
Black-Box Policy Search with Probabilistic Programs

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Abstract

In this work we show how to represent and learn policies that are themselves programs, i.e. stateful procedures with learnable parameters. Towards learning the parameters of such policies we develop connections between black box variational inference and existing policy learning approaches. We then explain how such learning can be implemented in a probabilistic programming system. Using our own novel implementation of such a system we demonstrate both conciseness of policy representation and automatic policy parameter learning for a range of canonical reinforcement learning problems.

1 Introduction

In planning under uncertainty one objective is to find a policy that selects actions, given currently available information, in a way that maximizes expected reward. In many cases an optimal policy can neither be represented compactly nor learned exactly. Online approaches to planning, such as Monte Carlo Tree Search [Kocsis and Szepesvári, 2006], are nonparametric policies that select actions based on simulations of future outcomes and rewards, also known as rollouts. While policies like this are often able to achieve near optimal performance, it is computationally intensive to perform rollouts at every step, and do not have compact parameterizations. Policy search methods (see Deisenroth et al. [2011] for a review) learn parameterized policies offline, which then can be used without performing rollouts at test time, trading off improved test-time computation against having to choose a policy parameterization that may be insufficient to represent the optimal policy.

In this work we show how probabilistic programs can be used to represent parametric policies in both more

general and more compact manner. We also develop automatic inference techniques for probabilistic programming systems that can be used to do model-agnostic offline policy search. Our principal contributions are thus: methods for representing policies as short parameterized programs, simplifying policy family specification, and techniques for automatic black-box probabilistic programming inference that automate search over the same.

Our proposed approach, which we call black box policy learning (BBPL), is a probabilistic programming formulation of Bayesian policy search [Wingate et al., 2011] in which policy learning is cast as stochastic gradient ascent on the marginal likelihood. In contrast to languages that target a single domain-specific algorithm [Andre and Russell, 2001, Srivastava et al., 2014, Nitti et al., 2015], our formulation emphasizes the use of general-purpose techniques for approximate Bayesian inference, in which learning is used for inference amortization. To this end, we adapt black-box variational inference (BBVI), a recent technique for approximation of the Bayesian posterior [Ranganath et al., 2014, Wingate and Weber, 2013] to perform (marginal) likelihood maximization in arbitrary programs. The resulting technique is general enough to allow implementation in a variety of probabilistic programming systems. We show that this same technique can be used to perform policy search under an appropriate planning as inference interpretation, in which a Bayesian model is weighted by the exponent of the reward. The resulting technique, BBPL is closely related to classic policy gradient methods such as REINFORCE [Williams, 1992].

We present case studies in the Canadian traveler problem, the rock sample domain, and introduce a setting inspired by Guess Who [Coster and Coster, 1979] as a benchmark for optimal diagnosis problems.

2 Policies as Programs

Probabilistic programming systems [Milch et al., 2007, Goodman et al., 2008, Minka et al., 2010, Pfeffer, 2009, Mansinghka et al., 2014, Wood et al., 2014, Gordon

```

(defquery ctp
  [problem base-prob make-policy]
  (let [graph (get problem :graph)
        start (get problem :s)
        target (get problem :t)
        sub-graph (sample-weather graph base-prob)
        [path dist counts]
          (dfs-agent sub-graph s t (make-policy))]
    (factor (- (or dist inf)))
    (predict :path path)
    (predict :distance dist)
    (predict :counts counts)))

(defm dfs-agent
  [graph start target policy]
  (loop [path [start]
        counts {}
        dist 0.0]
    (let [u (peek path)
          (if (= u target)
            [path dist counts]
            (let [unvisited
                  (filter
                   (fn [v] (not (get counts #{u v})))
                   (adjacent graph u))]
              (if (empty? unvisited)
                (if (empty? (pop path))
                  [nil dist counts]
                  (let [v (peek (pop path))]
                    (recur (pop path)
                          (assoc counts #{u v} 2)
                          (+ dist (distance graph u v))))))
                (let [v (policy u unvisited)]
                  (recur (conj path v)
                        (assoc counts #{u v} 1)
                        (+ dist
                         (distance graph u v))))))))))

(defm make-random-policy []
  (fn policy [u vs]
    (sample
     (categorical
      (zipmap vs (repeat (count vs) 1.)))))

(defm make-edge-policy []
  (let [Q (mem (fn [u v]
                 (sample
                  (learn [u v] (gamma 1. 1.)))))
        (fn policy [u vs]
          (argmax
           (zipmap vs (map (fn [v] (Q u v)) vs)))))

