

# Black-Box Optimization Benchmarking Template for the Comparison of Two Algorithms on the Noiseless Testbed

Draft version \*

Forename Name

## ABSTRACT

### Categories and Subject Descriptors

G.1.6 [Numerical Analysis]: Optimization—*global optimization, unconstrained optimization*; F.2.1 [Analysis of Algorithms and Problem Complexity]: Numerical Algorithms and Problems

### General Terms

Algorithms

### Keywords

Benchmarking, Black-box optimization

## 1. RESULTS

Results from experiments according to [2] on the benchmark functions given in [1, 3] are presented in Figures 1, 2 and 3 and in Tables 1. The **expected running time (ERT)**, used in the figures and table, depends on a given target function value,  $f_t = f_{\text{opt}} + \Delta f$ , and is computed over all relevant trials as the number of function evaluations executed during each trial while the best function value did not reach  $f_t$ , summed over all trials and divided by the number of trials that actually reached  $f_t$  [2, 4]. **Statistical significance** is tested with the rank-sum test for a given target  $\Delta f_t$  ( $10^{-8}$  as in Figure 1) using, for each trial, either the number of needed function evaluations to reach  $\Delta f_t$  (inverted and multiplied by  $-1$ ), or, if the target was not reached, the best  $\Delta f$ -value achieved, measured only up to the smallest number of overall function evaluations for any unsuccessful trial under consideration.

## 2. REFERENCES

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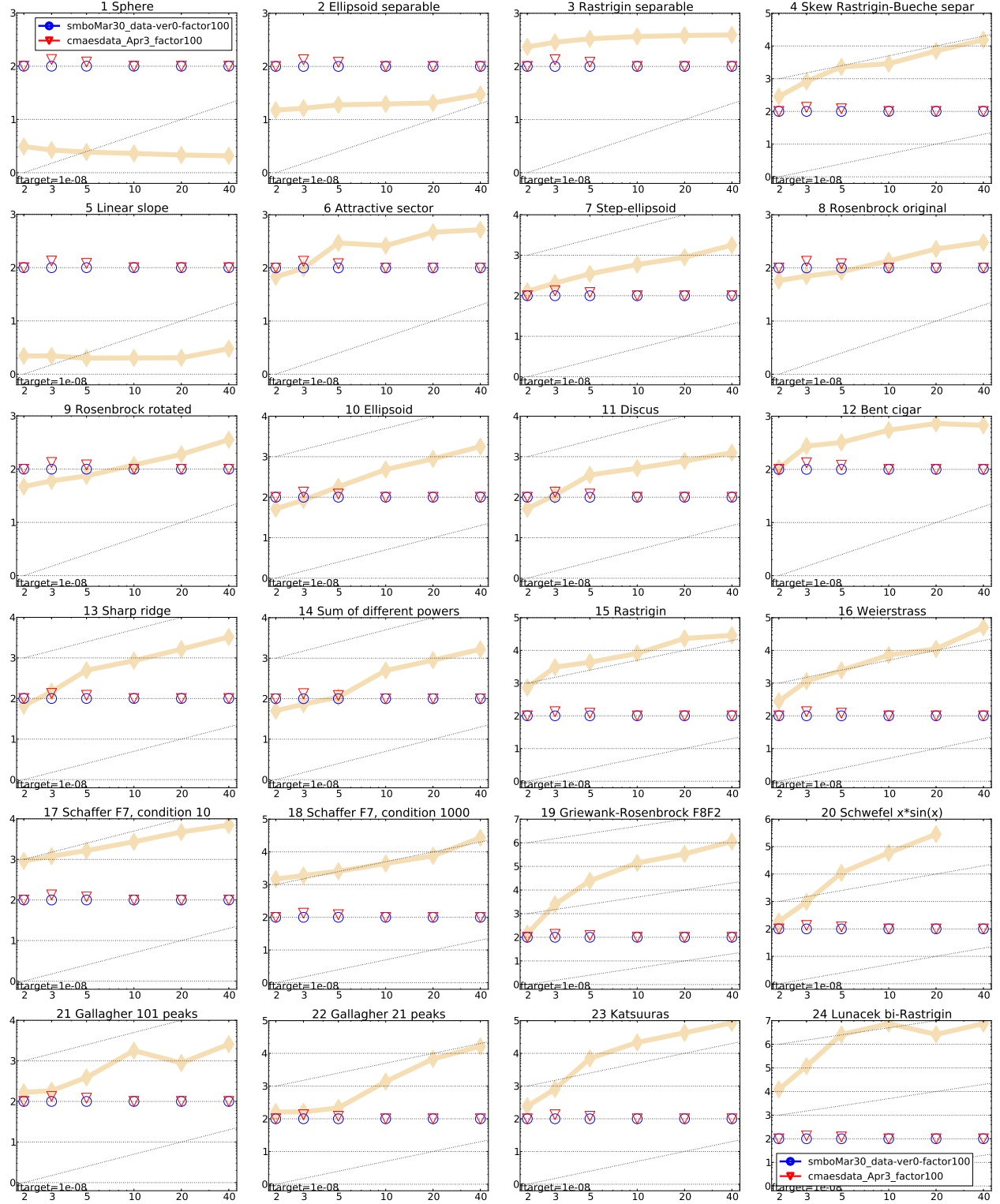


