

# Black-Box Optimization Benchmarking Template for Noiseless Function Testbed

Draft version \*

Forename Name

## ABSTRACT

### Categories and Subject Descriptors

G.1.6 [Numerical Analysis]: Optimization—*global optimization, unconstrained optimization*; F.2.1 [Analysis of Algorithms and Problem Complexity]: Numerical Algorithms and Problems

### General Terms

Algorithms

### Keywords

Benchmarking, Black-box optimization

## 1. RESULTS

Results of DIRECT from experiments according to [2] on the benchmark functions given in [1, 3] are presented in Figures 1, 2 and 4 and in Tables 1 and 3.

## 2. REFERENCES

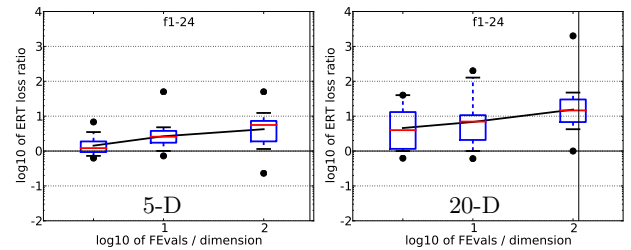
- [1] S. Finck, N. Hansen, R. Ros, and A. Auger. Real-parameter black-box optimization benchmarking 2009: Presentation of the noiseless functions. Technical Report 2009/20, Research Center PPE, 2009. Updated February 2010.
- [2] N. Hansen, A. Auger, S. Finck, and R. Ros. Real-parameter black-box optimization benchmarking 2012: Experimental setup. Technical report, INRIA, 2012.
- [3] N. Hansen, S. Finck, R. Ros, and A. Auger. Real-parameter black-box optimization benchmarking 2009: Noiseless functions definitions. Technical Report RR-6829, INRIA, 2009. Updated February 2010.

\*Submission deadline: March 28th.

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<i>f1-f24 in 5-D, maxFE/D=289</i>						
#FEs/D	best	10%	25%	med	75%	90%
2	0.63	0.70	0.91	1.2	2.0	3.7
10	0.73	1.0	1.7	2.6	3.9	5.1
100	0.23	1.1	1.7	5.6	7.4	16
1e3	0.41	2.7	5.0	21	40	95
RL <sub>US</sub> /D	1e2	1e2	1e2	1e2	1e2	1e2

<i>f1-f24 in 20-D, maxFE/D=114</i>						
#FEs/D	best	10%	25%	med	75%	90%
2	0.62	0.98	1.1	4.0	13	40
10	0.61	0.97	2.0	7.0	11	1.7e2
100	1.0	3.9	6.7	15	30	49
1e3	1.0	15	38	74	1.6e2	4.4e2
RL <sub>US</sub> /D	1e2	1e2	1e2	1e2	1e2	1e2

**Figure 3:** ERT loss ratio versus the budget (both in number of  $f$ -evaluations divided by dimension). The target value  $f_t$  for a given budget FEvals is the best target  $f$ -value reached within the budget by the given algorithm. Shown is the ERT of the given algorithm divided by best ERT seen in GECCO-BBOB-2009 for the target  $f_t$ , or, if the best algorithm reached a better target within the budget, the budget divided by the best ERT. Line: geometric mean. Box-Whisker error bar: 25-75%-ile with median (box), 10-90%-ile (caps), and minimum and maximum ERT loss ratio (points). The vertical line gives the maximal number of function evaluations in a single trial in this function subset. See also Figure 4 for results on each function subgroup.