```

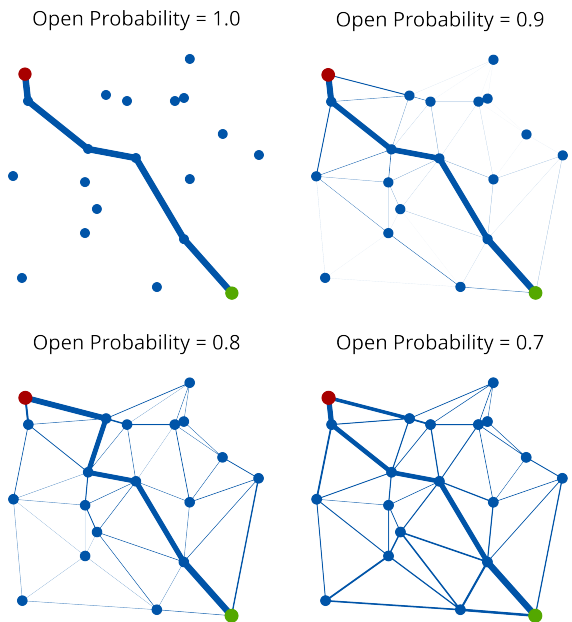


Figure 1: A Canadian traveler problem (CTP) implementation in Anglican. In the CTP, an agent must travel along a graph, which represents a network of roads, to get from the start node (green) to the target node (red). Due to bad weather some roads are blocked, but the agent does not know which in advance. Upon arrival at each node the agent observes the set of open edges. The function `dfs-agent` walks the graph by performing depth-first search, calling a function `policy` to choose the next destination based on the current and unvisited locations. The function `make-random-policy` returns a policy function that selects destinations uniformly at random, whereas `make-edge-policy` constructs a policy that selects according to sampled edge preferences ($Q u v$). By learning a distribution on each value ($Q u v$) through gradient ascent on the marginal likelihood, we obtain a heuristic offline policy that follows the shortest path when all edges are open, and explores more alternate routes as more edges are closed.

et al., 2014] represent generative models as programs in a language that provides specialized syntax to instantiate random variables, as well as syntax to impose conditions on these random variables. The goal of inference in a probabilistic program is to characterize the distribution on its random variables subject to the imposed conditions, which is done using one or more generic methods provided by an inference backend.

In sequential decision problems we must define a stochastic simulator of an agent, which chooses ac-

tions based on current contextual information, and a stochastic simulator of the world, which may have some internal variables that are opaque to the agent, but provides new contextual information after each action. For sufficiently simple problems, both the agent and the world simulator can be adequately described as graphical models. Here we are interested in using probabilistic programs as simulators of both the world and the agent. The trade-off made in this approach is that we can incorporate more detailed assumptions about the structure of the problem into our simulator

of the agent, which decreases the size of the search space, at the expense of having to treat these simulators as black boxes from the perspective of the learning algorithm.

In Figure 1 we show an example of a program, written in the language Anglican [Wood et al., 2014], which simulates an agent in the Canadian traveler problem (CTP) domain. This agent traverses a graph using depth first search (DFS) as a base strategy, choosing edges either at random, or according to sampled preferences. Probabilistic programs can describe a family of algorithmic policies, which may make use of programming constructs such as recursion, and higher-order functions and arbitrary deterministic operations. This allows us to define structured policies that enforce basic constraints, such as the rule that you should never travel the same edge twice.

Given a base policy program, we can define different parametrizations that encode additional structure, such as the typical travel distance starting from each edge. We can then formulate a Bayesian approach to policy learning, in which we place a prior on the policy parameters and optimize the hyperparameters of this prior to maximize the reward. To do so we employ a planning as inference interpretation [Toussaint et al., 2006, Rawlik et al., 2012, Neumann, 2011, Levine and Koltun, 2013] that casts policy search as stochastic gradient ascent on the marginal likelihood. This type of strategy, which is sometimes known as Bayesian policy search [Wingate et al., 2011, 2013], differs from probabilistic programming approaches tailored to Markov decision processes (MDPs) [Andre and Russell, 2001, Nitti et al., 2015] and partially observable MDPs (POMDPs) [Srivastava et al., 2014] in that it leverages generic methods for parameter learning, rather than domain-specific algorithms.

A challenge in devising methods for approximate inference in probabilistic programs is that such methods must deal gracefully with programs that may not instantiate the same set of random variables in each execution. For example, the random policy in Figure 1 will generate a different set of categorical variables in each execution, depending on the path followed through the graph. Similarly, the edge based policy samples values (Q u v) lazily, depending on the visited nodes.

In this paper we develop an approach to policy learning based on black box variational inference (BBVI) [Ranganath et al., 2014, Wingate and Weber, 2013], a recently introduced technique for variational approximation of the posterior in Bayesian models. We begin by reviewing planning as inference formulations of policy search. We show how BBVI can be adapted to

perform hyperparameter optimization. In an planning as inference interpretation this method, which we call black box policy learning (BBPL), is equivalent to classic policy gradient methods in a planning as inference interpretation. We then describe how BBPL may be implemented in the context of probabilistic programs with varying numbers of random variables, and provide a language-agnostic specification of the interface between the program and the inference back end.

