Animation of Fish Swimming

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Abstract

We present a simple, two-part model of the locomotion of slender-bodied aquatic animals designed specifically for the needs of computer animation. The first part of the model is kinematic and addresses body deformations for three swimming modes: steady swimming, rapid starting, and turning. The second part of the model is dynamic and addresses the resulting propulsion of the aquatic animal. While this approach is not as general as a fully dynamic model, it provides the animator with a small set of intuitive parameters that directly control how the fish model moves and is more efficient to simulate.

1 Introduction

The computer animation of fish swimming is challenging task. Traditional animation techniques require the explicit specification of motion, e.g., keyframing requires positioning control points of a geometric model at key frames (interpolation then determines the positions at in-between frames). The difficulty of realizing all the subtleties of the motion of swimming fish using these techniques motivates other approaches.

One approach is to use a mass-spring system as a model of the body of a fish where variations in spring tensions (representing muscle contractions) are governed by a behavioural model [8]. This approach is quite powerful, but it can be difficult to realize specific swimming motions that may be required in a scripted animation sequence.

Our approach is focused on animator control. We present a simple kinematic model of several common swimming motions (body deformations) that can be easily tuned by animators. We then present a dynamic model of the resulting propulsive forces that govern the movement of the fish in a liquid medium.

2 Previous Work

Not surprisingly, locomotion has received significant attention from researchers in computer animation. However, most of this research has focused on legged locomotion. Sun and Metaxas [7] provide a recent review of this research, including both dynamic and kinematic approaches.

More relevant to fish swimming, Miller [4] modeled the locomotion of snakes and worms using mass-spring systems. Muscle contractions are simulated by varying the spring tensions. Forward motion results from not allowing masses in contact with the ground to move backwards. Each segment of the snake or worm is modeled as a cube of masses with springs along each edge and across the diagonal of each face where the force f exerted on the masses at the end of each spring is

$$f = k(L - l) - D\frac{dl}{dt}$$

where k is the spring constant, D is the damping, l is the current length of the spring, and L is the minimum energy spring length. For worms, waves of muscle contraction are modelled by traveling sine waves that affect the values of L. Directional friction is modelled by not allowing segments near the floor to slide backwards. For snakes, compression waves are again sent down the mass-spring system, but the springs on the left hand side of the snake are 180 degrees out of phase with those on the right hand side.

Tu and Terzopoulos [8] presented a framework for animating fish that includes models of locomotion and behaviour. The body of a fish is modelled as a mass-spring system. Muscle contractions are simulated by changing the resting length of “muscle” springs. These contractions are governed by “motor controllers” which are driven by their behavioural model. The propulsive force generated by the action of these muscle contractions is computed and used to derive the motion of the fish. This approach allows their virtual fishes to act as autonomous agents. However, the animator can only indirectly control the swimming motions.

3 Body Deformations

Most aquatic animals locomote by undulatory body motions that impart momentum to the surrounding fluid [1, 2]. (Evolutionarily less successful propulsion methods including beating cilia and jet reaction [3].) In this section we present a simple model of these body deformations for three swimming modes: steady swimming, rapid starting, and turning. In the next section we consider the resulting propulsion.

We model body deformations using a parametric piecewise linear curve that represents the backbone of a slender-bodied aquatic animal. This curve determines the shape of an associated surface model (Figure 1 shows a polyhedral model.) It has length l and n segments. We use a two dimensional body coordinate system with the origin at the center of mass and the x-axis coinciding with the resting backbone, $B(u)$, $0 \leq u \leq l$,

$$B(u) = \begin{bmatrix} x(u) \\ y(u) \end{bmatrix}$$

that interpolates $n + 1$ control points $B_i(u), i = 1, \ldots, n + 1$ where the control points are constrained such that the Euclidean distance between adjacent points is l/n. For the

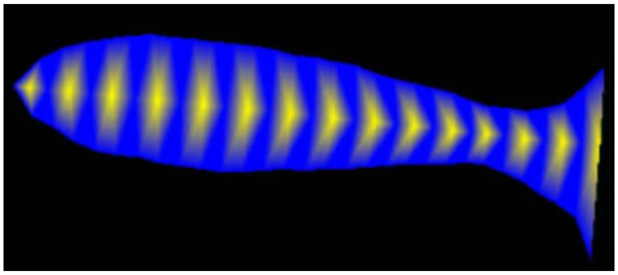


Figure 1: Polyhedral model of an aquatic animal.

