

# Black-Box Optimization Benchmarking Template for Noiseless Function Testbed

Draft version \*

Forename Name

## ABSTRACT

### Categories and Subject Descriptors

G.1.6 [Numerical Analysis]: Optimization—*global optimization, unconstrained optimization*; F.2.1 [Analysis of Algorithms and Problem Complexity]: Numerical Algorithms and Problems

### General Terms

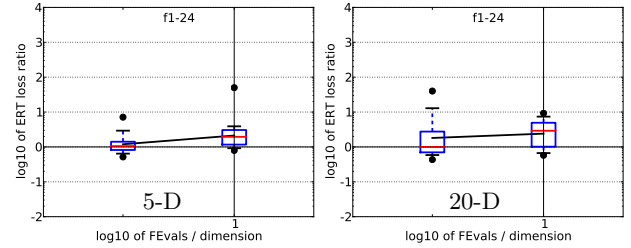
Algorithms

### Keywords

Benchmarking, Black-box optimization

## 1. RESULTS

Results of SMBO5 from experiments according to [?] on the benchmark functions given in [?, ?] are presented in Figures ??, ?? and ?? and in Tables ?? and ??.



| <i>f1–f24 in 5-D, maxFE/D=10</i> |      |      |      |     |     |     |
|----------------------------------|------|------|------|-----|-----|-----|
| #FEs/D                           | best | 10%  | 25%  | med | 75% | 90% |
| 2                                | 0.52 | 0.61 | 0.81 | 1.1 | 1.4 | 3.3 |
| 10                               | 0.79 | 0.90 | 1.1  | 1.9 | 3.1 | 4.2 |
| 100                              | 0.91 | 3.0  | 6.4  | 10  | 15  | 87  |
| RL <sub>US</sub> /D              | 10   | 10   | 10   | 10  | 10  | 10  |

| <i>f1–f24 in 20-D, maxFE/D=10</i> |      |      |      |     |     |     |
|-----------------------------------|------|------|------|-----|-----|-----|
| #FEs/D                            | best | 10%  | 25%  | med | 75% | 90% |
| 2                                 | 0.43 | 0.58 | 0.69 | 1.0 | 3.2 | 16  |
| 10                                | 0.58 | 0.66 | 1.0  | 2.9 | 5.2 | 7.7 |
| 100                               | 0.34 | 0.59 | 4.8  | 10  | 30  | 55  |
| RL <sub>US</sub> /D               | 5e-2 | 10   | 10   | 10  | 10  | 10  |

Figure 3: ERT loss ratio versus the budget (both in number of  $f$ -evaluations divided by dimension). The target value  $f_t$  for a given budget FEvals is the best target  $f$ -value reached within the budget by the given algorithm. Shown is the ERT of the given algorithm divided by best ERT seen in GECCO-BBOB-2009 for the target  $f_t$ , or, if the best algorithm reached a better target within the budget, the budget divided by the best ERT. Line: geometric mean. Box-Whisker error bar: 25-75%-ile with median (box), 10-90%-ile (caps), and minimum and maximum ERT loss ratio (points). The vertical line gives the maximal number of function evaluations in a single trial in this function subset. See also Figure ?? for results on each function subgroup.