3 Policy Search as Bayesian Inference

In sequential decision problems, an agent draws an action u_t from a policy distribution $\pi(u_t | x_t)$, which may be deterministic, conditioned on a context x_t . The agent then observes a new context x_{t+1} drawn from a distribution $p(x_{t+1} | u_t, x_t)$. In the finite horizon case, where an agent performs a fixed number of actions T , resulting in a sequence $\tau = (x_0, u_0, x_1, u_1, x_2, \dots, u_{T-1}, x_T)$, which is known as a trajectory, or roll-out. Each trajectory gets a reward $R(\tau)$. Policy search methods maximize the expected reward $J_\theta = \mathbb{E}_{p_\theta}[R(\tau)]$ for a family of stochastic policies π_θ with parameters θ

$$J_\theta = \int R(\tau) p_\theta(\tau) d\tau, \quad (1)$$

$$p_\theta(\tau) := p(x_0) \prod_{t=0}^{T-1} \pi(u_t | x_t, \theta) p(x_{t+1} | u_t, x_t). \quad (2)$$

We are interested in performing upper-level policy search, a variant of the problem defined in terms of the hyperparameters λ of a distribution $p_\lambda(\tau, \theta)$ that places a prior $p_\lambda(\theta)$ on the policy parameters

$$J_\lambda = \int R(\tau) p_\lambda(\tau, \theta) d\tau d\theta, \quad (3)$$

$$p_\lambda(\tau, \theta) := p_\lambda(\theta) p(\tau | \theta). \quad (4)$$

Upper-level policy search can be interpreted as maximization of the normalizing constant Z_λ of an unnormalized density

$$\gamma_\lambda(\tau, \theta) = p_\lambda(\tau, \theta) \exp(\beta R(\tau)), \quad (5)$$

$$Z_\lambda = \int \gamma_\lambda(\tau, \theta) d\tau d\theta \quad (6)$$

$$= \mathbb{E}_{p_\lambda}[\exp(\beta R(\tau))]. \quad (7)$$

The constant $\beta > 0$ has the interpretation of an ‘inverse temperature’ that controls how strongly the density penalizes sub-optimal actions. The normalization constant Z_λ is the expected value of the exponentiated reward $\exp(\beta R(\tau))$, which is known as the desirability in the context of optimal control [Kappen, 2005, Todorov, 2009]. It is perhaps not obvious that maximization of the expected reward J_λ yields the same

policy hyperparameters as maximization of the desirability Z_λ , but it turns out that the two are in fact equivalent, as we will explain in section 5.

In planning as inference formulations [Toussaint et al., 2006, Rawlik et al., 2012, Neumann, 2011, Levine and Koltun, 2013], $\gamma_\lambda(\tau, \theta)/Z_\lambda$ is often interpreted as a posterior $p_\lambda(\tau, \theta | r)$ conditioned on a pseudo observable $r = 1$ that is Bernoulli distributed with probability $p(r = 1 | \tau) \propto \exp(\beta R(\tau))$, resulting in a joint distribution that is proportional to $\gamma_\lambda(\tau, \theta)$,

$$p(r, \tau, \theta) \propto p_\lambda(\tau, \theta) \exp(\beta R(\tau)) = \gamma_\lambda(\tau, \theta). \quad (8)$$

Maximization of Z_λ is then equivalent to the maximization of the marginal likelihood $p_\lambda(r = 1)$ with respect to the hyperparameters λ . In a Bayesian context this is known as empirical Bayes [Maritz and Lwin, 1989], or type II maximum likelihood estimation.

4 Black-box Variational Inference

Variational Bayesian methods [Wainwright and Jordan, 2008] approximate an intractable posterior with a more tractable family of distributions. For purposes of exposition we consider the case of a posterior $p(z, \theta | y)$, in which y is a set of observations, θ is a set of model parameters, and z is a set of latent variables. We write $p(z, \theta | y) = \gamma(z, \theta)/Z$ with

$$\gamma(z, \theta) = p(y | z, \theta)p(z | \theta)p(\theta), \quad (9)$$

$$Z = \int \gamma(z, \theta) dz d\theta. \quad (10)$$

Variational methods approximate the posterior using a parametric family of distributions q_λ by maximizing a lower bound on $\log Z$ with respect to λ

$$\mathcal{L}_\lambda = \mathbb{E}_{q_\lambda}[\log \gamma(z, \theta) - \log q_\lambda(z, \theta)] \quad (11)$$

$$= \log Z - D_{\text{KL}}(q_\lambda(z) || \gamma(z)/Z) \leq \log Z. \quad (12)$$