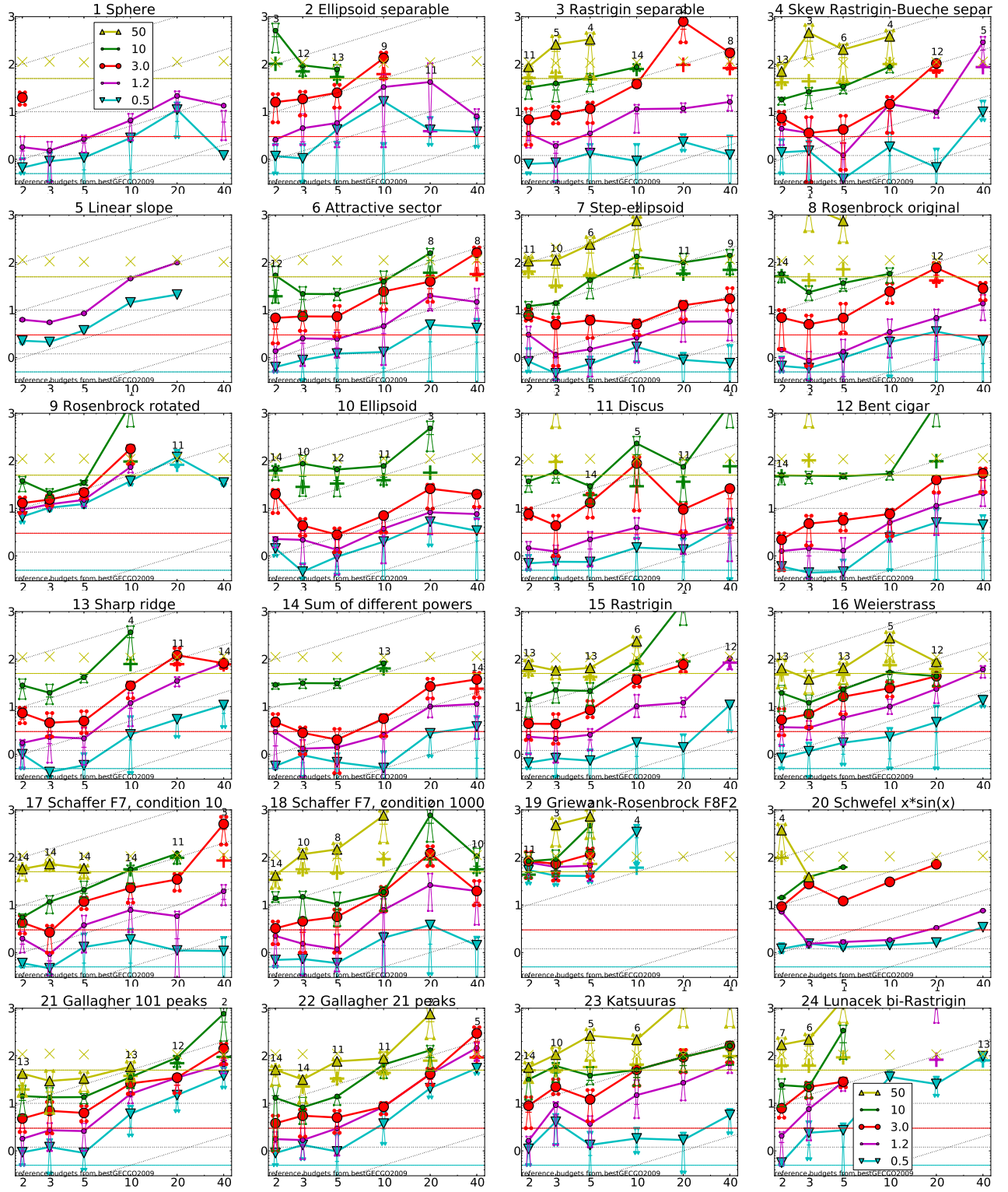
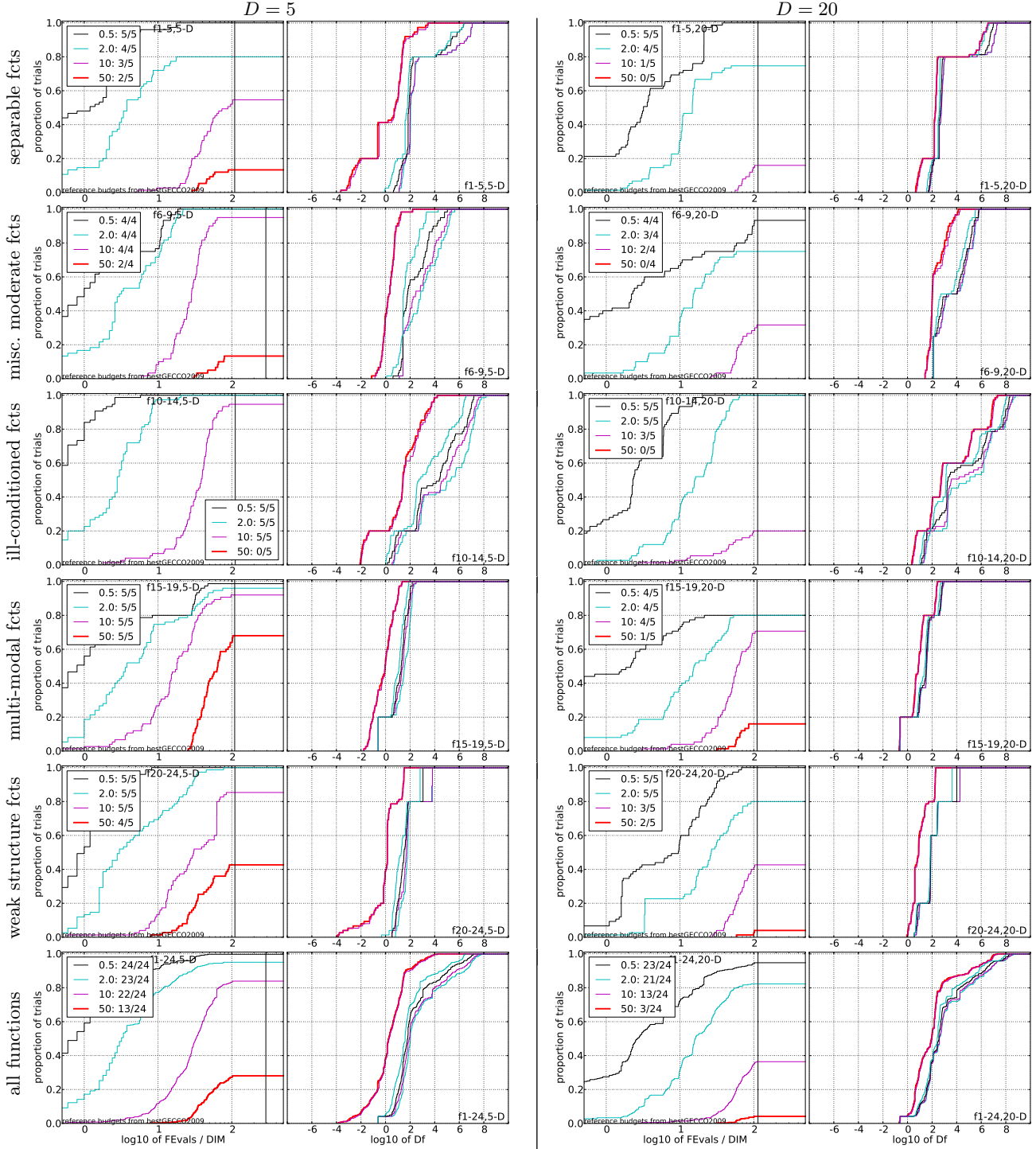


