'BEING THE GRAPH':USING HAPTIC AND KINESTHETIC INTERFACES TO ENGAGE STUDENTS LEARNING ABOUT FUNCTIONS

Susan Gerofsky, University of British Columbia

Matt Savage, University of British Columbia

Karon Maclean, University of British Columbia

This paper reports on a multidisciplinary collaborative project at UBC developing haptic/kinesthetic HCI for mathematics learning. It allows students to design landscapes using functions, and then to 'ride over' these landscapes via a force-feedback haptic device and, in future, with a programmed exercise bicycle. The project's rationale comes from the following hypothesis arising from Gerofsky's (2008) empirical research on gesture in the pedagogy of functions: That the 'top' (most capable and imaginative) mathematics students engage with graphs of mathematical functions in tactile/kinesthetic modes, rather than an exclusively visual mode.

INTRODUCTION

The study of mathematical functions is at the heart of most secondary school mathematics curricula worldwide, particularly when these curricula are designed as pre-calculus courses. Typically (and traditionally), the teaching of functions starts with word problems that generate tables of values, equations, and finally graphs that describe the function. Mathematical functions can be represented in multiple ways – as stories, tables of values, equations, graphs – but in school mathematics, words, numbers and algebraic equations take primary place, with graphic visuals treated as an endpoint, result or offshoot of the algebra.

As students advance in their study of functions, however, the emphasis shifts to a consideration of the graphs of functions *as* visual shapes with particular properties. Students' attention is meant to shift to salient graphic features of these shapes – their maximum and minimum points, the places they cross the *x*- and *y*- axes, their symmetries, the regions of the graphic plane they do or do not touch, etc. From these graphic features, one can 'read off' important information about the function. It is at this point that the graphs of functions take on a much more important role, and it is generally assumed that students will quickly become competent at 'reading' salient visual features of mathematical graphs through demonstration and verbal instruction.

Not all students, however, are able to make this leap from the algebraic to visual interpretation. In fact, many are left behind at an earlier stage, failing to cope with the abstraction of the algebraic equations that are initially given primary place when functions are introduced. Much recent research has addressed the need for an

2009. In Bardini,, C. Fortin, P. Oldknow, A. & Vagost D. (Eds.). *Proceedings of the 9th International Conference on Technology* ***in Mathematics Teaching,*** pp. XXX. Metz, France: ICTMT 9

embodied approach to mathematics learning, as groundwork for the subsequent generalization and abstraction (for example, see Nemirovsky 2003; Nuñez, Edwards & Matos, 1999; Tall 2003, 2004; Tall & Thomas, 1991). 'Embodiment' in this context refers both to actual physical movement and to imaginative engagement with technological representations of physical movement (through computer animations, for example). It has been theorized that, without embodied, perceptual referents, it is impossible for learners to (literally) 'make sense of' abstract formulations. Without a repertoire of sensory experiences as a basis, algebraic and even graphic abstractions can be learned by rote, but not understood on a deeper level. The stage where many students drop out of pre-calculus mathematics courses is precisely that point where the abstractions of algebra are introduced without a grounding in embodied experience (Ma 2001; Ma & Willms 1999).

The authors are in the early stages of a new project to introduce functions in school mathematics classes starting from a direct interpretation of graphs with a basis in embodied ways of knowing (through gesture and movement), and referring back to sensory experiences of graphs through haptic, design-based computer applications and interfaces. A key metaphor we use is the idea of 'reading the graph with your body' – that is, developing a repertoire of sensory, multimodal experiences through movement and engagement with technology that will help learners make sense of salient mathematical features of graphs in terms of their embodied experiences.