This objective may be optimized with stochastic gradient ascent [Hoffman et al., 2013]

$$\lambda_{k+1} = \lambda_k + \rho_k \nabla_\lambda \mathcal{L}_\lambda \Big|_{\lambda=\lambda_k}, \quad (13)$$

$$\nabla_\lambda \mathcal{L}_\lambda = \mathbb{E}_{q_\lambda(z)} \left[\nabla_\lambda \log q_\lambda(z) \log \frac{\gamma(z, \theta)}{q_\lambda(z, \theta)} \right]. \quad (14)$$

Here ρ_k is a sequence of step sizes that satisfies the conditions $\sum_{k=1}^{\infty} \rho_k = \infty$ and $\sum_{k=1}^{\infty} \rho_k^2 < \infty$. The calculation of the gradient $\nabla_\lambda \mathcal{L}_\lambda$ requires an integral over q_λ . For certain models, specifically those where the likelihood and prior are in the conjugate exponential family [Hoffman et al., 2013], this integral can be performed analytically.

Black box variational inference targets a much broader class of models by sampling $z^{[n]}, \theta^{[n]} \sim q_\lambda$ and replacing the gradient for each component i with a sample-based estimate [Ranganath et al., 2014]

$$\hat{\nabla}_{\lambda_i} \mathcal{L}_\lambda = \sum_{n=1}^N \nabla_{\lambda_i} \log q_\lambda(z^{[n]}, \theta^{[n]}) (\log w^{[n]} - \hat{b}_i), \quad (15)$$

$$w^{[n]} = \gamma(z^{[n]}, \theta^{[n]}) / q_\lambda(z^{[n]}, \theta^{[n]}), \quad (16)$$

in which \hat{b}_i is a control variate that reduces the variance of the estimator

$$\hat{b}_i = \frac{\sum_{n=1}^N (\nabla_{\lambda_i} \log q_\lambda(z^{[n]}, \theta^{[n]}))^2 w^{[n]}}{\sum_{n=1}^N (\nabla_{\lambda_i} \log q_\lambda(z^{[n]}, \theta^{[n]}))^2}. \quad (17)$$

5 Black-box Policy Search

The sample-based gradient estimator in BBVI resembles the one used in classic likelihood-ratio policy gradient methods [Deisenroth et al., 2011], such as REINFORCE [Williams, 1992], G(PO)MDP [Baxter and Bartlett, 1999, Baxter et al., 1999], and PGT [Sutton et al., 1999]. There is in fact a close connection between BBVI and these methods.

To make this connection precise, let us consider what it would mean to perform variational inference in a planning as inference setting. In this case, we can define a lower bound $\mathcal{L}_{\lambda, \lambda_0}$ on $\log Z_{\lambda_0}$ in terms of a variational distribution $q_\lambda(\tau, \theta)$ with parameters λ and an unnormalized density $\gamma_{\lambda_0}(\tau, \theta)$ of the form in equation 5, with parameters λ_0

$$\mathcal{L}_{\lambda, \lambda_0} = \mathbb{E}_{q_\lambda} [\log \gamma_{\lambda_0}(z, \theta) - \log q_\lambda(z, \theta)] \quad (18)$$

$$= E_{q_\lambda} \left[\beta R(\tau) + \log \frac{p_{\lambda_0}(\tau, \theta)}{q_\lambda(\tau, \theta)} \right] \quad (19)$$

If we now choose a variational distribution with the same form as the prior, then $q_\lambda(\tau, \theta) = p_{\lambda_0}(\tau, \theta)$ whenever $\lambda = \lambda_0$. Under this assumption, the lower bound at $\lambda = \lambda_0$ simplifies to

$$\mathcal{L}_{\lambda, \lambda_0} \Big|_{\lambda=\lambda_0} = E_{q_\lambda} [\beta R(\tau)] \Big|_{\lambda=\lambda_0} = \beta J_\lambda \Big|_{\lambda=\lambda_0}. \quad (20)$$

In other words, the lower bound $\mathcal{L}_{\lambda, \lambda_0}$ is proportional to the expected reward J_λ when the variational posterior is equal to the prior.