Figure 1: Expected number of  $f$ -evaluations (ERT, lines) to reach  $f_{\text{opt}} + \Delta f$ ; median number of  $f$ -evaluations (+) to reach the most difficult target that was reached not always but at least once; maximum number of  $f$ -evaluations in any trial (x); interquartile range with median (notched boxes) of simulated runlengths to reach  $f_{\text{opt}} + \Delta f$ ; all values are divided by dimension and plotted as  $\log_{10}$  values versus dimension. Shown is the ERT for targets just not reached by the GECCO-BBOB-2009 best algorithm within the given budget  $k\text{DIM}$ , where  $k$  is shown in the legend. Numbers above ERT-symbols indicate the number of trials reaching the respective target. Slanted grid lines indicate a scaling with  $\mathcal{O}(\text{DIM})$  compared to  $\mathcal{O}(1)$  when using the respective 2009 best algorithm.



**Figure 2:** Empirical cumulative distribution functions (ECDF), plotting the fraction of trials with an outcome not larger than the respective value on the  $x$ -axis. Left subplots: ECDF of number of function evaluations (FEvals) divided by search space dimension  $D$ , to fall below  $f_{\text{opt}} + \Delta f$  where  $\Delta f$  is the target just not reached by the GECCO-BBOB-2009 best algorithm within a budget of  $k \times \text{DIM}$  evaluations, where  $k$  is the first value in the legend. Legends indicate for each target the number of functions that were solved in at least one trial within the displayed budget. Right subplots: ECDF of the best achieved  $\Delta f$  for running times of  $0.5D, 1.2D, 3D, 10D, 100D, 1000D, \dots$  function evaluations (from right to left cycling cyan-magenta-black...) and final  $\Delta f$ -value (red), where  $\Delta f$  and  $Df$  denote the difference to the optimal function value.