Figure 1: Expected running time (ERT in number of  $f$ -evaluations) divided by dimension for target function value  $10^{-8}$  as  $\log_{10}$  values versus dimension. Different symbols correspond to different algorithms given in the legend of  $f_1$  and  $f_{24}$ . Light symbols give the maximum number of function evaluations from the longest trial divided by dimension. Horizontal lines give linear scaling, slanted dotted lines give quadratic scaling. Black stars indicate statistically better result compared to all other algorithms with  $p < 0.01$  and Bonferroni correction number of dimensions (six). Legend:  $\circ$ :SMBO,  $\nabla$ :CMAES.

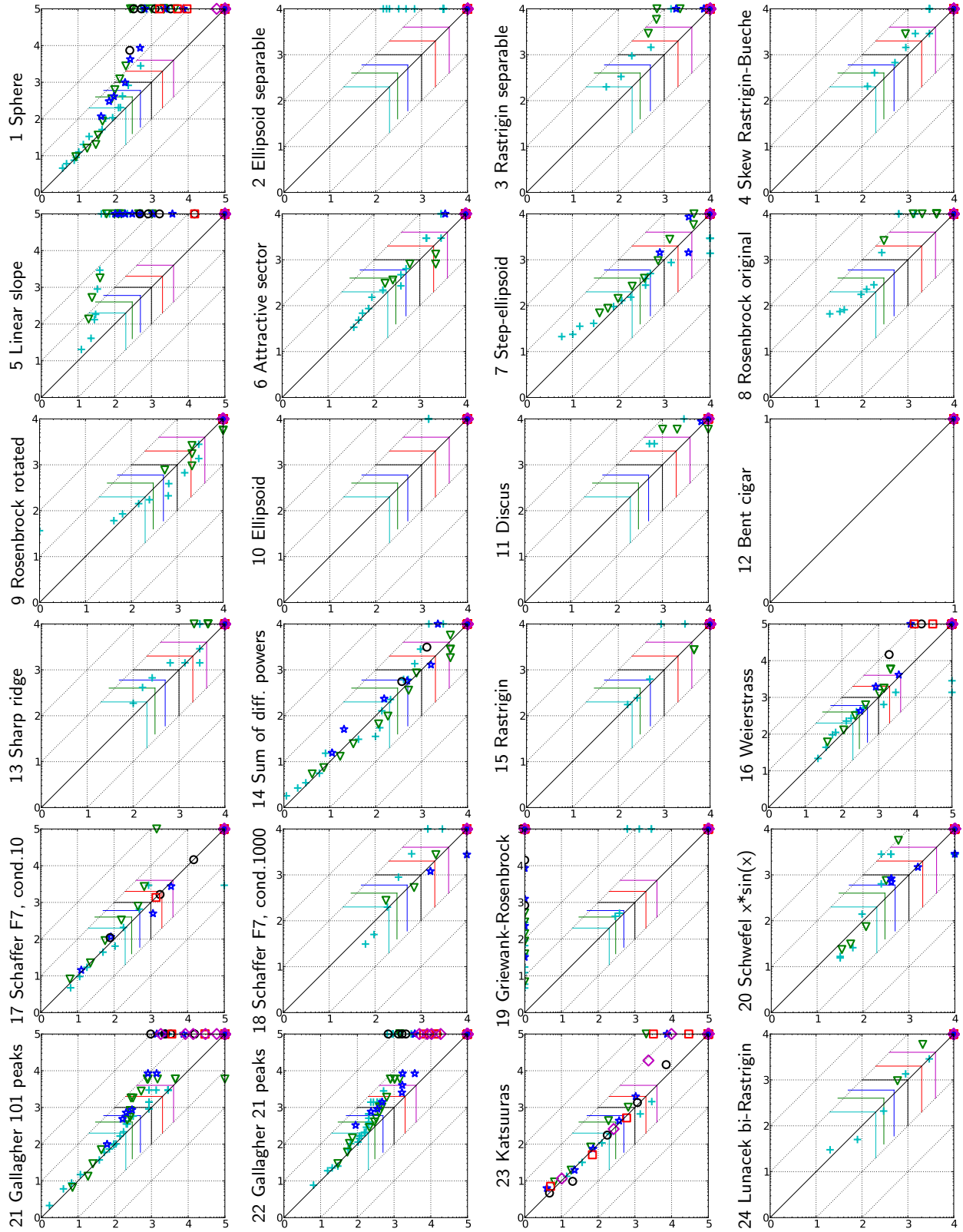
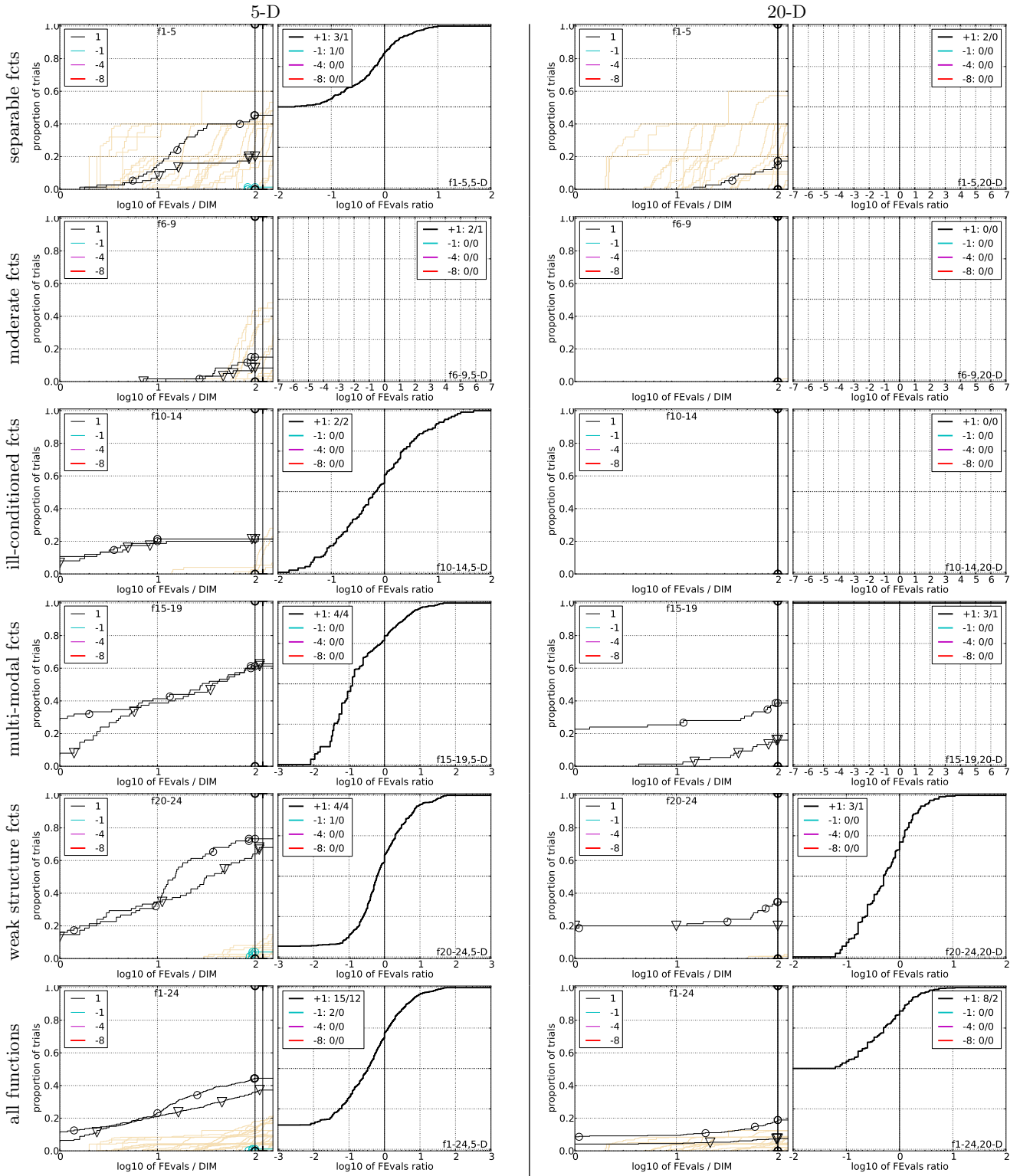


