Grammars

Machine Learning Reading Group

July 2016

Natural Language Processing



Figure 1: Adapted from [1].



Figure 2: Adapted from [10].

RNA Secondary Structure Prediction



Figure 3: Adapted from Wikimedia Commons.

- Non-Probabilistic Grammars
- Probabilistic Context Free Grammars (PCFGs)
 - PCFGs in Chomsky Normal Forms (CNFs)
 - Calculating Probability of a Sequence
 - Determining the Most likely Parse for a Sequence
 - Learning Rule Parameters for an Unparameterised PCFG
- Grammar Learning
 - The ADIOS Algorithm

- A grammar G is a 4-tuple $G = (N, \Sigma, R, S)$, where:
 - *N* is a finite set of **non-terminal** symbols.
 - Σ is a finite set of **terminal** symbols.
 - *R* is finite set of **rules** of the form $(\Sigma \cup N)^* N (\Sigma \cup N)^* \to (\Sigma \cup N)^*$
 - $S \in N$ is a distinguished start symbol.

Grammar Example

Grammar for $L = \{a_n b_n c_n : n > 0\}$

- $S \rightarrow ABCS$
- $S \rightarrow TC$
- $CA \rightarrow AC$
- $BA \rightarrow AB$
- $CB \rightarrow BC$
- $CTC \rightarrow TCc$
- $TC \rightarrow TB$
- $BTB \rightarrow TBb$
- $TB \rightarrow TA$
- $ATA \rightarrow TAa$
- $TA \rightarrow \epsilon$

- $N = \{A, B, C, S, T\}$
 - $\Sigma = \{c, a, b\}$

The Chomsky Hierarchy



- A CFG G is a 4-tuple $G = (N, \Sigma, R, S)$, where:
 - *N* is a finite set of **non-terminal** symbols.
 - Σ is a finite set of terminal symbols.
 - *R* is a finite set of **rules** of the form $X \to Y_1Y_2...Y_n$, where $X \in N, n \ge 0$ and $Y_i \in (N \bigcup \Sigma)^*$ for i = 1...n.
 - $S \in N$ is a distinguished start symbol.

CFG: $S \rightarrow ASB$ $A \rightarrow aAS|a|\epsilon$ $B \rightarrow SbS|A|bb$

Chomsky Normal Form (CNF)

- A context-free grammar is in Chomsky Normal Form when
 - each rule has the form $X \to X_1X_2$, $X \to a$, or $S \to \epsilon$ where X, X_1 and X_2 are non-terminals, a is a terminal, and S is the start symbol.
- Any context-free grammar can be converted into an equivalent grammar in Chomsky normal form.
 - **Disadvantages**: (1) more nonterminals & rules (up to quadratic blowup in size), (2) less obvious relation to problem domain.
 - Advantages: (1) easy implementation of parsers, (2) CNF conversion is used in some algorithms as a preprocessing step, e.g., CYK.

CNF Example

CFG:	CNF:
$S \rightarrow ASB$	$S_0 \rightarrow AU_1 SB AS$
A → aAS a ε	$S \rightarrow AU_2 SB AS$
$B \rightarrow SbS A bb$	$A \rightarrow V_1 U_3 a V_1 S$
	$B \rightarrow SU_4 V_2 V_2 V_1 U_5 a V_1 S$
	$U_2 \rightarrow SB$
	U₃ → AS
	$U_4 \rightarrow V_2S$
	U₅ → AS
	$V_1 \rightarrow a$
	$V_2 \rightarrow b$

• Non-probabilistic grammars either generate a string x or not.

- Patterns have to be modified to allow the language to grow.
- It is difficult to create a specific pattern. In some cases (e.g., some protein families) it is impossible to produce a discriminative pattern.
- As a pattern is loosened, it is possible to match random unrelated sequences.

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- It is difficult to create a specific pattern. In some cases (e.g., some protein families) it is impossible to produce a discriminative pattern.
- As a pattern is loosened, it is possible to match random unrelated sequences.
- **Probabilistic (stochastic) grammars** generate different strings x with probability $P(x|\theta)$.
 - Allows exceptions, but can give exceptions less probability.

Hidden Markov Models (HMMs) are Probabilistic Regular Grammars [3]

• HMMs are probabilistic regular grammars.

- HMMs are Moore machines (emit symbols on states).
- **Probabilistic regular grammars** are **Mealy machines** (emit a terminal on transition to a non-terminal).
- Moore and Mealy machines are interchangeable.
- Algorithms for scoring, training, aligning of probabilistic regular grammars are the same used as in HMM.