#FEs/D	5-D					#succ
	0.5	1.2	3.0	10	50	
<b>f<sub>1</sub></b>	<i>2.5e+1:4.8</i> 1.1(1)	<i>1.6e+1:7.6</i> 1.7(2)	<i>1.0e-8:12</i> ∞	<i>1.0e-8:12</i> ∞	<i>1.0e-8:12</i> ∞520	15/15 0/15
<b>f<sub>2</sub></b>	<i>1.6e+6:2.9</i> 7.4(15)	<i>4.0e+5:11</i> 2.6(4)	<i>4.0e+4:15</i> 8.2(9)	<i>6.3e+2:58</i> 6.8(5)	<i>1.0e-8:95</i> ∞510	15/15 0/15
<b>f<sub>3</sub></b>	<i>1.6e+2:4.1</i> 1.6(2)	<i>1.0e+2:15</i> 1.2(1)	<i>6.3e+1:23</i> 2.5(2)	<i>2.5e+1:73</i> 3.6(4)	<i>1.0e+1:716</i> 2.3(3)	15/15 4/15
<b>f<sub>4</sub></b>	<i>2.5e+2:2.6</i> 0.74(1)	<i>1.6e+2:10</i> 0.63(0.5)	<i>1.0e+2:19</i> 1.1(2)	<i>4.0e+1:65</i> 2.6(1)	<i>1.6e+1:434</i> 2.3(3)	15/15 6/15
<b>f<sub>5</sub></b>	<i>6.3e+1:4.0</i> 4.7(0.1)	<i>4.0e+1:10</i> 4.3(0.1)	<i>1.0e-8:10</i> ∞	<i>1.0e-8:10</i> ∞	<i>1.0e-8:10</i> ∞520	15/15 0/15
<b>f<sub>6</sub></b>	<i>1.0e+5:3.0</i> 2.0(2)	<i>2.5e+4:8.4</i> 1.5(2)	<i>1.0e+2:16</i> 2.3(2)	<i>2.5e+1:54</i> 2.0(1)	<i>2.5e-1:254</i> ∞520	15/15 0/15
<b>f<sub>7</sub></b>	<i>1.6e+2:4.2</i> 0.87(0.8)	<i>1.0e+2:6.2</i> 1.2(1)	<i>2.5e+1:20</i> 1.5(1)	<i>4.0e+0:54</i> 3.9(5)	<i>1.0e+0:324</i> 3.7(4)	15/15 6/15
<b>f<sub>8</sub></b>	<i>1.0e+4:4.6</i> 1.1(0.8)	<i>6.3e+3:6.8</i> 0.98(1)	<i>1.0e+3:18</i> 1.9(2)	<i>6.3e+1:54</i> 3.4(2)	<i>1.6e+0:258</i> 15(15)	15/15 2/15
<b>f<sub>9</sub></b>	<i>2.5e+1:20</i> 3.1(0.9)	<i>1.6e+1:26</i> 2.9(1)	<i>1.0e+1:35</i> 3.1(0.9)	<i>4.0e+0:62</i> 2.8(1)	<i>1.6e-2:256</i> ∞520	15/15 0/15
<b>f<sub>10</sub></b>	<i>2.5e+6:2.9</i> 1.7(2)	<i>6.3e+5:7.0</i> 0.96(1)	<i>2.5e+5:17</i> 0.82(0.8)	<i>6.3e+3:54</i> 6.1(7)	<i>2.5e+1:297</i> ∞520	15/15 0/15
<b>f<sub>11</sub></b>	<i>1.0e+3:3.0</i> 1.2(0.7)	<i>6.3e+4:6.2</i> 1.8(3)	<i>6.3e+2:16</i> 4.1(3)	<i>6.3e+1:74</i> 2.0(1)	<i>6.3e-1:298</i> ∞510	15/15 0/15
<b>f<sub>12</sub></b>	<i>4.0e+7:3.6</i> 0.65(0.6)	<i>1.6e+7:7.6</i> 0.85(0.9)	<i>1.0e+6:19</i> 1.5(0.8)	<i>1.6e+1:52</i> 4.6(0.7)	<i>1.0e+0:268</i> ∞520	15/15 0/15