The gradient of the lower bound similarly simplifies to

$$\begin{aligned} \nabla_\lambda \mathcal{L}_{\lambda, \lambda_0} \Big|_{\lambda=\lambda_0} &= E_{q_\lambda} \left[\nabla_\lambda \log q_\lambda(\tau, \theta) \log \frac{\gamma_{\lambda_0}(\tau, \theta)}{q_\lambda(\tau, \theta)} \right] \Big|_{\lambda=\lambda_0} \\ &= E_{q_{\lambda_0}} \left[\nabla_\lambda \log q_\lambda(\tau, \theta) \Big|_{\lambda=\lambda_0} \beta R(\tau) \right] \\ &= \int d\tau d\theta \nabla_\lambda q_\lambda(\tau, \theta) \Big|_{\lambda=\lambda_0} \beta R(\tau) \\ &= \nabla_\lambda J_\lambda \Big|_{\lambda=\lambda_0}. \end{aligned}$$

The implication of this identity is that we can perform gradient ascent on J_λ by making a slight modification to the update equation

$$\lambda_{k+1} = \lambda_k + \rho_k \hat{\nabla}_\lambda \mathcal{L}_{\lambda, \lambda_k} \Big|_{\lambda=\lambda_k}. \quad (21)$$

The difference in these updates is that instead of calculating the gradient $\hat{\nabla}_\lambda \mathcal{L}_{\lambda, \lambda_0}$ estimate relative to a fixed set of prior parameters λ_0 , we update the parameters of the prior $p_{\lambda_k}(\tau, \theta)$ after each gradient step, and calculate the gradient $\nabla_\lambda \mathcal{L}_{\lambda, \lambda_k}$. We note that the constant β is simply a scaling factor on the step sizes ρ_k , and will from here on assume that $\beta = 1$.

When BBVI is performed using the update step in equation 21, and the variational family q_λ is chosen to have the same form as the prior p_λ , we obtain a procedure for EB estimation, which maximizes the normalizing constant Z_λ with respect to the parameters λ of the prior. The difference between the EB and maximum likelihood (ML) methods is that the first calculates the gradient relative to hyperparameters λ , whereas the other calculates the gradient relative to the parameters θ . Because this difference relates only to the assumed model structure, EB estimation is sometimes referred to as Type II maximum likelihood.

As is evident from equation 20, EB estimation in the context of planning as inference formulations maximizes the expected reward J_λ . In the context of a probabilistic programming system this means that we can effectively get three algorithms for the price of one: If we can provide an implementation of BBVI, then this implementation can be adapted to perform EB estimation, which in turn allows us to perform policy search by simply defining models where exponent of the reward takes the place of the likelihood terms. This results in a method that we call black box policy learning (BBPL), which is equivalent to variants of REINFORCE applied to upper-level policy search.

6 Learning Probabilistic Programs

An implementation of BBVI and BBPL for probabilistic program inference needs to address two domain-specific issues. The first is that probabilistic programs need not always instantiate the same number of random variables, the second is that we need to distinguish between distributions that define model parameters θ and those that define latent variables z , or variables that are part of the context x in the case of decision problems.

Let us refer back to the program in Figure 1. The function `dfs-agent` performs a recursive loop until a stopping criterion is met: either the target node is reached,

or there are no more paths left to try. At each step `dfs-agent` makes a call to `policy`, which is created by either calling `make-random-policy` or `make-edge-policy`. A random policy samples the next destination uniformly from unexplored directions. When depth first search is performed with this policy, we are defining a model in which the number of context variables is random, since the number of steps required to reach the goal state will vary. In the case of the edge policy, we use a memoized function to sample edge preference values as needed, choosing the unexplored edge with the highest preference at each step. In this case the number of parameter variables is random, since we only instantiate preferences for edges that are (a) open and (b) connect to the current location of the agent.

As has been noted by Wingate and Weber [2013], BBVI can deal with varying sets of random variables quite naturally. Since the gradient is computed from a sample estimate, we can compute gradients for a given random variable by simply averaging over those executions of a program in which the variable exists. Sampling variables as needed can in fact be more statistically efficient, since irrelevant variables that never affect the trajectory of the agent will not contribute to the gradient estimate. BBVI has the additional advantage of having relatively light-weight implementation requirements. In BBVI only derivatives of a limited number of primitive distribution types are needed, rather than a full (forward-mode) automatic differentiation implementation [Pearlmutter and Siskind, 2008], as is used to perform variational inference in Stan [Kucukelbir et al., 2015]. BBVI and related techniques could therefore principle be implemented in many of the probabilistic programming systems that are currently in development.

To provide a language-agnostic definition of BBVI and BBPL, we formalize learning in probabilistic programs as the interaction between a program \mathcal{P} and an inference back end \mathcal{B} . The program \mathcal{P} represents all deterministic parts of the computation and has internal state (e.g. its environment variables). The back end \mathcal{B} keeps track of the probability of a program execution, learned parameters, and other inference-related variables.

A program \mathcal{P} executes as normal, but delegates to the inference back end whenever it needs to instantiate a random variable, or evaluate a conditioning statement. The back end \mathcal{B} then supplies a value for the random variable, or makes note of the probability associated with the conditioning statement, and then delegates back to \mathcal{P} to continue execution. We will assume that the programming language provides some way to differentiate between latent variables z , which are simply to be sampled, and parameters θ for which a distri-

bution is to be learned. In the program in Figure 1, (`sample (learn b d)`) signifies that a variable sampled from a distribution `d` with address `b` is a model parameter.