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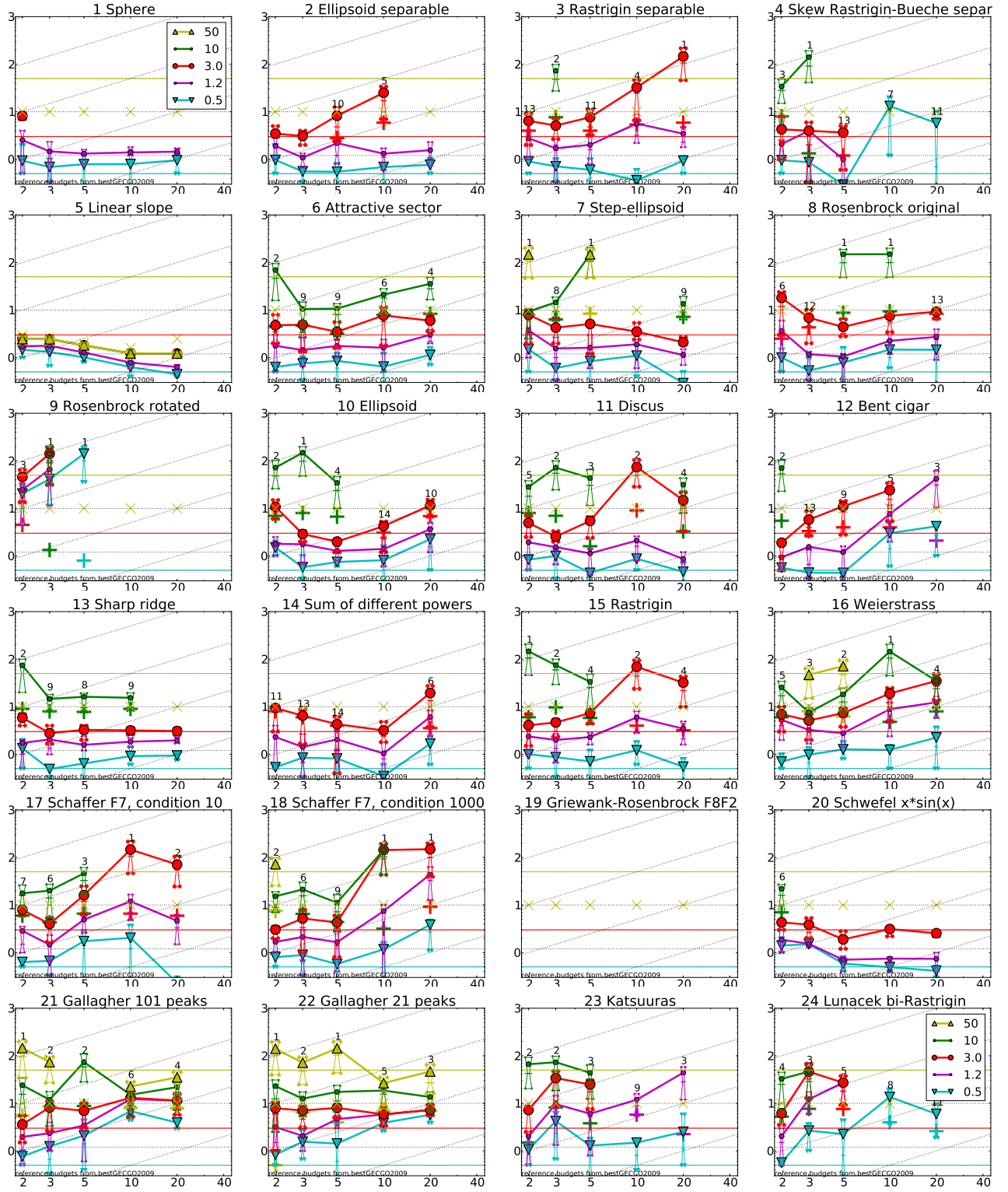
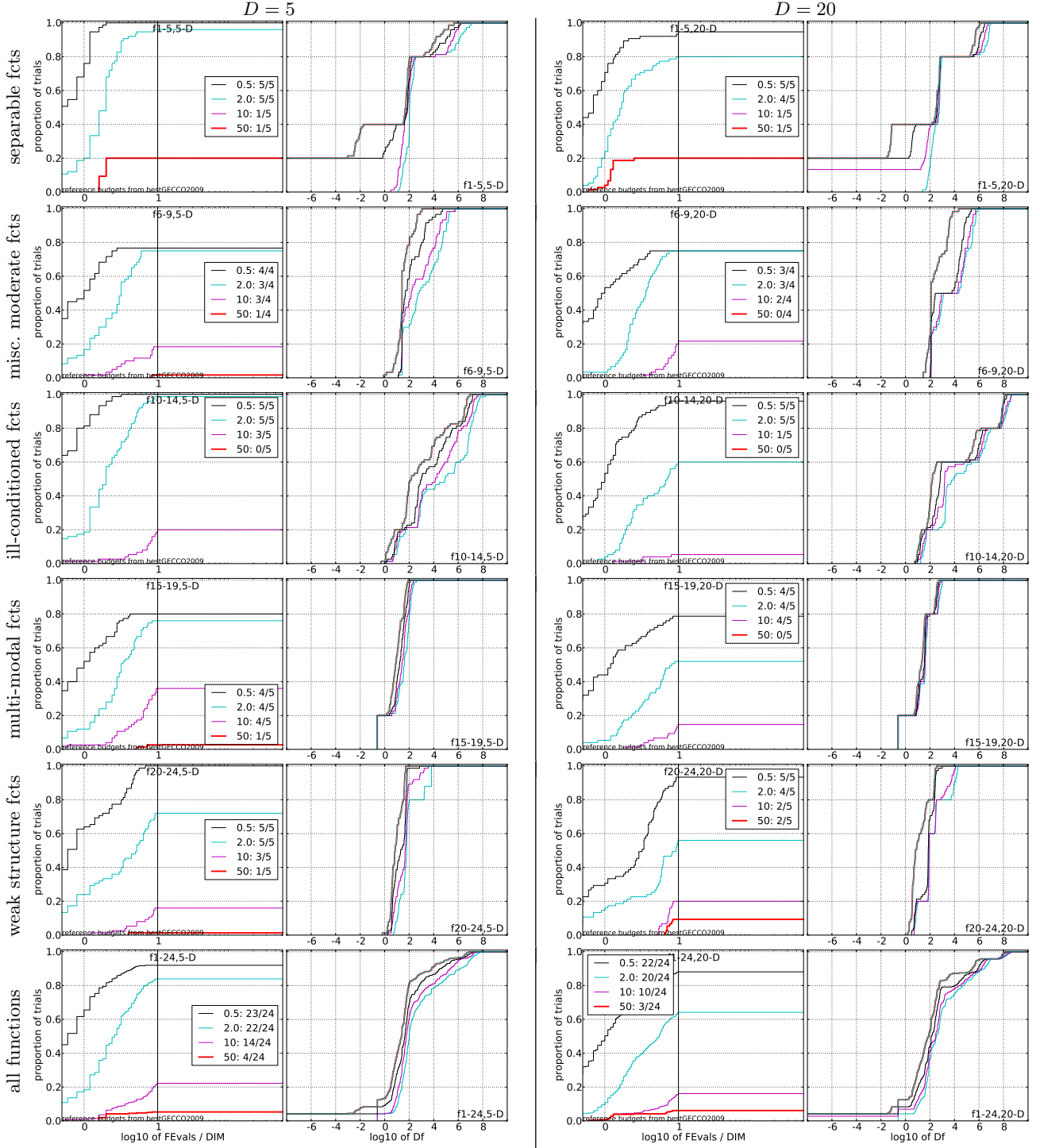