• A PCFG

- extends context-free grammars by defining a multinomial distribution over the set of derivation rules over each terminal symbol.
- has a parameter P(X → γ) for each rule X → γ, where probabilities are normalized at the level of each nonterminal,
 ∀X ∈ N, ∑_{X→2} P(X → γ) = 1.

•
$$\sum_{x} P(x|\theta) = 1$$

PCFG Example

PCFG

- $S \rightarrow NP VP 1.0$
- $PP \rightarrow P NP 1.0$
- $VP \rightarrow V NP 0.7$
- $VP \rightarrow VP PP 0.3$
- $P \rightarrow with 1.0$
- V → saw 1.0
- $NP \rightarrow NP PP 0.4$
- NP → astronomers 0.1
- NP \rightarrow ears 0.18
- NP → saw 0.04
- NP \rightarrow stars 0.18
- NP → telescopes 0.1

• Scoring problem: calculate the probability of a sequence according to a parametrised PCFG:

$$P(x|\theta) = ? \tag{1}$$

 Alignment problem: Determine most likely parse (alignment) of a sequence according to a parametrised PCFG:

$$\underset{t}{\operatorname{argmax}} P(t|x,\theta) = ? \tag{2}$$

• **Training** problem: Learn **rule probability parameters** for an unparameterised PCFG, given a set of sequences:

$$\underset{\theta}{\operatorname{argmax}} P(x_1 \dots x_n | \theta) =? \tag{3}$$

• The **probability** of a sequence x according to grammar G with parameters θ :

$$P(x|\theta) = \sum_{t} P(x,t)$$
(4)

where t is a parse (alignment) of the sequence.

- **Trivial** solution: find all parse trees, calculate and sum up their probabilities.
 - Problem: exponential time complexity in general
- Efficient solution: using inside and outside probabilities.

Probability of Sequence



 $\begin{array}{rcl} P(X,t_1) = & 1.0 \times 0.1 \times 0.7 \times 1.0 \times 0.4 \times 0.18 \times 1.0 \times 1.0 \times 0.18 \\ = & 0.0009072 \end{array}$

Probability of Sequence



 $\begin{array}{rcl} P(x,t_2) &=& 1.0 \times 0.1 \times 0.3 \times 0.7 \times 1.0 \times 0.18 \times 1.0 \times 1.0 \times 0.18 \\ &=& 0.0006804 \end{array}$

- For the sentence *x* = astronomers saw stars with ears, we can construct 2 parse trees.
- $P(x|\theta) = P(x,t_1) + P(x,t_2) = 0.0009072 + 0.0006804 = 0.0015876$

• We want to calculate the probability of a sequence $x = x_1, ..., x_L$ according to grammar G with parameters θ , i.e., $P(x|\theta)$.

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- $P(x|\theta) = \alpha(1, L, 1)$
- $e_v(x_i)$ is the probability of rule $X_v \to x_i$.
- $t_v(y,z)$ is the probability of rule $X_v \to X_y X_z$.
- M = |X| and L = |x|.



procedure INSIDE

Initialization:

for i = 1 to L, v = 1 to M do $\alpha(i, i, v) = e_v(x_i)$

end for

Iteration:

for
$$i = 1$$
 to $L - 1, j = i + 1$ to $L, v = 1$ to M do
 $\alpha(i, j, v) = \sum_{y=1}^{M} \sum_{z=1}^{M} \sum_{k=i}^{j-1} \alpha(i, k, y) \alpha(k+1, j, z) t_v(y, z)$

end for

Termination:

 $P(x|\theta) = \alpha(1, L, 1)$

end procedure

Illustration of Recursion



Figure 4: Illustration of the recursion calculation of $\alpha(i, j, v)$.

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 (S) for the complete sequence sequence x, excluding all parse subtrees for the subsequence x_i, ..., x_j rooted at nonterminal X_v.

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procedure INSIDE Initialization:

$$\begin{split} \beta(1,L,1) &= 1 \\ \text{for} \quad v = 2 \text{ to } M \quad \text{do} \\ \beta(1,L,v) &= 0 \end{split}$$

end for

Iteration:

for
$$i = 1$$
 to $L, j = L$ to $i, v = 1$ to M do

$$\beta(i, j, v) = \sum_{y,z} \sum_{k=1}^{i-1} \alpha(k, i - 1, z) \beta(k, j, y) t_y(y, z) + \sum_{y,z} \sum_{k=j+1}^{L} \alpha(j + 1, k, z) \beta(i, k, y) t_y(v, z)$$

end for

Termination:

$$P(x|\theta) = \sum_{v=1}^{M} \beta(i, i, v) e_v(x_i)$$
 for any *i*.