In order for the learning algorithm to be well-defined in programs that instantiate varying numbers of random variables, we require that the each random variable z_α is uniquely identified by an address α , which may either be generated automatically by the language runtime, or specified by the programmer. Each model parameter θ_β is similarly identified by an address β .

In BBVI, the interface between a program \mathcal{P} and the back end \mathcal{B} can be formalized with the following rules:

- Initially \mathcal{B} calls \mathcal{P} with no arguments $\mathcal{P}()$
- A call to \mathcal{P} returns one of four responses to \mathcal{B} :
 1. (`sample`, α, f, ϕ): Identifies a random variable z_α with unique address α , distributed according to $f_\alpha(\cdot | \phi_\alpha)$. The back end generates a value $z_\alpha \sim f_\alpha(\cdot | \phi_\alpha)$ and calls $\mathcal{P}(z_\alpha)$.
 2. (`learn`, β, f, η): The address β identifies a random variable θ_β in the model, distributed according to a distribution f_β with parameters η_β . The back end generates $\theta_\beta \sim f_\beta(\cdot | \lambda_\beta)$ conditioned on a learned variational parameter λ_β and registers an importance weight $w_\beta = f_\beta(\theta_\beta | \eta_\beta) / f_\beta(\theta_\beta | \lambda_\beta)$. Execution continues by calling $\mathcal{P}(\theta_\beta)$.
 3. (`factor`, γ, l): Here γ is a unique address for a factor with log probability l_γ and importance weight $w_\gamma = \exp(l_\gamma)$. Execution continues by calling $\mathcal{P}()$.
 4. (`return`, v): Execution completes, returning a value v .

Because each call to \mathcal{P} is deterministic, an execution history is fully characterized by the values for each random variable that are generated by \mathcal{B} . However the set of random variables that is instantiated may vary from execution to execution. We write A, B, Γ for the set of addresses of each type visited in a given execution. The program \mathcal{P} now defines an unnormalized density $\gamma_{\mathcal{P}}$ of the form

$$\gamma_{\mathcal{P}}(z, \theta) := p_{\mathcal{P}}(z, \theta) \prod_{\gamma \in \Gamma} \exp(l_\gamma), \quad (22)$$

$$p_{\mathcal{P}}(z, \theta) := \prod_{\alpha \in A} f_\alpha(z_\alpha | \phi_\alpha) \prod_{\beta \in B} f_\beta(\theta_\beta | \eta_\beta). \quad (23)$$

Implicit in this notation is the fact that the distribution types $f_\alpha(\cdot | \phi_\alpha)$ and $f_\beta(\cdot | \eta_\beta)$ are return values from calls to \mathcal{P} , which implies that both the parameter

values and the distribution type may vary from execution to execution. While $f_\alpha(\cdot | \phi_\alpha)$ and $f_\beta(\cdot | \eta_\beta)$ are fully determined by preceding values for z and θ , we assume they are opaque to the inference algorithm, in the sense that no analysis is performed to characterize the conditional dependence of each ϕ_α or η_β on other random variables in the program.

Given the above definition of a target density $\gamma_{\mathcal{P}}(z, \theta)$, we are now in a position to define the density of a variational approximation \mathcal{Q}_λ to the program. In this density, the runtime values η_β are replaced by variational parameters λ_β

$$p_{\mathcal{Q}_\lambda}(z, \theta) := \prod_{\alpha \in A} f_\alpha(z_\alpha | \phi_\alpha) \prod_{\beta \in B} f_\beta(\theta_\beta | \lambda_\beta). \quad (24)$$

This density corresponds to that of a mean-field probabilistic program, where the dependency of each θ_β on other random variables is ignored.

Repeated execution of \mathcal{P} given the interface described above results in a sequence of weighted samples $(w^{[n]}, \theta^{[n]}, z^{[n]})$, whose importance weight $w^{[n]}$ is defined as

$$w^{[n]} := \gamma_{\mathcal{P}}(z^{[n]}, \theta^{[n]}) / p_{\mathcal{Q}_\lambda}(z^{[n]}, \theta^{[n]}) \quad (25)$$

$$= \prod_{\beta \in B} \frac{f(\theta_\beta | \eta_\beta)}{f(\theta_\beta | \lambda_\beta)} \prod_{\gamma \in \Gamma} \exp l_\gamma. \quad (26)$$

With this notation in place, it is clear that we can define a lower bound $\mathcal{L}_{\mathcal{Q}_\lambda, \mathcal{Q}_{\lambda_k}}$ analogous to that of Equation 19, and a gradient estimator analogous to that of Equation 15, in which the latent variables z take the role of the trajectory variables τ . In summary, we can describe a sequential decision problem as a probabilistic program \mathcal{P} in which the log probabilities l_γ are interpreted as rewards, parameters θ_β define the policy and all other latent variables z_α are trajectory variables. EB inference can then be used to learn the hyperparameters λ that maximize the expected reward.