<b>f<sub>13</sub></b>	<i>1.0e+3:2.8</i> 1.0(1)	<i>6.3e+2:8.4</i> 1.3(2)	<i>6.3e+2:17</i> 1.5(0.9)	<i>6.3e+1:52</i> 4.0(1)	<i>6.3e-2:264</i> ∞520	15/15 0/15
<b>f<sub>14</sub></b>	<i>1.6e+1:3.0</i> 1.1(1)	<i>1.0e+1:10</i> 0.99(1)	<i>6.3e+0:15</i> 0.66(0.6)	<i>2.5e-1:53</i> 2.9(1.0)	<i>1.0e-5:251</i> ∞520	15/15 0/15
<b>f<sub>15</sub></b>	<i>1.6e+2:3.0</i> 1.2(2)	<i>1.0e+2:13</i> 0.9(1)	<i>6.3e+1:24</i> 1.7(1)	<i>4.0e+1:55</i> 1.9(1)	<i>1.6e+1:289</i> 1.1(1.0)	5/5 13/15
<b>f<sub>16</sub></b>	<i>4.0e+1:4.8</i> 1.8(1)	<i>2.5e+1:16</i> 1.8(2)	<i>1.6e+1:46</i> 1.7(1)	<i>1.0e+1:120</i> 0.96(0.5)	<i>4.0e+0:334</i> 0.98(0.8)	15/15 13/15
<b>f<sub>17</sub></b>	<i>1.0e+1:5.2</i> 1.3(1)	<i>6.3e+0:26</i> 0.73(0.7)	<i>1.0e+0:57</i> 1.0(0.5)	<i>2.5e+0:110</i> 0.95(0.5)	<i>6.3e-1:412</i> 0.72(0.4)	15/15 14/15
<b>f<sub>18</sub></b>	<i>6.3e+1:3.4</i> 0.88(1)	<i>4.0e+1:7.2</i> 0.81(1.0)	<i>2.5e+1:20</i> 1.4(2)	<i>1.6e+1:58</i> 0.90(1)	<i>1.6e+0:318</i> 2.3(2)	15/15 8/15
<b>f<sub>19</sub></b>	<i>1.6e-1:172</i> 1.2(0.3)	<i>1.0e-1:242</i> 1.4(1)	<i>6.3e-2:675</i> 0.86(0.8)	<i>4.0e-2:3078</i> 0.76(0.8)	<i>2.5e-2:4946</i> 0.74(0.8)	15/15 2/15
<b>f<sub>20</sub></b>	<i>6.3e+3:5.1</i> 1.3(0.1)	<i>4.0e+3:8.4</i> 0.99(0.1)	<i>4.0e+1:15</i> 4.0(0.0)	<i>2.5e+0:69</i> 4.5(0.0)	<i>1.0e+0:851</i> ∞520	15/15 0/15
<b>f<sub>21</sub></b>	<i>4.0e+1:3.9</i> 1.2(1)	<i>2.5e+1:11</i> 1.2(1)	<i>1.6e+1:31</i> 1.0(1)	<i>6.3e+0:73</i> 0.93(0.4)	<i>1.6e+0:347</i> 0.48(0.3)	5/5 15/15
<b>f<sub>22</sub></b>	<i>6.3e+1:3.6</i> 1.4(2)	<i>4.0e+1:15</i> 1(0.9)	<i>2.5e+1:32</i> 0.78(0.6)	<i>1.0e+1:71</i> 0.98(0.8)	<i>1.6e+0:341</i> 1.1(1)	5/5 11/15
<b>f<sub>23</sub></b>	<i>1.0e+1:3.0</i> 2.2(2)	<i>6.3e+0:9.0</i> 2.1(2)	<i>4.0e+0:33</i> 1.8(2)	<i>2.5e+0:84</i> 2.3(1)	<i>1.0e+0:518</i> 2.6(3)	15/15 5/15
<b>f<sub>24</sub></b>	<i>6.3e+1:15</i> 0.93(1)	<i>4.0e+1:37</i> 3.9(2)	<i>4.0e+1:37</i> 3.9(2)	<i>2.5e+1:118</i> 14(16)	<i>1.6e+1:692</i> 11(13)	15/15 1/15

