UBC MLRG (Summer2017): Online, Active, and Causal Learning

# Machine Learning Reading Group (MLRG)

- Machine learning reading group (MLRG) format:
  - Each semester we pick a general topic.
  - Each week someone leads us through a tutorial-style lecture/discussion.
  - So it's organized a bit more like a "topics course" than reading group.
- We use this format because ML has become a huge field.

# Machine Learning Reading Group (MLRG)

- I've tried to pack as much as possible into the two ML courses:
  - CPSC 340 covers most of the most-useful methods.
  - CPSC 540 covers most of the background needed to read research papers.
- This reading group covers topics that aren't yet in these course.
  - Aimed at people who have taken CPSC 340, and are comfortable with 540-level material.

### **Recent MLRG History**

- Topics covered in recent tutorial-style MLRG sessions:
  - Summer 2015: Probabilistic graphical models.
  - Fall 2015: Convex optimization.
  - Winter 2016: Bayesian statistics.
  - Summer 2016: Miscellaneous.
  - Fall 2016: Deep learning.
  - Winter 2017: Reinforcement learning.
  - Summer 2017: Online, Active, and Causal Learning ("Time and Actions").

# **Topic 1: Online Learning**

- Usual supervised learning setup:
  - Training phase:
    - Build a model 'w' based on IID training examples  $(x_t, y_t)$ .
  - Testing phase:
    - Use the model to make predictions  $\hat{y}_t$  on new IID testing examples  $\hat{x}_t$ .
    - Our "score" is the total difference between predictions  $\hat{y}_t$  and true test labels  $y_t$ .
- In online learning there is no separate training/testing phase:
  - We receive a sequence of features  $x_t$ .
  - You make prediction  $\hat{y}_t$  on each example  $x_t$  as it arrives.
    - You only get to see  $y_t$  after you've made prediction  $\hat{y}_t$ .
  - Our "score" is the total difference between predictions  $\hat{y}_t$  and true labels  $y_t$ .
    - We need to predict well as we go (not just at the end).
    - You pay a penalty for having a bad model as you are learning.

## **Topic 1: Online Learning**

- In online learning, we typically don't assume data is IID.
  - Often analyze a weaker notion of performance called "regret".
- Main applications: online ads and spam filtering.
- A common variation is with **bandit feedback**:
  - There may be multiple possible  $y_t$ , we only observe loss for action we choose.
    - You only observe whether they clicked on your ad, not which ads they would have clicked on.
  - Here we have an exploration vs. exploitation trade-off:
    - Should we explore by picking a y<sub>t</sub> we don't know much about?
    - Should we exploit by picking a y<sub>t</sub> that is likely to be clicked?

### **Topic 2: Active Learning**

- Supervised learning trains on labeled examples (X,y).
  - The doctor has labeled thousands of images for you.
- Semi-supervised learning trains on (X,y) and unlabeled examples  $\tilde{X}$ .
  - The doctor has labeled 20 images for you.
  - You have a database of thousands of images.
- Active learning trains only on unlabeled examples  $\tilde{X}$ .
  - But you can ask the doctor to label 20 images for you.

#### **Topic 2: Active Learning**



• Which x<sup>t</sup> should we label to learn the most?

• Closely-related to optimal experimental design in statistics.

## **Topic 3: Causal Learning**

- The difference between observational and interventional data:
  - If I see that my watch says 10:55, class is almost over (observational).
  - If I set my watch to say 10:55, it doesn't help (interventional).
- In 340 and 540, we only considered observational data.
  - If our model performs actions, we need to learn effects of actions.
  - Otherwise, it may make stupid predictions.
- We may want to discover direction of causality.
  - "Watch" only predicts of "time" in observational setting (so it's not causal).
  - We can design experiments or make assumptions that find directions.
    - Randomized controledl trials used in medicine.

## Topic 3: Causal Learning

- Levels of causal inference:
  - Observational prediction:
    - Do people who take Cold-FX have shorter colds?
  - Causal prediction:
    - Does taking Cold-FX cause you to have shorter colds?
  - Counter-factual prediction:
    - You didn't take Cold-FX and had long cold, would taking it have made it shorter?
- Counter-factuals condition on imaginary pasts.

# (pause)

# **Online Classification with Perceptron**

- Perceptron for online linear binary classification [Rosenblatt, 1952]
  - Start with  $w_0 = 0$ .
  - At time time 't' we receive features  $x_t$ .
  - We predict  $\hat{y}_t = \text{sign}(w_t^T x_t)$ .
  - If  $\hat{y}_t \neq y_t$ , then set  $w_{t+1} = w_t + y_t x_t$ .
    - Otherwise, set w<sub>t+1</sub> = w<sub>t</sub>.
- Perceptron mistake bound [Novikoff, 1962]:
  - Assume data is linearly-separable with a "margin":
    - There exists w\* with  $||w^*||=1$  such that sign $(x_t^T w^*) = sign(y_t)$  for all 't' and  $|x^T w^*| \ge \gamma$ .
  - Then the number of total mistakes is bounded.
    - No requirement that data is IID.

#### Perceptron Mistake Bound

• Let's normalize each  $x_t$  so that  $||x_t|| = 1$ .

Length doesn't change label.