An assumption that we made when deriving BBPL is that the variational distribution $q_\lambda(\tau, \theta)$ must have the same analytical form as the prior $p_{\lambda_0}(\tau, \theta)$. Practically this requirement means that a program \mathcal{P} must be written in such a way that the values of the hyperparameters η_β have the same constant values in every execution, since their values may not depend on those of random variables. One way to enforce this is to pass η as a parameter in the initial call $\mathcal{P}(\eta)$ by \mathcal{B} , though we do not formalize such a requirement here.

7 Case Studies

We demonstrate the use of programs for policy search in three problem domains: (1) the Canadian Traveller

Problem, (2) a modified version of the RockSample POMDP, and (3) an optimal diagnosis benchmark inspired by the classic children’s game Guess Who.

These three domains are examples of *deterministic* POMDPs, in which the initial state of the world is not known, and observations may be noisy, but the state transitions are deterministic. Even for discrete variants of such problems, the number of possible information states $x_t = (u_0, o_1, \dots, u_{t-1}, o_t)$ grows exponentially with the horizon T , meaning that it is not possible to fully parameterize a distribution $\pi(u | x, \theta)$ in terms of a conditional probability table $\theta_{x,u}$. In our probabilistic program formulations for these problems, the agent is modeled as an algorithm with a number of random parameters, and we use BBPL to learn the distribution on parameters that maximizes the reward.

We implement our case studies using the probabilistic programming system Anglican [Wood et al., 2014]. We use the same experimental setup in each of the three domains. A trial begins with a learning phase, in which BBPL is used to learn the policy hyperparameters, followed by a number of testing episodes in which the agent chooses actions according to a fixed learned policy. At each gradient update step, we use 1000 samples to calculate a gradient estimate. Each testing phase consists of 1000 episodes. All shown results are based on test-phase simulations.

Stochastic gradient methods can be sensitive to the learning rate parameters. Results reported here use a RMSProp style rescaling of the gradient [Hinton et al.], which normalizes the gradient by a discounted rolling average of its magnitude with decay factor 0.9. We use a step size schedule $\rho_k = \rho_0 / (\tau + k)^\kappa$ as reported in [Hoffman et al., 2013], with $\tau = 1$, $\kappa = 0.5$ in all experiments. We use a relatively conservative base learning rate $\rho_0 = 0.1$ in all reported experiments. For independent trials performed across a range 1, 2, 5, 10, \dots , 1000 of total gradient steps, consistent convergence was observed in all runs using over 100 gradient steps.

7.1 Canadian Traveller Problem

In the Canadian Traveller Problem (CTP) [Papadimitriou and Yannakakis, 1991], an agent must traverse a graph $G = (V, E)$, in which edges may be missing at random. It is assumed the agent knows the distance $d : E \rightarrow \mathbb{R}^+$ associated with each edge, as well as the probability $p : E \rightarrow (0, 1]$ that the edge is open, but has no advance knowledge of the edges that are blocked. The problem is NP-hard [Fried et al., 2013], and heuristic online and offline approaches [Eyerich et al., 2010] are used to solve problem instances.

The results in Figure 1 show that the learned pol-

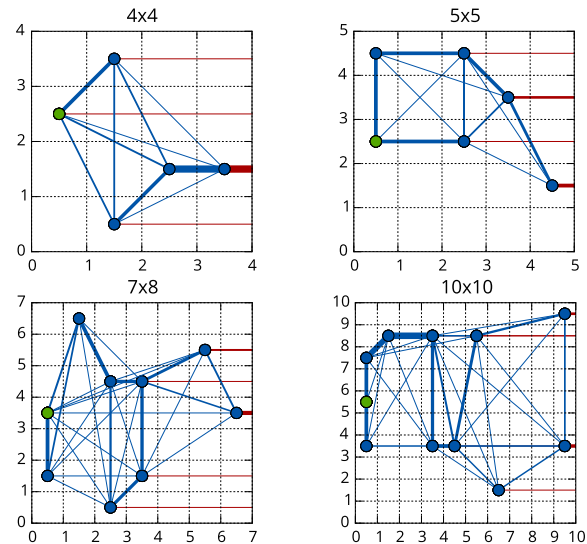


Figure 2: Learned policies for the Rock Sample domain. Edge weights indicate the frequency at which the agent moves between each pair of rocks. Starting points are in green, exit paths in red.

icy behaves in a reasonable manner. When edges are open with high probability, the policy takes the shortest path from the start node, marked in green, to the target node, marked in red. As the number of closed edges increases, the policy makes use of more alternate routes, whilst avoiding peripheral edges of the graph.