#FEs/D	20-D					#succ
	0.5	1.2	3.0	10	50	
<b>f<sub>1</sub></b>	<i>6.3e+1:24</i> 9.1(11)	<i>4.0e+1:42</i> 10(7)	<i>1.0e-8:43</i> ∞	<i>1.0e-8:43</i> ∞	<i>1.0e-8:43</i> ∞2100	15/15 0/15
<b>f<sub>2</sub></b>	<i>4.0e+6:29</i> 2.9(3)	<i>2.5e+6:42</i> 20(26)	<i>1.0e+5:65</i> ∞	<i>1.0e+4:207</i> ∞	<i>1.0e-8:412</i> ∞2100	15/15 0/15
<b>f<sub>3</sub></b>	<i>6.3e+2:33</i> 1.4(1)	<i>4.0e+2:44</i> 5.3(3)	<i>1.6e+2:109</i> 144(162)	<i>1.0e+2:255</i> ∞	<i>2.5e+1:3277</i> ∞2100	15/15 0/15
<b>f<sub>4</sub></b>	<i>6.3e+2:22</i> 0.62(0.9)	<i>4.0e+2:91</i> 2.2(1)	<i>2.5e+2:250</i> 8.3(5)	<i>1.6e+2:332</i> ∞	<i>6.3e+1:1927</i> ∞2100	15/15 0/15
<b>f<sub>5</sub></b>	<i>2.5e+2:19</i> 22(0.0)	<i>1.6e+2:34</i> 58(0.0)	<i>1.0e-8:41</i> ∞	<i>1.0e-8:41</i> ∞	<i>1.0e-8:41</i> ∞2300	15/15 0/15
<b>f<sub>6</sub></b>	<i>2.5e+5:16</i> 6.2(12)	<i>6.3e+4:43</i> 9.3(11)	<i>1.6e+4:62</i> 13(15)	<i>1.6e+2:353</i> 8.9(7)	<i>1.6e+1:1078</i> ∞2100	15/15 0/15
<b>f<sub>7</sub></b>	<i>1.0e+3:11</i> 1.7(2)	<i>4.0e+2:39</i> 2.9(3)	<i>2.5e+2:74</i> 3.4(4)	<i>6.3e+1:319</i> 6.3(5)	<i>1.0e+1:1351</i> ∞2100	15/15 0/15
<b>f<sub>8</sub></b>	<i>4.0e+4:19</i> 3.8(6)	<i>2.5e+4:35</i> 3.9(5)	<i>4.0e+3:67</i> 23(19)	<i>2.5e+2:231</i> ∞	<i>1.6e+1:1470</i> ∞2100	15/15 0/15
<b>f<sub>9</sub></b>	<i>1.0e+2:357</i> 6.7(4)	<i>6.3e+1:560</i> ∞	<i>4.0e+1:684</i> ∞	<i>2.5e+1:756</i> ∞	<i>1.0e+1:1716</i> ∞2100	15/15 0/15
<b>f<sub>10</sub></b>	<i>1.6e+6:15</i> 7.0(10)	<i>1.0e+6:27</i> 6.0(7)	<i>4.0e+5:70</i> 7.4(6)	<i>6.3e+4:231</i> 42(46)	<i>4.0e+3:1015</i> ∞2100	15/15 0/15
<b>f<sub>11</sub></b>	<i>4.0e+4:11</i> 2.4(2)	<i>2.5e+3:27</i> 2.0(2)	<i>1.6e+2:313</i> 0.61(0.6)	<i>1.0e+2:481</i> 3.1(3)	<i>1.0e+1:1002</i> ∞2100	15/15 0/15
<b>f<sub>12</sub></b>	<i>1.0e+8:23</i> 4.3(6)	<i>6.3e+7:39</i> 5.7(7)	<i>2.5e+7:76</i> 11(7)	<i>4.0e+6:209</i> 153(155)	<i>1.0e+1:1042</i> ∞2200	15/15 0/15