Figure 1: Expected number of  $f$ -evaluations (ERT, lines) to reach  $f_{\text{opt}} + \Delta f$ ; median number of  $f$ -evaluations (+) to reach the most difficult target that was reached not always but at least once; maximum number of  $f$ -evaluations in any trial ( $\times$ ); interquartile range with median (notched boxes) of simulated runlengths to reach  $f_{\text{opt}} + \Delta f$ ; all values are divided by dimension and plotted as  $\log_{10}$  values versus dimension. Shown is the ERT for targets just not reached by the GECCO-BBOB-2009 best algorithm within the given budget  $k\text{DIM}$ , where  $k$  is shown in the legend. Numbers above ERT-symbols indicate the number of trials reaching the respective target. Slanted grid lines indicate a scaling with  $\mathcal{O}(\text{DIM})$  compared to  $\mathcal{O}(1)$  when using the respective 2009 best algorithm.



**Figure 2: Empirical cumulative distribution functions (ECDF), plotting the fraction of trials with an outcome not larger than the respective value on the  $x$ -axis. Left subplots: ECDF of number of function evaluations (FEvals) divided by search space dimension  $D$ , to fall below  $f_{\text{opt}} + \Delta f$  where  $\Delta f$  is the target just not reached by the GECCO-BBOB-2009 best algorithm within a budget of  $k \times \text{DIM}$  evaluations, where  $k$  is the first value in the legend. Legends indicate for each target the number of functions that were solved in at least one trial within the displayed budget. Right subplots: ECDF of the best achieved  $\Delta f$  for running times of  $0.5D, 1.2D, 3D, 10D, 100D, 1000D, \dots$  function evaluations (from right to left cycling cyan-magenta-black...) and final  $\Delta f$ -value (red), where  $\Delta f$  and  $Df$  denote the difference to the optimal function value.**