Figure 2: Expected running time (ERT in  $\log_{10}$  of number of function evaluations) of SMBO ( $x$ -axis) versus CMAES ( $y$ -axis) for 46 target values  $\Delta f \in [10^{-8}, 10]$  in each dimension on functions  $f_1$ – $f_{24}$ . Markers on the upper or right edge indicate that the target value was never reached. Markers represent dimension: 2: +, 3:  $\nabla$ , 5: \*, 10:  $\circ$ , 20:  $\square$ , 40:  $\diamond$ .



**Figure 3: Empirical cumulative distributions (ECDF) of run lengths and speed-up ratios in 5-D (left) and 20-D (right).** Left sub-columns: ECDF of the number of function evaluations divided by dimension  $D$  (FEvals/ $D$ ) to reach a target value  $f_{\text{opt}} + \Delta f$  with  $\Delta f = 10^k$ , where  $k \in \{1, -1, -4, -8\}$  is given by the first value in the legend, for SMBO ( $\circ$ ) and CMAES ( $\nabla$ ). Light beige lines show the ECDF of FEvals for target value  $\Delta f = 10^{-8}$  of all algorithms benchmarked during BBOB-2009. Right sub-columns: ECDF of FEval ratios of SMBO divided by CMAES, all trial pairs for each function. Pairs where both trials failed are disregarded, pairs where one trial failed are visible in the limits being  $> 0$  or  $< 1$ . The legends indicate the number of functions that were solved in at least one trial (SMBO first).