<b>f<sub>13</sub></b>	<i>1.6e+3:28</i> 3.9(3)	<i>1.0e+3:64</i> 11(5)	<i>6.3e+2:79</i> 31(18)	<i>4.0e+1:211</i> ∞	<i>2.5e+0:1724</i> ∞2100	15/15 0/15
<b>f<sub>14</sub></b>	<i>2.5e+1:15</i> 3.8(5)	<i>1.6e+1:42</i> 4.8(3)	<i>1.0e+1:75</i> 7.1(4)	<i>1.6e+0:219</i> ∞	<i>6.3e-4:1106</i> ∞2100	15/15 0/15
<b>f<sub>15</sub></b>	<i>6.3e+2:15</i> 1.8(2)	<i>4.0e+2:67</i> 3.6(3)	<i>2.5e+2:292</i> 5.3(4)	<i>1.6e+2:846</i> 38(40)	<i>1.0e+2:1671</i> ∞2200	15/15 0/15
<b>f<sub>16</sub></b>	<i>4.0e+1:26</i> 3.6(4)	<i>2.5e+1:127</i> 3.7(3)	<i>1.6e+1:540</i> 1.6(0.8)	<i>1.6e+1:540</i> 1.6(0.8)	<i>1.0e+1:1384</i> 1.3(0.9)	15/15 12/15
<b>f<sub>17</sub></b>	<i>1.6e+1:11</i> 2.1(6)	<i>1.0e+1:63</i> 1.9(2)	<i>6.3e+0:305</i> 2.3(2)	<i>4.0e+0:468</i> 5.1(3)	<i>1.0e+0:1030</i> ∞2100	15/15 0/15
<b>f<sub>18</sub></b>	<i>4.0e+1:116</i> 0.65(1)	<i>2.5e+1:252</i> 2.1(2)	<i>1.6e+1:430</i> 5.7(5)	<i>1.0e+1:621</i> 25(24)	<i>4.0e+0:1096</i> ∞2100	15/15 0/15
<b>f<sub>19</sub></b>	<i>1.6e-1:2.5e5</i> ∞	<i>1.0e-1:3.4e5</i> ∞	<i>6.3e-2:3.4e5</i> ∞	<i>4.0e-2:3.4e5</i> ∞	<i>2.5e-2:3.4e5</i> ∞2000	3/15 0/15
<b>f<sub>20</sub></b>	<i>1.6e+4:38</i> <b>0.84(0.0)</b> ↓	<i>1.0e+4:42</i> 1.6(0.0)	<i>2.5e+2:62</i> 23(0.0)	<i>2.5e+0:250</i> ∞	<i>1.6e+0:2536</i> ∞2000	15/15 0/15
<b>f<sub>21</sub></b>	<i>6.3e+1:36</i> 8.3(7)	<i>4.0e+1:77</i> 9.1(6)	<i>4.0e+1:77</i> 9.1(6)	<i>1.6e+1:456</i> 4.0(3)	<i>4.0e+0:1094</i> ∞2100	15/15 0/15
<b>f<sub>22</sub></b>	<i>6.3e+1:45</i> 9.2(4)	<i>4.0e+1:68</i> 12(8)	<i>4.0e+1:68</i> 12(8)	<i>1.6e+1:231</i> 11(10)	<i>6.3e+0:1219</i> 12(13)	15/15 2/15
<b>f<sub>23</sub></b>	<i>6.3e+0:29</i> 1.2(1)	<i>4.0e+0:118</i> 4.6(4)	<i>2.5e+0:306</i> 6.1(4)	<i>2.5e+0:306</i> 6.1(4)	<i>1.0e+0:1614</i> 19(21)	15/15 1/15
<b>f<sub>24</sub></b>	<i>2.5e+2:208</i> 2.5(2)	<i>1.6e+2:918</i> 33(37)	<i>1.0e+2:6628</i> ∞	<i>6.3e+1:9885</i> ∞	<i>4.0e+1:31629</i> ∞2000	15/15 0/15