• Whenever we make a mistake, we have sign( $y_t$ )  $\neq$  sign( $w_t^T x_t$ ) and

$$||w_{t+1}||^{2} = ||w_{t} + yx_{t}||^{2}$$
  
=  $||w_{t}||^{2} + 2 \underbrace{y_{t}w_{t}^{T}x_{t}}_{<0} + 1$   
 $\leq ||w_{t}||^{2} + 1$   
 $\leq ||w_{t-1}||^{2} + 2$   
 $\leq ||w_{t-2}||^{2} + 3.$ 

• So after 'k' errors we have  $||w_t||^2 \le k$ .

#### Perceptron Mistake Bound

- Let's consider a solution  $w^*$ , so sign $(y_t) = sign(x_t^T w^*)$ .
- Whenever we make a mistake, we have:

 $||w_{t+1}|| = ||w_{t+1}|| ||w_*||$   $\geq w_{t+1}^T w_*$   $= (w_t + y_t x_t)^T w_*$   $= w_t^T w_* + y_t x_t^T w_*$   $= w_t^T w_* + |x_t^T w_*|$  $\geq w_t^T w_* + \gamma.$ 

• So after 'k' mistakes we have  $||w_t|| \ge \gamma k$ .

#### Perceptron Mistake Bound

- So our two bounds are  $||w_t|| \leq sqrt(k)$  and  $||w_t|| \geq \gamma k$ .
- This gives  $\gamma k \leq sqrt(k)$ , or a maximum of  $1/\gamma^2$  mistakes.
- Note that  $\gamma$  is upper-bounded by one due to  $||x|| \le 1$ .

# Beyond Separable Problems: Follow the Leader

- Perceptron can find perfect classifier for separable data.
- What should we do for non-separable data?
  - And assuming we're not using kernels...
- An obvious strategy is called follow the leader (FTL):
  - At time 't', find the best model from the previous (t-1) examples.
  - Use this model to predict  $y_t$ .
- Problems:
  - It might be expensive to find the best model.
    - NP-hard to find best linear classifier for non-separable.
  - It can perform very poorly.

## Follow the Leader Counter-Example

- Consider this online convex optimization scenario:
  - At iteration 't', we make a prediction  $w_t$ .
  - We then receive a convex function  $f_t$  and pay the penalty  $f_t(w_t)$ .
    - f<sub>t</sub> could be the logistic loss on example 't'.
- In this setting, follow the leader (FTL) would choose:  $w_t \in \operatorname{argmin}_{w} \sum_{i=1}^{t-1} f_i(w).$
- The problem is convex but the performance can be arbitrarily bad...

# Follow the Leader Counter Example

- Assume  $x \in [-1,1]$  and: FTL objective:
  - $f_1(x_1) = (1/2)x^2.$
  - $f_2(x_2) = -x.$
  - $f_3(x_3) = x.$
  - $f_4(x_4) = -x.$
  - $f_5(x_5) = x.$
  - $f_6(x_6) = -x.$
  - $f_7(x_7) = x.$

— ...

- $F_1(x_1) = (1/2)x^2.$ 
  - $-F_2(x_2) = -(1/2)x^2.$
- $-F_3(x_3) = (1/2)x^2.$
- $F_4(x_4) = -(1/2)x^2.$
- $-F_5(x_5) = (1/2)x^2.$
- $F_6(x_6) = -(1/2)x^2.$
- $-F_7(x_7) = (1/2)x^2.$

— ...

- FTL predictions:
  - $x_1 = (initial guess)$
  - $-x_{2}=0$

— ...

- $-x_3 = 1$  (worst possible)
- $x_4 = -1$  (worst possible)
- $x_5 = 1$  (worst possible)
- $-x_6 = -1$  (worst possible)
- $-x_7 = 1$  (worst possible)

# **Regularized FTL and Regret**

- Worst possible sequence:
  - {+1,-1,+1,-1,+1,-1,+1,-1,...}
- FTL produces the sequence:
  - {x0,0,+1,-1,+1,-1,+1,-1,...}, which is close to the worst possible.
- Best possible sequence:
  - {0,+1,-1,+1,-1,+1,-,1,+1,...}
- Best sequence with a fixed prediction:
  - $\ \{0,0,0,0,0,0,0,0,...\}$
- We have no way to bound error compared to best sequence: could have adversary.
- We instead consider a weaker notion of "success" called regret:
  - How much worse is our total error than optimal fixed prediction at time 't'.
  - Note that fixed prediction might change with 't'.
- Next week we'll see algorithms with optimal regret.

## Schedule

Date	Торіс	Presenter
Jun 6	Motivation/overview, perceptron, follow the leader.	Mark
Jun 13	Online convex optimization, mirror descent	Julie
Jun 20	Multi-armed bandits, contextual bandits	Alireza
Jun 27	Heavy hitters	Michael
Jul 4	Regularized FTL, AdaGrad, Adam, online-to-batch	Raunak
Jul 11	Best-arm identification, dueling bandits	Glen
Jul 18	Uncertainty sampling, variance/error reduction, QBC	Nasim
Jul 25	A/B testing, Optimal experimental design	Mohamed
Aug 1	Randomized controlled trials, do-calculus	Sanna
Aug 8	Granger causality, independent component analysis	Issam
Aug 15	Counterfactuals	Eric
Aug 22	MPI causality	Julieta
Aug 29	Instrumental variables	Jimmy