| 5-D                   |                 |                   |             |             |                  |       | 20-D                  |                   |          |          |          |                 |       |
|-----------------------|-----------------|-------------------|-------------|-------------|------------------|-------|-----------------------|-------------------|----------|----------|----------|-----------------|-------|
| $\Delta f$            | 1e+1            | 1e-1              | 1e-3        | 1e-5        | 1e-7             | #succ | $\Delta f$            | 1e+1              | 1e-1     | 1e-3     | 1e-5     | 1e-7            | #succ |
| <b>f<sub>1</sub></b>  | 11              | 12                | 12          | 12          | 12               | 15/15 | <b>f<sub>1</sub></b>  | 43                | 43       | 43       | 43       | 43              | 15/15 |
| 1: SMBO               | 3.9(3)          | <b>608(625)*2</b> | $\infty^*2$ | $\infty^*2$ | $\infty^{500*2}$ | 0/15  | 1: SMBO               | <b>39(35)*3</b>   | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| 2: CMAES              | 11(19)          | $\infty$          | $\infty$    | $\infty$    | $\infty^{600}$   | 0/15  | 2: CMAES              | $\infty$          | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| <b>f<sub>2</sub></b>  | 83              | 88                | 90          | 92          | 94               | 15/15 | <b>f<sub>2</sub></b>  | 385               | 387      | 390      | 391      | 393             | 15/15 |
| 1: SMBO               | $\infty^*2$     | $\infty^*2$       | $\infty^*2$ | $\infty^*2$ | $\infty^{500*2}$ | 0/15  | 1: SMBO               | $\infty$          | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| 2: CMAES              | $\infty$        | $\infty$          | $\infty$    | $\infty$    | $\infty^{600}$   | 0/15  | 2: CMAES              | $\infty$          | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| <b>f<sub>3</sub></b>  | 716             | 1637              | 1646        | 1650        | 1654             | 15/15 | <b>f<sub>3</sub></b>  | 5066              | 7635     | 7643     | 7646     | 7651            | 15/15 |
| 1: SMBO               | <b>2.5(2)*3</b> | $\infty^*3$       | $\infty^*3$ | $\infty^*3$ | $\infty^{500*3}$ | 0/15  | 1: SMBO               | $\infty$          | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| 2: CMAES              | $\infty$        | $\infty$          | $\infty$    | $\infty$    | $\infty^{600}$   | 0/15  | 2: CMAES              | $\infty$          | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| <b>f<sub>4</sub></b>  | 809             | 1688              | 1817        | 1886        | 1903             | 15/15 | <b>f<sub>4</sub></b>  | 4722              | 7666     | 7700     | 7758     | 1.4e5           | 9/15  |
| 1: SMBO               | $\infty^*3$     | $\infty^*3$       | $\infty^*3$ | $\infty^*3$ | $\infty^{500*3}$ | 0/15  | 1: SMBO               | $\infty$          | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| 2: CMAES              | $\infty$        | $\infty$          | $\infty$    | $\infty$    | $\infty^{600}$   | 0/15  | 2: CMAES              | $\infty$          | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| <b>f<sub>5</sub></b>  | 10              | 10                | 10          | 10          | 10               | 15/15 | <b>f<sub>5</sub></b>  | 41                | 41       | 41       | 41       | 41              | 15/15 |
| 1: SMBO               | <b>10(5)*3</b>  | $\infty^*3$       | $\infty^*3$ | $\infty^*3$ | $\infty^{500*3}$ | 0/15  | 1: SMBO               | <b>369(357)*3</b> | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| 2: CMAES              | $\infty$        | $\infty$          | $\infty$    | $\infty$    | $\infty^{600}$   | 0/15  | 2: CMAES              | $\infty$          | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| <b>f<sub>6</sub></b>  | 114             | 281               | 580         | 1038        | 1332             | 15/15 | <b>f<sub>6</sub></b>  | 1296              | 3413     | 5220     | 6728     | 8409            | 15/15 |