end procedure

 \triangleright

Illustration of Recursion



Figure 5: Illustration of the recursion calculation of $\beta(i, j, v)$. The first diagram corresponds to the first part of the recursion and the second diagram corresponds to the second part of the recursion. 27/57

procedure CYK \triangleright Finds the most likely parse \hat{t} of a sequence x. Initialization:

for
$$i = 1$$
 to $L, v = 1$ to M do
 $\gamma(i, i, v) = \log e_v(x_i)$
 $\tau(i, i, v) = (0, 0, 0)$

end for

Iteration:

for
$$i = 1$$
 to $L - 1, j = i + 1$ to $L, v = 1$ to M do
 $\gamma(i, j, v) = \max_{y, z} \max_{k=i..j-1} \gamma(i, k, y) + \gamma(k + 1, j, z) + \log t_v(y, z)$
 $\tau(i, j, v) = \operatorname{argmax}_{(y, z, k), k=i..j-1} \gamma(i, k, y) + \gamma(k + 1, j, z) + \log t_v(y, z)$
end for

Termination:

$$\log P(x, \hat{t}|\theta) = \tau(1, L, 1).$$

end procedure
procedure CYK TRACEBACK Initialization:

Push (1, L, 1) on the stack.

Iteration:

Pop (i, j, v).

 $(y,z,k)=\tau(i,j,v).$

if $\tau(i,j,v) = (0,0,0)$ (implying i == j) then

Attach x_i as the child of v.

else

```
Attach y, z to parse tree as children of v.
Push (k + 1, j, z).
Push (i, k, y).
end if
```

end procedure

General Schema for Certain (Multinomial Distributions) EM Algorithms [6] [8]

• Given two events, *x* and *y*, the maximum likelihood estimation (MLE) for their conditional probability is:

$$P(x|y) = \frac{count(x,y)}{count(x)}$$
(5)

 If they are observable, it is easy to see what to do: just count the events in a representative corpus and use the MLE or a smoothed distribution.

General Schema for Certain (Multinomial Distributions) EM Algorithms [6] [8]

- What if these are hidden variables that cannot be observed directly?
- Use a model θ and iteratively improve the model based on a corpus of observable data (O) generated by the hidden variables:

$$P_{\hat{\theta}}(x|y) = \frac{E_{\mu}[(count(x,y)|O]]}{E_{\mu}[count(x)|O]]}$$
(6)

- It is worth noting that if you know how to calculate the numerator, the denominator is trivially derivable.
- By updating θ and iterating, the model converges to at least a local maximum.

The Inside-Outside Algorithm for CNFs (Parameter Re-estimation by Expectation Maximization)

 Begin with a given grammar topology and some initial probability estimates for rules.

• $\forall X \in N, \sum_{X \to \gamma} P(X \to \gamma) = 1, P(X \to \gamma) \ge 0$

- The probability of each parse of a training sequence according to G will act as our confidence in it.
- Sum the probabilities of each rule being used in each place to give an expectation of how often each rule was used.
- Use the expectations to refine the probability estimates increase the likelihood of the training corpus according to G.

• The expected number of times that X_v is used in a derivation:

$$c(v) = \frac{1}{P(x|\theta)} \sum_{i=1}^{L} \sum_{j=i}^{L} \alpha(i, j, v) \beta(i, j, v)$$
(7)

 The expected number of times that X_v is occupied and then production rules X_v → X_yX_z is used:

$$c(v \to yz) = \frac{1}{P(x|\theta)} \sum_{i=1}^{L-1} \sum_{j=i+1}^{L} \sum_{k=i}^{j-1} \beta(i,j,v) \alpha(i,k,y) \alpha(k+1,j,z) t_v(y,z)$$
(8)

• MLE for $v \rightarrow yz$ given v:

$$\hat{t}_{v}(y,z) = \frac{c(v \to yz)}{c(v)}$$

=
$$\frac{\sum_{i=1}^{L-1} \sum_{j=i+1}^{L} \sum_{k=i}^{j-1} \beta(i,j,v) \alpha(i,k,y) \alpha(k+1,j,z) t_{v}(y,z)}{\sum_{i=1}^{L} \sum_{j=i}^{L} \alpha(i,j,v) \beta(i,j,v)}$$

(9)

• MLE for $v \rightarrow yz$ given v:

$$\hat{t}_{v}(y,z) = \frac{c(v \to yz)}{c(v)} \\
= \frac{\sum_{i=1}^{L-1} \sum_{j=i+1}^{L} \sum_{k=i}^{j-1} \beta(i,j,v) \alpha(i,k,y) \alpha(k+1,j,z) t_{v}(y,z)}{\sum_{i=1}^{L} \sum_{j=i}^{L} \alpha(i,j,v) \beta(i,j,v)} \tag{9}$$