| 5-D                   |                                |                                 |                                 |                                |                               |                |
|-----------------------|--------------------------------|---------------------------------|---------------------------------|--------------------------------|-------------------------------|----------------|
| #FEs/D                | 0.5                            | 1.2                             | 3.0                             | 10                             | 50                            | #succ          |
| <b>f<sub>1</sub></b>  | <i>2.5e+1:4.8</i><br>0.82(0.8) | <i>1.6e+1:7.6</i><br>0.87(0.6)  | <i>1.0e-8:12</i><br>∞           | <i>1.0e-8:12</i><br>∞          | <i>1.0e-8:12</i><br>∞50       | 15/15<br>0/15  |
| <b>f<sub>2</sub></b>  | <i>1.6e+6:2.9</i><br>0.95(0.9) | <i>4.0e+5:11</i><br>0.98(0.7)   | <i>4.0e+4:15</i><br>2.7(3)      | <i>6.3e+2:58</i><br>∞          | <i>1.0e-8:95</i><br>∞50       | 15/15<br>0/15  |
| <b>f<sub>3</sub></b>  | <i>1.6e+2:4.1</i><br>0.73(0.8) | <i>1.0e+2:15</i><br>0.69(0.6)   | <i>6.3e+1:23</i><br>1.6(2)      | <i>2.5e+1:73</i><br>∞          | <i>1.0e+1:716</i><br>∞50      | 15/15<br>0/15  |
| <b>f<sub>4</sub></b>  | <i>2.5e+2:2.6</i><br>0.56(0.4) | <i>1.6e+2:10</i><br>0.52(0.4)   | <i>1.0e+2:19</i><br>0.97(1)     | <i>4.0e+1:65</i><br>∞          | <i>1.6e+1:434</i><br>∞50      | 15/15<br>0/15  |
| <b>f<sub>5</sub></b>  | <i>6.3e+1:4.0</i><br>1.3(0.2)  | <i>4.0e+1:10</i><br>0.65(0.1)   | <i>1.0e-8:10</i><br>0.91(0.1)   | <i>1.0e-8:10</i><br>0.91(0.1)  | <i>1.0e-8:10</i><br>0.91(0.1) | 15/15<br>15/15 |
| <b>f<sub>6</sub></b>  | <i>1.0e+5:3.0</i><br>1.4(2)    | <i>2.5e+4:8.4</i><br>1.0(0.9)   | <i>1.0e+2:16</i><br>1.1(1)      | <i>2.5e+1:54</i><br>0.99(1)    | <i>2.5e-1:254</i><br>∞50      | 15/15<br>0/15  |
| <b>f<sub>7</sub></b>  | <i>1.6e+2:4.2</i><br>1(0.8)    | <i>1.0e+2:6.2</i><br>1.3(1)     | <i>2.5e+1:20</i><br>1.3(0.9)    | <i>4.0e+0:54</i><br>14(15)     | <i>1.0e+0:324</i><br>2.3(3)   | 15/15<br>1/15  |
| <b>f<sub>8</sub></b>  | <i>1.0e+4:4.6</i><br>0.86(1.0) | <i>6.3e+3:6.8</i><br>0.77(0.8)  | <i>1.0e+3:18</i><br>1.2(0.7)    | <i>6.3e+1:54</i><br>14(14)     | <i>1.6e+0:258</i><br>∞50      | 15/15<br>0/15  |
| <b>f<sub>9</sub></b>  | <i>2.5e+1:20</i><br>36(39)     | <i>1.6e+1:26</i><br>∞           | <i>1.0e+1:35</i><br>∞           | <i>4.0e+0:62</i><br>∞          | <i>1.6e-2:256</i><br>∞50      | 15/15<br>0/15  |
| <b>f<sub>10</sub></b> | <i>2.5e+6:2.9</i><br>1.3(0.9)  | <i>6.3e+5:7.0</i><br>0.91(0.7)  | <i>2.5e+5:17</i><br>0.59(0.7)   | <i>6.3e+3:54</i><br>3.2(3)     | <i>2.5e+1:297</i><br>∞50      | 15/15<br>0/15  |
| <b>f<sub>11</sub></b> | <i>1.0e+6:3.0</i><br>0.73(0.8) | <i>6.3e+4:6.2</i><br>0.91(0.9)  | <i>6.3e+2:16</i><br>1.7(2)      | <i>6.3e+1:74</i><br>2.9(4)     | <i>6.3e-1:298</i><br>∞50      | 15/15<br>0/15  |