| 1: SMBO               | 31(31)          | $\infty$          | $\infty$    | $\infty$    | $\infty^{500}$   | 0/15  | 1: SMBO               | $\infty$          | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| 2: CMAES              | $\infty$        | $\infty$          | $\infty$    | $\infty$    | $\infty^{600}$   | 0/15  | 2: CMAES              | $\infty$          | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| <b>f<sub>7</sub></b>  | 24              | 1171              | 1572        | 1572        | 1597             | 15/15 | <b>f<sub>7</sub></b>  | 1351              | 9503     | 16524    | 16524    | 16969           | 15/15 |
| 1: SMBO               | 34(35)          | $\infty$          | $\infty$    | $\infty$    | $\infty^{500}$   | 0/15  | 1: SMBO               | $\infty$          | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| 2: CMAES              | 61(70)          | $\infty$          | $\infty$    | $\infty$    | $\infty^{600}$   | 0/15  | 2: CMAES              | $\infty$          | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| <b>f<sub>8</sub></b>  | 73              | 336               | 391         | 410         | 422              | 15/15 | <b>f<sub>8</sub></b>  | 2039              | 4040     | 4219     | 4371     | 4484            | 15/15 |
| 1: SMBO               | $\infty$        | $\infty$          | $\infty$    | $\infty$    | $\infty^{500}$   | 0/15  | 1: SMBO               | $\infty$          | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| 2: CMAES              | $\infty$        | $\infty$          | $\infty$    | $\infty$    | $\infty^{600}$   | 0/15  | 2: CMAES              | $\infty$          | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| <b>f<sub>9</sub></b>  | 35              | 214               | 300         | 335         | 369              | 15/15 | <b>f<sub>9</sub></b>  | 1716              | 3277     | 3455     | 3594     | 3727            | 15/15 |
| 1: SMBO               | $\infty^*3$     | $\infty^*3$       | $\infty^*3$ | $\infty^*3$ | $\infty^{500*3}$ | 0/15  | 1: SMBO               | $\infty$          | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| 2: CMAES              | $\infty$        | $\infty$          | $\infty$    | $\infty$    | $\infty^{600}$   | 0/15  | 2: CMAES              | $\infty$          | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| <b>f<sub>10</sub></b> | 349             | 574               | 626         | 829         | 880              | 15/15 | <b>f<sub>10</sub></b> | 7413              | 107351   | 492017   | 7073     | 17476           | 15/15 |
| 1: SMBO               | $\infty$        | $\infty$          | $\infty$    | $\infty$    | $\infty^{500}$   | 0/15  | 1: SMBO               | $\infty$          | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| 2: CMAES              | $\infty$        | $\infty$          | $\infty$    | $\infty$    | $\infty^{600}$   | 0/15  | 2: CMAES              | $\infty$          | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| <b>f<sub>11</sub></b> | 143             | 763               | 1177        | 1467        | 1673             | 15/15 | <b>f<sub>11</sub></b> | 1002              | 6278     | 9762     | 12285    | 14831           | 15/15 |
| 1: SMBO               | 49(58)          | $\infty$          | $\infty$    | $\infty$    | $\infty^{500}$   | 0/15  | 1: SMBO               | $\infty$          | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| 2: CMAES              | 62(72)          | $\infty$          | $\infty$    | $\infty$    | $\infty^{600}$   | 0/15  | 2: CMAES              | $\infty$          | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| <b>f<sub>12</sub></b> | 108             | 371               | 461         | 1303        | 1494             | 15/15 | <b>f<sub>12</sub></b> | 1042              | 2740     | 4140     | 12407    | 13827           | 15/15 |
| 1: SMBO               | $\infty$        | $\infty$          | $\infty$    | $\infty$    | $\infty^{500}$   | 0/15  | 1: SMBO               | $\infty$          | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| 2: CMAES              | $\infty$        | $\infty$          | $\infty$    | $\infty$    | $\infty^{600}$   | 0/15  | 2: CMAES              | $\infty$          | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| <b>f<sub>13</sub></b> | 132             | 250               | 1310        | 1752        | 2255             | 15/15 | <b>f<sub>13</sub></b> | 652               | 2751     | 18749    | 24455    | 30201           | 15/15 |