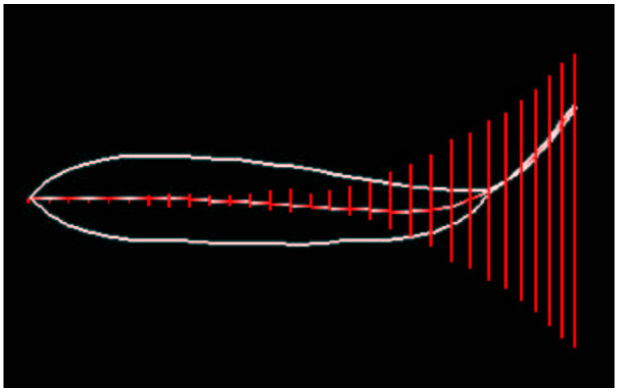


Figure 2: The kinematic model of steady swimming, based on a traveling sine wave, deforms the aquatic animal about its center of mass. The vertical red lines show the magnitude of the amplitude of the traveling sine wave at each vertebra joint.

first n control points B_i there is associated angle θ_i specifying the position of the next control point B_{i+1} given that the vertebra length is constant. The maximum difference between adjacent angles is user-defined constraint corresponding to the flexibility of the backbone. Note that given the position of B_1 and $\theta_i, i = 1, \ldots, n$, the backbone is completed determined.

3.1 Steady motion

Steady undulatory body motion can be simulated by modulating θ_i according to a general traveling sine wave (see Figure 2):

$$F\left(x(u), t\right) = A\left(x(u)\right) sin\left(\frac{2\pi}{\lambda}\left(x(u) - \omega t\right)\right)$$

$$\theta_i = \tan^{-1}\left(\frac{\partial F\left(x_i(u), t\right)}{\partial x}\right)$$

Propulsion mode	**Amplitude**
Anguilliform	Increases posteriorly, large over whole body
Subcarangiform	Increases rapidly over posterior third to half of body
Carangiform	Increases rapidly over posterior third

Table 1: Classification of propulsion modes of fish [1].

where the wavelength λ and the amplitude function $A\left(x(u)\right)$ determine the propulsion mode. Table 1 shows a qualitative classification scheme [1].

3.2 Rapid starting

Aquatic animals can exhibit rapid starting or lunging motion when fleeing from a predator or pursuing a prey. The kinematics of an aquatic animal making a rapid start differ from its geometry during steady swimming: some common species of fish quickly bend into a characteristic "L" shape and then generate a large thrust force by pushing against the water mostly with their tail fin [10]. We model of rapid starting by allowing the user to define backbone angles θ_i for this 'L' shape and then simulating rapid starting motion by varying the angles from their initial values to their "L" shape values and then back to their initial values.

3.3 Turns

Similarly, fish exhibit a characteristic maximum body deformation shape during a turning motion [9]. We model the body deformation associated with turning in a manner similar to rapid starting based on maximal angle values specified by the user.

4 Propulsion

Now we consider the propulsive effects of the body deformations just discussed. The hydrodynamics of aquatic animal propulsion is complex and not completely understood. The following assumptions simplify the problem of dynamically modeling aquatic animal locomotion:

- The fluid is incompressible and inviscid.
- The Reynolds number is large ($Re \gg 1$) so inertial forces dominate viscous forces.
- The aquatic animal is slender-bodied and has a streamlined shape.
- The aquatic animals swims by undulatory body motions only, swimming using fins, cilia, or jet propulsion is not considered.

Our model of the dynamics of aquatic animal swimming is based on the models of Newman and Wu [5] and Weihs [9]. The law of conservation of momentum dictates that the changes in the momentum of the fluid surrounding the aquatic animal and the momentum changes in the animal must sum to zero. Two components of momentum change in the fluid surrounding the aquatic animal are considered. First, the lateral motions of the animal impart momentum changes in the fluid in a direction normal to the x-axis (body coordinates) of the animal. Second, since the fluid is assumed inviscid and only sharp-edged surfaces contribute to the wake, the momentum changes in the wake may be modeled by summing the momentum shedding forces on the fins [9]. Thus, the propulsive force generated by the aquatic animal, $\mathbf{f}$, is the sum of the momentum shed from lateral motions of the body and from sharp-edged surfaces $\mathbf{L}_i$ such as the caudal (tail) fin

$$\mathbf{f} = -\frac{d}{dt}\int_0^l m(u)w(u)\mathbf{n}du - \sum_{i=1}^{k}\mathbf{L}_i$$

where $w(u)$ is the normal component of the velocity of the aquatic animal at u, $\mathbf{n}$ is the unit normal vector, and $m(u)$ is the added mass of the aquatic animal at u, and

$$\mathbf{L}_i = c\mathbf{v}_i^2 A_i$$

where $\mathbf{v}_i$ is the velocity of fin i, A_i is the area of fin i, and c is user-defined constant to allow control over the dynamics. The $\mathbf{L}_i$ term is derived from classical unsteady aerodynamics.