| <b>f<sub>12</sub></b> | <i>4.0e+7:3.6</i><br>0.61(0.4) | <i>1.6e+7:7.6</i><br>0.79(0.9)  | <i>4.0e+6:19</i><br>2.9(3)      | <i>1.6e+4:52</i><br>∞          | <i>1.0e+0:268</i><br>∞50      | 15/15<br>0/15  |
| <b>f<sub>13</sub></b> | <i>1.0e+3:2.8</i><br>1.1(1)    | <i>6.3e+2:8.4</i><br>0.94(0.8)  | <i>4.0e+2:17</i><br>0.97(0.4)   | <i>6.3e+1:52</i><br>1.6(1)     | <i>6.3e-2:264</i><br>∞50      | 15/15<br>0/15  |
| <b>f<sub>14</sub></b> | <i>1.6e+1:3.0</i><br>1.4(2)    | <i>1.0e+1:10</i><br>1.0(2)      | <i>6.3e+0:15</i><br>1.4(1)      | <i>2.5e-1:53</i><br>∞          | <i>1.0e-5:251</i><br>∞50      | 15/15<br>0/15  |
| <b>f<sub>15</sub></b> | <i>1.6e+2:3.0</i><br>1.2(2)    | <i>1.0e+2:13</i><br>0.88(0.7)   | <i>6.3e+1:24</i><br>1.5(1)      | <i>4.0e+1:55</i><br>3.0(3)     | <i>1.6e+1:289</i><br>∞50      | 5/5<br>0/15    |
| <b>f<sub>16</sub></b> | <i>4.0e+1:4.8</i><br>1.3(1)    | <i>2.5e+1:16</i><br>0.88(0.5)   | <i>1.6e+1:46</i><br>0.79(0.6)   | <i>1.0e+1:120</i><br>0.76(0.7) | <i>4.0e+0:334</i><br>1.1(1)   | 15/15<br>2/15  |
| <b>f<sub>17</sub></b> | <i>1.0e+1:5.2</i><br>1.7(2)    | <i>6.3e+0:26</i><br>0.95(0.8)   | <i>4.0e+0:57</i><br>1.4(2)      | <i>2.5e+0:110</i><br>2.1(2)    | <i>6.3e-1:412</i><br>∞50      | 15/15<br>0/15  |
| <b>f<sub>18</sub></b> | <i>6.3e+1:3.4</i><br>0.84(0.6) | <i>4.0e+1:7.2</i><br>1.1(1)     | <i>2.5e+1:20</i><br>1.1(1)      | <i>1.6e+1:58</i><br>0.96(1)    | <i>1.6e+0:318</i><br>∞50      | 15/15<br>0/15  |
| <b>f<sub>19</sub></b> | <i>1.6e-1:172</i><br>∞         | <i>1.0e-1:242</i><br>∞          | <i>6.3e-2:675</i><br>∞          | <i>4.0e-2:3078</i><br>∞        | <i>2.5e-2:4946</i><br>∞50     | 15/15<br>0/15  |
| <b>f<sub>20</sub></b> | <i>6.3e+3:5.1</i><br>0.57(0.2) | <i>4.0e+3:8.4</i><br>0.42(0.2)↓ | <i>4.0e+1:15</i><br>0.62(0.3)↓2 | <i>2.5e+0:69</i><br>∞          | <i>1.0e+0:851</i><br>∞50      | 15/15<br>0/15  |
| <b>f<sub>21</sub></b> | <i>4.0e+1:3.9</i><br>2.7(3)    | <i>2.5e+1:11</i><br>1.6(2)      | <i>1.6e+1:31</i><br>1.1(0.8)    | <i>6.3e+0:73</i><br>5.1(5)     | <i>1.6e+0:347</i><br>∞50      | 5/5<br>0/15    |
| <b>f<sub>22</sub></b> | <i>6.3e+1:3.6</i><br>2.0(2)    | <i>4.0e+1:15</i><br>1.6(1)      | <i>2.5e+1:32</i><br>1.2(1)      | <i>1.0e+1:71</i><br>1.2(1)     | <i>1.6e+0:341</i><br>2.1(2)   | 5/5<br>1/15    |
| <b>f<sub>23</sub></b> | <i>1.0e+1:3.0</i><br>2.2(2)    | <i>6.3e+0:9.0</i><br>3.4(4)     | <i>4.0e+0:33</i><br>3.8(4)      | <i>2.5e+0:84</i><br>2.6(3)     | <i>1.0e+0:518</i><br>∞50      | 15/15<br>0/15  |
| <b>f<sub>24</sub></b> | <i>6.3e+1:15</i><br>0.78(0.8)  | <i>4.0e+1:37</i><br>3.7(3)      | <i>4.0e+1:37</i><br>3.7(4)      | <i>2.5e+1:118</i><br>∞         | <i>1.6e+1:692</i><br>∞50      | 15/15<br>0/15  |