Table 1: Expected running time (ERT in number of function evaluations) divided by the best ERT measured during BBOB-2009. The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear in the second row of each cell, the best ERT (preceded by the target  $\Delta f$ -value in *italics*) in the first. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Bold entries are statistically significantly better (according to the rank-sum test) compared to the best algorithm in BBOB-2009, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k > 1$  is following the ↓ symbol, with Bonferroni correction by the number of functions.

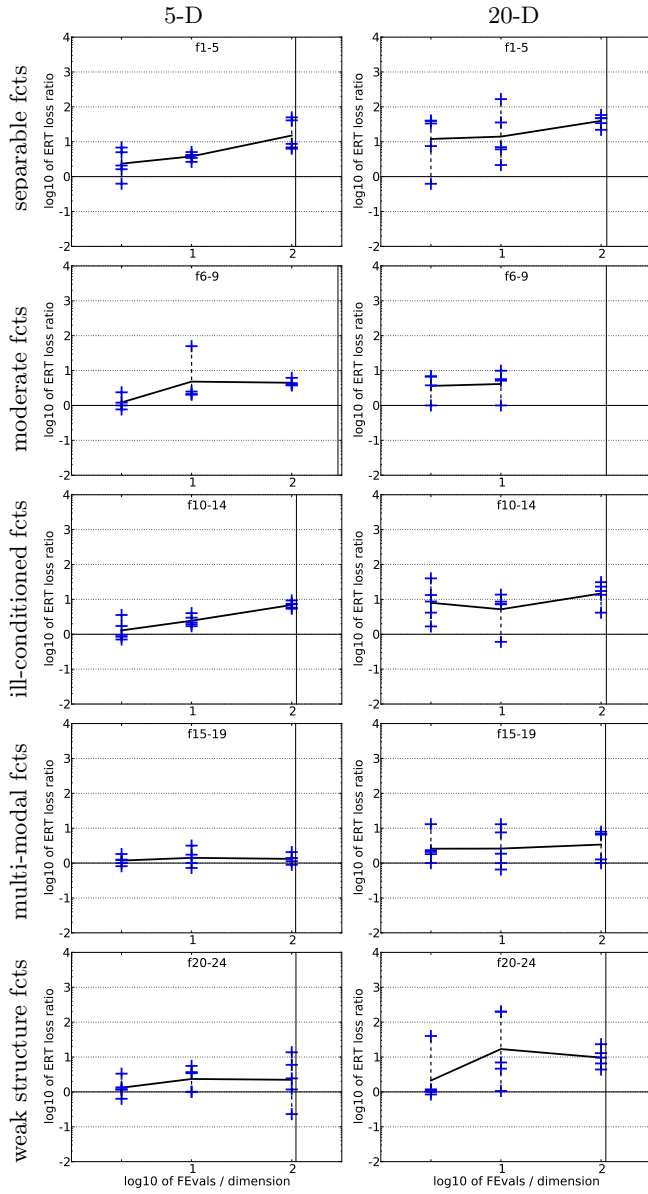


Figure 4: ERT loss ratios (see Figure 3 for details). Each cross (+) represents a single function, the line is the geometric mean.