'Reading the graph with your body': Findings from the Graphs & Gestures project

The Graphs & Gestures project (Gerofsky, forthcoming) is a longitudinal study of the use of gesture and movement as a diagnostic tool and an intervention in helping school mathematics students learn about functions from the starting point of an embodied 'reading' of graphs. The project is now in its third of a planned five-year trajectory. In pilot projects in the first two years, Gerofsky worked first with adults and then with high school students in three Vancouver, Canada schools, videotaping subjects as they gestured the shapes of given graphs. In follow-up interviews while rewatching the video footage, participants were prompted to discuss the extent of their gestures (just a finger moving, or engaging whole arm, two arms, spine?) their choices regarding placement of the *x*- and *y*-axes against their bodies, symmetrical or non-symmetrical movements, acceleration or non-acceleration of gestures, etc. In the high school, teachers were also videotaped and interviewed, and their comments about student engagement and achievement in mathematics were noted.

The following hypothesis arose from the first two years' pilot projects:

Hypothesis: That the 'top' (most capable and imaginative) mathematics students engage with graphs of mathematical functions in tactile/kinesthetic modes, rather than an exclusively visual mode. They:

- gesture the shapes of graphs in ways that engage their whole spine;

•place graphs in a gestural space 'within reach' rather than 'within sight';

•don't engage in eye-tracking of the imagined graph;

•describe their actions in terms of 'being the graph' or 'riding on the graph', as opposed to 'seeing the graph'.

These results imply it may be worthwhile to use student gestures of the graphs of functions as both a diagnostic and a remedial tool, to teach all students to 'read graphs with their bodies' through a variety of early gesture- and movement-based interventions. In the third year of this project, now in progress, the research team has done just that. Working with choreographer Kathryn Ricketts, Gerofsky has planned and carried out six movement-based interventions with a Grade 8 (age 13) mathematics class, designed to help all the students engage in the practices observed in 'top' students in the pilot projects. Videotaped pre- and post-testing of students' gestured graphs, triangulated with the classroom teacher's anecdotal comments about individual student attitudes and understanding throughout the course of the project, and with student writing and post-interviews will offer evidence on the efficacy (or non-efficacy) of such interventions. Initial informal observations from this year's project support the idea that a physical, bodily experience focusing attention on salient mathematical features of these graphs (extrema and roots of a function) through movement and sound enabled these young students to learn patterns in polynomial functions quickly and with understanding, without having already learned the algebra related to these functions.

'Haptic math': Developing multisensory, whole-body HCI to support embodied mathematics learning

In the related Haptic Math project, Gerofsky, Maclean and Savage have begun work developing software and haptic HCI to support student learning of mathematical functions at the high school level. Consistent with the findings of the Graphs & Gestures research, the Haptic Math researchers have been experimenting with ways to offer students immediate, physical experiences of the graphs of functions that will help build an experiential repertoire basis for mathematical generalization.

The software application developed in the first year of this project offers mathematics students at the Grade 11 and 12 level (aged 16 and 17, the last two years of secondary school) the opportunity to use mathematical functions as a design tool, in this first case to design the shape of an on-screen roller coaster. Future plans are to develop software that would allow students to do other kinds of design with functions and their graphs, including the design of music, computer graphics and animations, physical sculptures, furniture and woven textiles. The concept of learning mathematics for use as a design tool is a central idea in this project (Gerofsky 2007; Shaffer 1997). The integration of design with haptic/kinaesthetic feedback is intended to give learners the ability to 'feel their way over the graph' they have created during the design process, to get a sense of the function(s) in a multisensory, multimodal way, before realizing their final version of the design in physical or on-screen form.

Initially we were interested in using a whole-body kinaesthetic interface like a programmable exercycle, to offer the most physically-engaging HCI possible (in keeping with the Graphs & Gestures project hypotheses). This may indeed be the next step in the Haptic Math project, and consultations with exercycles-in-education researchers Darren Warburton (UBC Cardiovascular Physiology and Rehabilitation Laboratory) and Sharon Bredin (UBC School of Human kinetics) are ongoing. However, for our first year piloting the Haptic Math project, the team decided to forego the technical hurdles of converting an exercycle to our purposes, and to begin instead by using the Twiddler, a low-cost motor-controlled haptic feedback knob invented by Maclean and Shaver (Shaver & Maclean 2003).