| 1: SMBO               | $\infty^*3$     | $\infty^*3$       | $\infty^*3$ | $\infty^*3$ | $\infty^{500*3}$ | 0/15  | 1: SMBO               | $\infty$          | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| 2: CMAES              | $\infty$        | $\infty$          | $\infty$    | $\infty$    | $\infty^{600}$   | 0/15  | 2: CMAES              | $\infty$          | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| <b>f<sub>14</sub></b> | 10              | 58                | 139         | 251         | 476              | 15/15 | <b>f<sub>14</sub></b> | 75                | 304      | 932      | 1648     | 15661           | 15/15 |
| 1: SMBO               | 1.1(1)          | $\infty$          | $\infty$    | $\infty$    | $\infty^{500}$   | 0/15  | 1: SMBO               | $\infty$          | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| 2: CMAES              | 1.6(2)          | $\infty$          | $\infty$    | $\infty$    | $\infty^{600}$   | 0/15  | 2: CMAES              | $\infty$          | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| <b>f<sub>15</sub></b> | 511             | 19369             | 20073       | 20769       | 21359            | 14/15 | <b>f<sub>15</sub></b> | 30378             | $3.1e5$  | $3.2e5$  | $4.5e5$  | $4.6e5$         | 15/15 |
| 1: SMBO               | $\infty$        | $\infty$          | $\infty$    | $\infty$    | $\infty^{500}$   | 0/15  | 1: SMBO               | $\infty$          | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| 2: CMAES              | $\infty$        | $\infty$          | $\infty$    | $\infty$    | $\infty^{600}$   | 0/15  | 2: CMAES              | $\infty$          | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| <b>f<sub>16</sub></b> | 120             | 2662              | 10449       | 11644       | 12095            | 15/15 | <b>f<sub>16</sub></b> | 1384              | 77015    | $1.9e5$  | $2.0e5$  | $2.2e5$         | 15/15 |
| 1: SMBO               | 2.7(3)          | $\infty$          | $\infty$    | $\infty$    | $\infty^{500}$   | 0/15  | 1: SMBO               | <b>6.8(7)*3</b>   | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| 2: CMAES              | 3.6(4)          | $\infty$          | $\infty$    | $\infty$    | $\infty^{600}$   | 0/15  | 2: CMAES              | $\infty$          | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| <b>f<sub>17</sub></b> | 5.2             | 899               | 3669        | 6351        | 7934             | 15/15 | <b>f<sub>17</sub></b> | 63                | 4005     | 30677    | 56288    | 80472           | 15/15 |
| 1: SMBO               | 2.4(3)          | $\infty$          | $\infty$    | $\infty$    | $\infty^{500}$   | 0/15  | 1: SMBO               | 22(28)            | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| 2: CMAES              | 2.8(2)          | $\infty$          | $\infty$    | $\infty$    | $\infty^{600}$   | 0/15  | 2: CMAES              | 22(23)            | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| <b>f<sub>18</sub></b> | 103             | 3968              | 9280        | 10905       | 12469            | 15/15 | <b>f<sub>18</sub></b> | 621               | 19561    | 67569    | $1.3e5$  | $1.5e5$         | 15/15 |
| 1: SMBO               | 16(18)          | $\infty$          | $\infty$    | $\infty$    | $\infty^{500}$   | 0/15  | 1: SMBO               | $\infty$          | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| 2: CMAES              | 12(12)          | $\infty$          | $\infty$    | $\infty$    | $\infty^{600}$   | 0/15  | 2: CMAES              | $\infty$          | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| <b>f<sub>19</sub></b> | 1               | 242               | $1.2e5$     | $1.2e5$     | $1.2e5$          | 15/15 | <b>f<sub>19</sub></b> | 1                 | $3.4e5$  | $6.2e6$  | $6.7e6$  | $6.7e6$         | 15/15 |