The moment around the aquatic animal's centre of mass is

$$\mathbf{M} = -\frac{d}{dt}\int_0^l m(u)(\mathbf{r}\times w(u)\mathbf{n})du - \sum_{i=1}^{k}(\mathbf{r}\times\mathbf{L}_i)$$

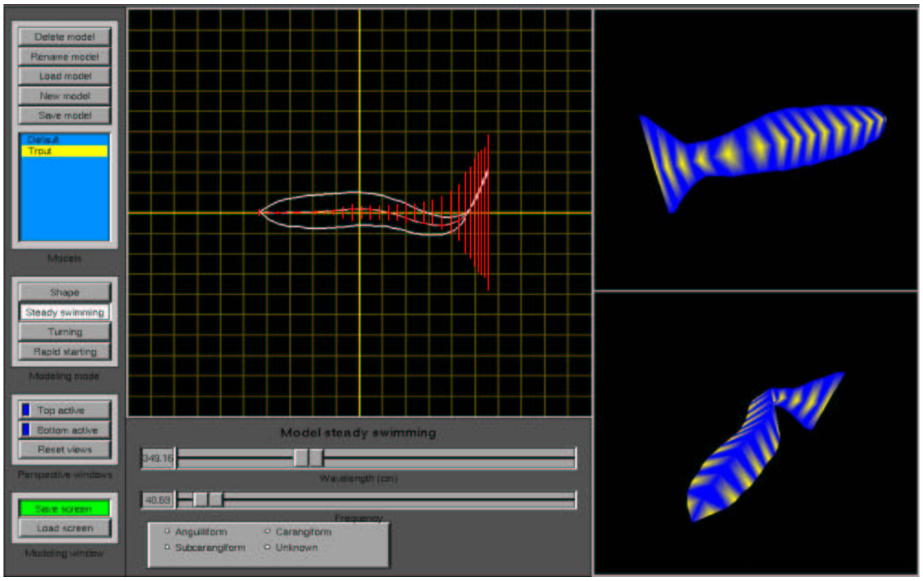
where $\mathbf{r}$ is the vector to the center of mass.

5 Implementation

A simulation system was implemented to test our models. This system has two main modes: a modelling mode, for interactively adjusting the parameters of body deformation model, and an animation mode, for interactively adjusting the dynamic simulation of propulsion.

5.1 Modelling Mode

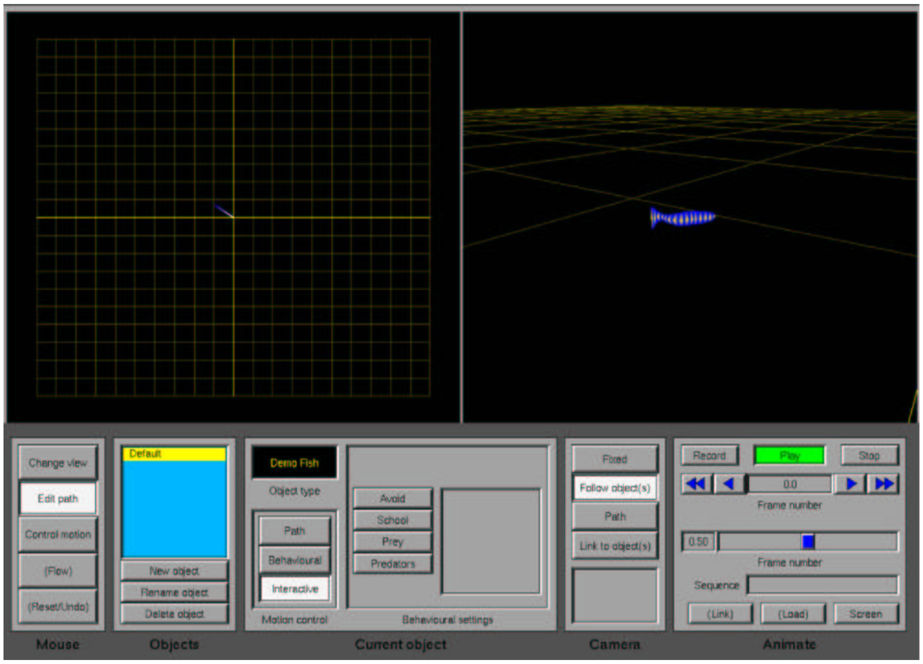
In the modelling mode the user can interactively model the shape of the fish, the amplitude and wavelength of the traveling sine wave that drives the model of steady swimming motion and the maximum displacement of the backbone during rapid staring and turning motions. Figure 3 shows the system in modelling mode.



349.16 Wavelength (cm)

40.69 Frequency

Figure 3: Modelling mode of the simulation system.



0.0 Frame number

0.50 Frame number

Figure 4: Animation mode of the simulation system.

5.2 Animation Mode

In the animation mode, the animator can interactively explore the behaviour of the propulsion dynamics by “driving” the model using the mouse to specify steady swimming and turning directions. Additional animation tools are also provided. Figure 4 shows the system in modelling mode.

6 Conclusions

To facilitate the computer animation of swimming fish we have presented simple models of both the body deformations of slender-bodied aquatic animals commonly used for locomotion and the resulting propulsive forces. These models provide a small set of intuitive parameters for animating fish. The values of these parameters can be directly specified by the animator or can be automatically generated from a behaviour model such as the well-known model of flocking, schooling, and herding behaviour presented by Reynolds [6].

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