| 20-D                  |                                 |                                 |                                 |                                |                                 |                |
|-----------------------|---------------------------------|---------------------------------|---------------------------------|--------------------------------|---------------------------------|----------------|
| #FEs/D                | 0.5                             | 1.2                             | 3.0                             | 10                             | 50                              | #succ          |
| <b>f<sub>1</sub></b>  | <i>6.3e+1:24</i><br>0.79(0.5)   | <i>4.0e+1:42</i><br>0.69(0.2)↓4 | <i>1.0e-8:43</i><br>∞           | <i>1.0e-8:43</i><br>∞          | <i>1.0e-8:43</i><br>∞200        | 15/15<br>0/15  |
| <b>f<sub>2</sub></b>  | <i>4.0e+6:29</i><br>0.54(0.6)   | <i>2.5e+6:42</i><br>0.74(0.8)   | <i>1.0e+5:65</i><br>∞           | <i>1.0e+4:207</i><br>∞         | <i>1.0e-8:412</i><br>∞200       | 15/15<br>0/15  |
| <b>f<sub>3</sub></b>  | <i>6.3e+2:33</i><br>0.57(0.5)   | <i>4.0e+2:44</i><br>1.6(1)      | <i>1.6e+2:109</i><br>27(28)     | <i>1.0e+2:255</i><br>∞         | <i>2.5e+1:3277</i><br>∞200      | 15/15<br>0/15  |
| <b>f<sub>4</sub></b>  | <i>6.3e+2:22</i><br>5.3(9)      | <i>4.0e+2:91</i><br>∞           | <i>2.5e+2:250</i><br>∞          | <i>1.6e+2:332</i><br>∞         | <i>6.3e+1:1927</i><br>∞200      | 15/15<br>0/15  |
| <b>f<sub>5</sub></b>  | <i>2.5e+2:19</i><br>0.47(0.2)↓2 | <i>1.6e+2:34</i><br>0.37(0.1)↓4 | <i>1.0e-8:41</i><br>0.60(0.1)   | <i>1.0e-8:41</i><br>0.60(0.1)  | <i>1.0e-8:41</i><br>0.60(0.1)   | 15/15<br>15/15 |
| <b>f<sub>6</sub></b>  | <i>2.5e+5:16</i><br>1.4(1)      | <i>6.3e+4:43</i><br>1.4(1.0)    | <i>1.6e+4:62</i><br>1.9(1)      | <i>1.6e+2:353</i><br>2.0(2)    | <i>1.6e+1:1078</i><br>∞200      | 15/15<br>0/15  |
| <b>f<sub>7</sub></b>  | <i>1.0e+3:11</i><br>0.55(0.6)   | <i>4.0e+2:39</i><br>0.58(0.4)↓  | <i>2.5e+2:74</i><br>0.57(0.4)↓  | <i>6.3e+1:319</i><br>0.85(0.7) | <i>1.0e+1:1351</i><br>∞200      | 15/15<br>0/15  |
| <b>f<sub>8</sub></b>  | <i>4.0e+4:19</i><br>1.6(2)      | <i>2.5e+4:35</i><br>1.6(1)      | <i>4.0e+3:67</i><br>2.7(2)      | <i>2.5e+2:231</i><br>∞         | <i>1.6e+1:1470</i><br>∞200      | 15/15<br>0/15  |
| <b>f<sub>9</sub></b>  | <i>1.0e+2:357</i><br>∞          | <i>6.3e+1:560</i><br>∞          | <i>4.0e+1:684</i><br>∞          | <i>2.5e+1:756</i><br>∞         | <i>1.0e+1:1716</i><br>∞200      | 15/15<br>0/15  |
| <b>f<sub>10</sub></b> | <i>1.6e+6:15</i><br>3.0(3)      | <i>1.0e+6:27</i><br>2.7(2)      | <i>4.0e+5:70</i><br>3.3(3)      | <i>6.3e+4:231</i><br>∞         | <i>4.0e+3:1015</i><br>∞200      | 15/15<br>0/15  |
| <b>f<sub>11</sub></b> | <i>4.0e+4:11</i><br>0.82(0.6)   | <i>2.5e+3:27</i><br>0.64(0.5)   | <i>1.6e+2:313</i><br>0.95(1)    | <i>1.0e+2:481</i><br>1.3(1)    | <i>1.0e+1:1002</i><br>∞200      | 15/15<br>0/15  |