| 1: SMBO               | <b>1*3</b>      | $\infty^*3$       | $\infty^*3$ | $\infty^*3$ | $\infty^{500*3}$ | 0/15  | 1: SMBO               | <b>1*3</b>        | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| 2: CMAES              | 33(42)          | $\infty$          | $\infty$    | $\infty$    | $\infty^{600}$   | 0/15  | 2: CMAES              | $\infty$          | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| <b>f<sub>20</sub></b> | 16              | 38111             | 54470       | 54861       | 55313            | 14/15 | <b>f<sub>20</sub></b> | 82                | $3.1e6$  | $5.5e6$  | $5.6e6$  | $5.6e6$         | 14/15 |
| 1: SMBO               | 26(31)          | $\infty$          | $\infty$    | $\infty$    | $\infty^{500}$   | 0/15  | 1: SMBO               | $\infty$          | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| 2: CMAES              | 44(48)          | $\infty$          | $\infty$    | $\infty$    | $\infty^{600}$   | 0/15  | 2: CMAES              | $\infty$          | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| <b>f<sub>21</sub></b> | 41              | 1674              | 1705        | 1729        | 1757             | 14/15 | <b>f<sub>21</sub></b> | 561               | 14103    | 14643    | 15567    | 17589           | 15/15 |
| 1: SMBO               | 1.5(1.0)        | 1.5(1)            | $\infty$    | $\infty$    | $\infty^{500}$   | 0/15  | 1: SMBO               | <b>6.4(6)*3</b>   | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| 2: CMAES              | 2.4(3)          | $\infty$          | $\infty$    | $\infty$    | $\infty^{600}$   | 0/15  | 2: CMAES              | $\infty$          | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| <b>f<sub>22</sub></b> | 71              | 938               | 1008        | 1040        | 1068             | 14/15 | <b>f<sub>22</sub></b> | 467               | 23491    | 24948    | 26847    | $1.3e5$         | 12/15 |
| 1: SMBO               | 1.3(1)          | $\infty$          | $\infty$    | $\infty$    | $\infty^{500}$   | 0/15  | 1: SMBO               | <b>14(15)*3</b>   | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| 2: CMAES              | 4.6(5)          | $\infty$          | $\infty$    | $\infty$    | $\infty^{600}$   | 0/15  | 2: CMAES              | $\infty$          | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| <b>f<sub>23</sub></b> | 3.0             | 14249             | 31654       | 33030       | 34256            | 15/15 | <b>f<sub>23</sub></b> | 3.2               | 67457    | $4.9e5$  | $8.1e5$  | $8.4e5$         | 15/15 |
| 1: SMBO               | 1.3(1)          | $\infty$          | $\infty$    | $\infty$    | $\infty^{500}$   | 0/15  | 1: SMBO               | 1.6(1)            | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| 2: CMAES              | 2.1(2)          | $\infty$          | $\infty$    | $\infty$    | $\infty^{600}$   | 0/15  | 2: CMAES              | 2.2(2)            | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| <b>f<sub>24</sub></b> | 1622            | $6.4e6$           | $9.6e6$     | $1.3e7$     | $1.3e7$          | 3/15  | <b>f<sub>24</sub></b> | $1.3e6$           | $5.2e7$  | $5.2e7$  | $5.2e7$  | $5.2e7$         | 3/15  |
| 1: SMBO               | $\infty^*$      | $\infty^*$        | $\infty^*$  | $\infty^*$  | $\infty^{500*}$  | 0/15  | 1: SMBO               | $\infty$          | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |
| 2: CMAES              | $\infty$        | $\infty$          | $\infty$    | $\infty$    | $\infty^{600}$   | 0/15  | 2: CMAES              | $\infty$          | $\infty$ | $\infty$ | $\infty$ | $\infty^{2000}$ | 0/15  |

Table 1: ERT in number of function evaluations divided by the best ERT measured during BBOB-2009 given in the respective first row with the central 80% range divided by two in brackets for different  $\Delta f$  values. #succ is the number of trials that reached the final target  $f_{\text{opt}} + 10^{-8}$ . 1:SMBO is SMBO and 2:CMAES is CMAES. Bold entries are statistically significantly better compared to the other algorithm, with  $p = 0.05$  or  $p = 10^{-k}$  where  $k \in \{2, 3, 4, \dots\}$  is the number following the  $\star$  symbol, with Bonferroni correction of 48. A  $\downarrow$  indicates the same tested against the best BBOB-2009.