The Twiddler allows for choices in modelling the haptic feedback offered to the user's hand. After experimenting with several models, including a gravity-based force feedback, the team chose to use a spring model for haptic representation. Studies with users will give further indication about whether this model is the optimum one, but it does allow the user to feel resistance when sliding the cursor along the graph and going 'uphill', and a 'pull' when going 'downhill'. It also allows cursor movement in two directions (right-to-left and left-to-right) and allows for feeling the precarious balance of the cursor at the peak of a maximum point, and the 'trapped' sensation at the trough of a minimum.

In terms of the HCI component, the following research questions have arisen:

•do haptics of any kind enhance the intended mathematics learning experience?

•if so, would a whole body/kinaesthetic interface (like the exercycle, or a larger version of the Twiddler) offer a better learning experience than the present Twiddler?

•could haptic feedback be enhanced, or even replaced, by auditory feedback (like the sounds created by students in the Graphs & Gestures project)?

•do we want to provide the student with active or passive haptic feedback? (That is, are students to be 'in control' or 'passengers' with regard to the haptic experience?)

Thinking about the design-based software also generates many questions:

•would a collaborative learning model be more pedagogically sound than the individual learning model that informed our first prototype?

•What are the benefits of an open-ended design approach versus a goal-oriented task or game? Could both approaches be offered as options in one application?

•What might motivate students and teachers to use this software in school mathematics?

•How to deliver this hardware and software to schools at low cost, and in a sturdy enough form to survive repeated use by students?

•Would it be beneficial to collaborate with local games developers to accelerate development and bring together insights from both the world of academic research and the highly-developed, highly-appealing world of commercial video games?

References

Gerofsky, S. (2007). 'Because you can make things with it': A rationale for a project to teach mathematics as a multimodal design tool in secondary education. *Journal of Teaching and Learning* 5(1), 23-32.

Gerofsky, S. (2008). Gesture as diagnosis & intervention in the pedagogy of graphing: Pilot studies & next steps. In *International Group for the Psychology of Mathematics Education: Proceedings of the Joint Meeting of PME 32 and PME-NA XXX,* July 17-21, 2008, Morelia, Mexico.

Gerofsky. S. (forthcoming). 'Seeing' the graph vs. 'being' the graph: Gesture, engagement and awareness in school mathematics. In G. Stam (Ed.), *Integrating gesture.* Amsterdam: John Benjamins.

Ma, S. (2001). Participation in advanced mathematics: Do expectation and influence of students, peers, teachers, and parents matter? *Contemporary educational psychology* 26(1), 132-146.

Ma, X. & Willms, J.D. (1999). Dropping out of advanced mathematics: How much do students and schools contribute to the problem? *Educational evaluation and policy* analysi 21(4), 365-383.

Nemirovsky, R. (2003). Three conjectures concerning the relationship between body activity and understanding mathematics. *Proceedings of the 27th Annual Meeting of the International Group for the Psychology of Mathematics Education* (PME24), Hiroshima, Japan, 3-22.

Nuñez, R. E., Edwards, L., & Matos, J.F. (1999). Embodied cognition as grounding for situatedness and context in mathematics education. *Educational studies in mathematics*, 39, 45-65.

Shaffer, D.W. (1997). Learning mathematics through design: The anatomy of Escher's World. *Journal of mathematical behaviour* 16(2), 95-112.

Shaver, M. J. & MacLean, K. (2003). The Twiddler: A haptic teaching tool for low-cost communication and mechanical design. *Technical Report TR-2005-09*, University of British Columbia Department of Computer Science, Vancouver, Canada.

Tall, D. O. (2003). Using technology to support an embodied approach to learning concepts in mathematics. In L. M. Carvalho & L. C. Guimaraes (Eds.) *Historia e Tecnololgia no Ensino da Matematica* (Vol. 1, 1-28). Rio de Janeiro, Brasil.

Tall, D. O. (2004). Building theories: The three worlds of mathematics. *For the learning of mathematics,* 24(1), 29-32.

Tall, D. O. & Thomas, M. (1991). Encouraging versatile thinking in algebra using the computer. *Educational studies in mathematics,* 22(2), 125-147.