| <b>f<sub>12</sub></b> | <i>1.0e+8:23</i><br>3.6(5)      | <i>6.3e+7:39</i><br>21(26)      | <i>2.5e+7:76</i><br>∞           | <i>4.0e+6:209</i><br>∞         | <i>1.0e+1:1042</i><br>∞200      | 15/15<br>0/15  |
| <b>f<sub>13</sub></b> | <i>1.6e+3:28</i><br>0.68(0.3)   | <i>1.0e+3:64</i><br>0.61(0.1)↓4 | <i>6.3e+2:79</i><br>0.78(0.1)↓2 | <i>4.0e+1:211</i><br>∞         | <i>2.5e+0:1724</i><br>∞200      | 15/15<br>0/15  |
| <b>f<sub>14</sub></b> | <i>2.5e+1:15</i><br>2.3(2)      | <i>1.6e+1:42</i><br>2.9(3)      | <i>1.0e+1:75</i><br>5.2(5)      | <i>1.6e+0:219</i><br>∞         | <i>6.3e-4:1106</i><br>∞200      | 15/15<br>0/15  |
| <b>f<sub>15</sub></b> | <i>6.3e+2:15</i><br>0.71(0.8)   | <i>4.0e+2:67</i><br>1.0(0.8)    | <i>2.5e+2:292</i><br>2.2(2)     | <i>1.6e+2:846</i><br>∞         | <i>1.0e+2:1671</i><br>∞200      | 15/15<br>0/15  |
| <b>f<sub>16</sub></b> | <i>4.0e+1:26</i><br>1.7(2)      | <i>2.5e+1:127</i><br>1.9(2)     | <i>1.6e+1:540</i><br>1.3(1)     | <i>1.6e+1:540</i><br>1.3(1)    | <i>1.0e+1:1384</i><br>∞200      | 15/15<br>0/15  |
| <b>f<sub>17</sub></b> | <i>1.6e+1:11</i><br>0.43(0.6)   | <i>1.0e+1:63</i><br>1.5(2)      | <i>6.3e+0:305</i><br>4.6(5)     | <i>4.0e+0:468</i><br>∞         | <i>1.0e+0:1030</i><br>∞200      | 15/15<br>0/15  |
| <b>f<sub>18</sub></b> | <i>4.0e+1:116</i><br>0.66(0.7)  | <i>2.5e+1:252</i><br>3.5(4)     | <i>1.6e+1:430</i><br>6.9(7)     | <i>1.0e+1:621</i><br>∞         | <i>4.0e+0:1090</i><br>∞200      | 15/15<br>0/15  |
| <b>f<sub>19</sub></b> | <i>1.6e-1:2.5e5</i><br>∞        | <i>1.0e-1:3.4e5</i><br>∞        | <i>6.3e-2:3.4e5</i><br>∞        | <i>4.0e-2:3.4e5</i><br>∞       | <i>2.5e-2:3.4e5</i><br>∞200     | 3/15<br>0/15   |
| <b>f<sub>20</sub></b> | <i>1.6e+4:38</i><br>0.22(0.1)↓  | <i>1.0e+4:42</i><br>0.35(0.2)↓4 | <i>2.5e+2:62</i><br>0.82(0.2)   | <i>2.5e+0:250</i><br>∞         | <i>1.6e+0:2536</i><br>∞200      | 15/15<br>0/15  |
| <b>f<sub>21</sub></b> | <i>6.3e+1:36</i><br>2.2(0.8)    | <i>4.0e+1:77</i><br>3.0(2)      | <i>4.0e+1:77</i><br>3.0(3)      | <i>1.6e+1:456</i><br>0.96(0.9) | <i>4.0e+0:1094</i><br>0.64(0.6) | 15/15<br>4/15  |
| <b>f<sub>22</sub></b> | <i>6.3e+1:45</i><br>2.6(1)      | <i>4.0e+1:68</i><br>2.1(0.7)    | <i>4.0e+1:68</i><br>2.1(0.7)    | <i>1.6e+1:231</i><br>1.2(1.0)  | <i>6.3e+0:1219</i><br>0.77(0.8) | 15/15<br>3/15  |
| <b>f<sub>23</sub></b> | <i>6.3e+0:29</i><br>1.8(2)      | <i>4.0e+0:118</i><br>7.5(8)     | <i>2.5e+0:306</i><br>∞          | <i>2.5e+0:306</i><br>∞         | <i>1.0e+0:1614</i><br>∞200      | 15/15<br>0/15  |
| <b>f<sub>24</sub></b> | <i>2.5e+2:208</i><br>0.58(0.6)  | <i>1.6e+2:918</i><br>∞          | <i>1.0e+2:6628</i><br>∞         | <i>6.3e+1:9885</i><br>∞        | <i>4.0e+1:31629</i><br>∞200     | 15/15<br>0/15  |