• Similarly, MLE for $v \rightarrow a$ given v:

$$\hat{e}_{v}(a) = \frac{c(v \to a)}{c(v)} = \frac{\sum_{i|x_{i}=a} \beta(i, i, v) e_{v}(a)}{\sum_{i=1}^{L} \sum_{j=1}^{L} \alpha(i, j, v) \beta(i, j, v)}$$
(10)

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= \frac{\sum_{i=1}^{L-1} \sum_{j=i+1}^{L} \sum_{k=i}^{j-1} \beta(i,j,v) \alpha(i,k,y) \alpha(k+1,j,z) t_{v}(y,z)}{\sum_{i=1}^{L} \sum_{j=i}^{L} \alpha(i,j,v) \beta(i,j,v)} \tag{9}$$

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(10)

 In the case of multiple independent observed sequences, expected counts are summed over all sequences.

- **Grammar learning** refers to the learning of a formal grammar from a set of observations, thus constructing a model which accounts for the characteristics of the observed objects.
- Presentation set:
 - Text (only positive examples).
 - Informant (both positive and negative examples).
- Learning methods:
 - Supervised (use structured sequences).
 - Unsupervised (uses unstructured sequences).
 - Semi-supervised (uses unstructured data to improve supervised learning tasks).

A Survey of Grammatical Inference Methods for Natural Language Learning [2]

	Presentation set		Type of information		
	Text	Informant	Supervised	Unsupervised	Semi-super- vised
ADIOS	X			X	
EMILE		х	Х		
e-GRIDS	х			Х	
CLL	х			X	
CDC	х			Х	
INDUCTIVE CYK		х		х	
LAgtS	х			Х	
GA-based		Х	X		
ALLis	х		Х		
ABL	х			х	
UnsuParse	х			Х	
Incremental parsing	х			Х	
Self-training	х				Х
Co-training	Х				Х

- Given a corpus of raw text (unstructured positive examples) separated into sequences, we want to derive a specification of the underlying grammar.
- This means we want to
 - Create new unseen grammatically correct sequences.
 - Accept new unseen grammatically correct sequences and reject ungrammatical ones.

Automatic Distillation of Structure (ADIOS) [9] [5] [7]



- ADIOS uses statistical information present in raw sequential data to identify significant segments and to distill rule-like regularities that support structured generalization.
- ADIOS has three main elements
 - A representational data structure.
 - A segmentation criterion (MEX).
 - A generalization ability.

Graph Representation

• Words as vertices and sequences as paths.



Is that a dog? Is that a cat? Where is the dog? And is that a horse?

Figure 6: Image adapted from [5]

Automatic Distillation of Structure (ADIOS)



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 - A representational data structure.
 - A segmentation criterion (MEX).
 - A generalization ability.

Detecting Significant Patterns



Figure 7: Image adapted from [5]

• Identifying patterns becomes easier on a graph.

Motif EXtraction (MEX)



Figure 8: Image adapted from [5]

Rewiring the graph



Once a pattern is identified as significant, the sub-paths it subsumes are merged into a new vertex and the graph is rewired accordingly. Repeating this process, leads to the formation of complex, hierarchically structured patterns.



Figure 9: Image adapted from [5]

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- ADIOS has three main elements
 - A representational data structure.
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Generalization: Defining an Equivalence Class

show me flights from philadelphia to san francisco on wednesdays



Figure 10: Image adapted from [7]



Bootstrapping



Figure 11: Image adapted from [7]



Figure 12: Image adapted from [5]

- Initialization load all data into a pseudograph
- Until no more patterns are found
 - For each path P
 - Create generalized search paths from P
 - Detect significant patterns using MEX
 - If found, add best new pattern and equivalence classes and rewire the graph

ADIOS:Context-free Mode Example



Figure 13: Adapted from [4]

- **Recall**: the probability of ADIOS recognizing an unseen gramatical sequence.
 - Can be assessed by leaving out some of the training corpus.
- **Precision**: the proportion of grammatical ADIOS productions.
 - Can be evaluated by external referee (e.g., by a human subject).

- ADIOS is a heuristic.
 - Once a pattern is created it remains forever errors conflate
 - Sequence ordering affects outcome.
- Running ADIOS with different orderings gives patterns that *cover* different parts of the grammar.

- Train multiple learners on the corpus.
 - Each on a different sequence ordering.
 - Create a forest of learners.
- To create a new sequence
 - Pick one learner at random.
 - Use it to produce sequence.
- To check grammaticality of given sequence
 - If any learner accepts sequence, declare as grammatical.

- These are some of the sources I used to prepare these slides!
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