	20	25
23	30	26	21	30
26	32	28	22	31
27	34	33	24	32
28	38	34	26	34
31	39	36	28	35
32	41	38	29	37
33	42	40	30	38
35	47	43	31	43
36	48	44	33	45
37	49	45	35	46
39	50	46	36	47
40	52	47	37	48
42	53	50	40	50
43	56	51	41	51
44	57	52	43	52
45	58	53	44	53
46	59	54	45	54
48	62	55	46	55
51	64	56	49	57
54	66	57	51	58
55	68	58	53	59
60	69	59	54	60
61	70	61	55	62
62	71	65	56	63
63	72	66	60	64
64	73	68	61	67
65	74	69	63	68

67	75	70	65	69
72	76	71	66	70
73	77	77	67	75
76	78	79	71	77
78	80	80	74	78
79	81	81	75	80
80	83	82	79	81
82	84	83	82	83

Table 2. Subsets Used in Second Part of Subset Study. Listed are the numbers of all the stimuli used in each of the 7 additional subsets given to participants in the second part of the subset study described in Section 7.2.4

Subset 6	Subset 7	Subset 8	Subset 9	Subset 10	Subset 11	Subset 12
2	1	1	2	1	5	1
5	3	2	6	2	6	3
6	4	8	7	3	7	4
9	5	9	8	4	8	5
11	6	10	9	5	9	7
12	7	13	11	6	11	9
13	8	15	12	10	12	10
14	10	17	14	11	13	11
15	11	19	16	15	14	13
16	14	21	18	16	15	16
19	16	23	21	18	17	17
21	17	24	22	19	18	18
25	20	25	23	20	19	19
26	22	26	24	25	21	20
27	23	27	25	26	22	24
28	27	28	27	28	23	25
29	29	30	29	29	26	27
33	30	31	30	31	28	31
34	31	32	33	34	29	36
36	32	34	34	38	30	37
38	33	35	35	39	32	38
39	35	37	36	40	33	39
40	36	38	37	41	35	41
41	37	40	39	42	36	42
42	39	44	42	45	37	43
44	41	47	43	46	38	44
47	42	48	47	47	40	45
49	43	50	48	49	41	46
50	45	52	50	50	44	49
52	46	53	51	52	48	51
57	48	57	53	54	49	52
58	49	58	56	55	52	53
59	51	59	58	56	56	54
60	54	61	60	57	57	55
61	55	62	61	58	59	56
65	56	63	64	59	60	57

68	62	64	65	63	61	59
69	63	67	66	68	62	60
70	64	68	67	69	64	62
72	65	69	68	70	65	63
73	66	70	71	71	66	66
74	67	72	72	73	69	67
76	71	75	73	74	70	69
77	72	77	74	76	71	70
79	73	78	76	77	74	72
80	74	79	77	79	75	73
81	75	80	80	80	78	75
82	76	81	81	81	79	76
83	78	82	82	83	80	80
84	80	83	83	84	82	84

You hereby CONSENT to participate in this study and acknowledge RECEIPT of a copy of the consent form:

NAME ____________________________________
(please print)

SIGNATURE ________________________________ DATE ____________________

If you have any concerns regarding your treatment as a research subject you may contact the Research Subject Information Line in the UBC Office of Research Services at 604-822-8598.

You hereby CONSENT to participate in this study and acknowledge RECEIPT of a copy of the consent form:

NAME ____________________________________
(please print)

SIGNATURE ______________________________ DATE ___________________

If you have any concerns regarding your treatment as a research subject you may contact the Research Subject Information Line in the UBC Office of Research Services at 604-822-8598.

You hereby CONSENT to participate in this study and acknowledge RECEIPT of a copy of the consent form:

NAME ________________________________
(please print)

SIGNATURE ____________________________ DATE ________________

If you have any concerns regarding your treatment as a research subject you may contact the Research Subject Information Line in the UBC Office of Research Services at 604-822-8598.

Figure 4. Ethics Approval Form

The University of British Columbia
Office of Research Services
Behavioural Research Ethics Board
Suite 102, 6190 Agronomy Road, Vancouver, B.C. V6T 1Z3

CERTIFICATE OF APPROVAL- MINIMAL RISK RENEWAL

PRINCIPAL INVESTIGATOR:	DEPARTMENT:	UBC BREB NUMBER:
Karon E. MacLean	UBC/Science/Computer Science	H01-80470

INSTITUTION(S) WHERE RESEARCH WILL BE CARRIED OUT:

Institution	Site
UBC	Point Grey Site

Other locations where the research will be conducted:
N/A

CO-INVESTIGATOR(S):

Ricardo Pedrosa
Colin Swindells
Susan Gerofsky
Noorin Fazal
David Ternes
Matt Savage-LeBeau
Mario Enriquez
Steve Yohanan

SPONSORING AGENCIES:

Innovation and Science Council of British Columbia - "Physical and multimodal user interfaces - usability & psychophysics"
Natural Sciences and Engineering Research Council of Canada (NSERC) - "The design of multi-modal symbolic information displays" - "Orsil title - Physical user interfaces: Communication of information and affect"
Various Sources

PROJECT TITLE:
Orsil title - Physical user interfaces: Communication of information and affect

EXPIRY DATE OF THIS APPROVAL: June 14, 2008

APPROVAL DATE: June 14, 2007

The Annual Renewal for Study have been reviewed and the procedures were found to be acceptable on ethical grounds for research involving human subjects.

Approval is issued on behalf of the Behavioural Research Ethics Board