Table 1: Expected running time (ERT in number of function evaluations) divided by the best ERT measured during BBOB-2009. The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear in the second row of each cell, the best ERT (preceded by the target  $\Delta f$ -value in *italics*) in the first. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Bold entries are statistically significantly better (according to the rank-sum test) compared to the best algorithm in BBOB-2009, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k > 1$  is following the  $\downarrow$  symbol, with Bonferroni correction by the number of functions.

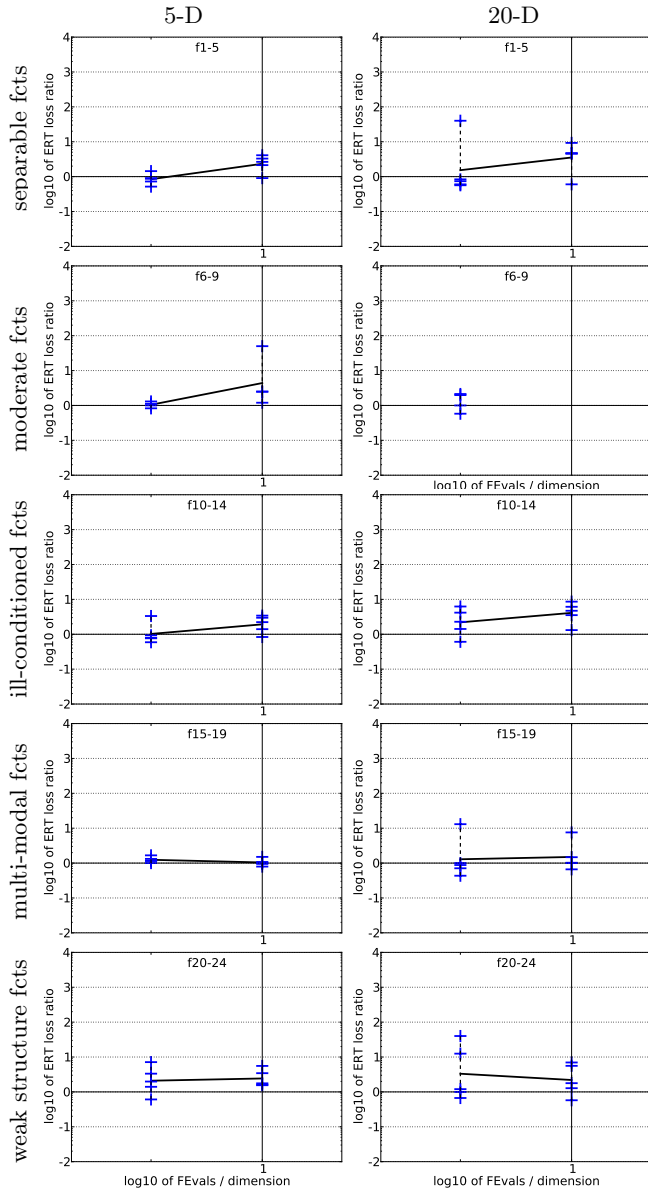


Figure 4: ERT loss ratios (see Figure ?? for details). Each cross (+) represents a single function, the line is the geometric mean.