7.2 RockSample POMDP

In the RockSample POMDP [Smith and Simmons, 2004], a square field $N \times N$ with M rocks is given. A rover is initially located in the middle of the left edge of the square. Each of the rocks can be either good or bad; and the rover must traverse the field and collect samples of good rocks. The rover can sense the quality of a rock remotely with an accuracy decreasing with the distance to the rock. The objective is to cross the field fast, while still sampling as many good rocks as possible.

Since RockSample problem was formulated as a benchmark for value iteration algorithms, it is normally evaluated in an infinite horizon setting where the discount factor penalizes sensing and movement. In the original formulation of the problem, movement and sensing incur no cost. The agent gets a reward of 10 for each good rock, as well as for reaching the right edge, but incurs a penalty of -10 when sampling a bad rock.

Here we adapt RockSample to a finite horizon setting, in which we assume sensing is free, and movement incurs a cost of -1. We learn parameters of a policy in which the robot moves along rocks in a left-to-right order. At each rock the agent selects the closest

next rock and senses it. Based on the reading it then chooses to either move to the rock, or discard it and consider the next closest rock. When the agent gets to a rock, it only samples the rock if the rock is good. The parameters describe the prior over the probability of moving to a rock conditioned on the current location and the sensor reading.

The policy plots in Figure 2 show that this simple policy results in sensible movement preferences. In particular we point out that in the 5×5 instance, the agent always visits the top-left rock when traveling to the top-middle rock, since doing so incurs no additional cost. Similarly, the agent follows an almost deterministic trajectory along the left-most 5 rocks in the 10×10 instance, but does not always make the detour towards the lower rocks afterwards.

7.3 Guess Who

Guess Who is a classic game in which players pick a card depicting a face, belonging to a set that is known to both players. The players then take turns asking questions until they identify the card of the other player [Coster and Coster, 1979]. We here consider a single-player setting where an agent asks a pre-determined number of questions, but the responses are inaccurate with some probability. This is sometimes known as a measurement selection, or optimal diagnosis problem. We make use of a feature set based on the original game, consisting of 24 individuals, characterized by 11 binary attributes and two multi-class attributes, resulting in a total of 19 possible questions. We assume a response accuracy of 0.9. By design, the structure of the domain is such that there is no clear winning opening question. However the best question at any point is highly contextual.

We assume that the agent knows the reliability of the response and has an accurate representation of the posterior belief $b_t(s) = p(s | x_t)$ for each candidate s in given questions and responses. The agent selects randomly among the highest ranked candidates after the final question. We consider 3 policy variants, two of which are parameter-free baselines. In the first baseline, questions are asked uniformly at random. In the second, questions are asked according to a myopic estimate of the value of information [Hay et al., 2012], i.e. the change in expected reward relative to the current best candidates, which is myopically optimal in this setting. Finally, we consider a policy that empirically samples questions q according to a weight $v_q = \gamma^{c_q} (Ab)_q$, based on the current belief b , a weight matrix A , and a discount factor γ^{n_q} based on the number of times n_q a question was previously asked. Intuitively, this algorithm can be understood as learning a small set of α -vectors, one for each question, similar

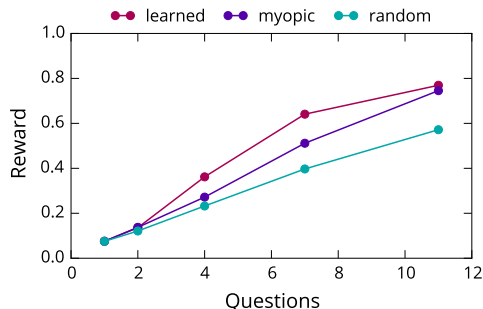


Figure 3: (left) Average reward in Guess Who as a function of number of questions. (right) Convergence of rewards as function number of gradient steps. Each dot marks an independent restart.

to those learned in point-based value iteration [Pineau et al., 2003]. The discounting effectively “shrinks” the belief-space volume associated with the α -vector of the current best question, allowing the agent to select the next-best question.

The results in Figure 3 show that the learned policy clearly outperforms both baselines, which is a surprising result given the complexity of the problem and the relatively simplistic form of this heuristic policy. While these results should not be expected to be in any way optimal, they are encouraging in that they illustrate how probabilistic programming can be used to implement and test policies that rely on transformations of the belief or information state in a straightforward manner.

8 Discussion

In this paper we put forward the idea that probabilistic programs can be a productive medium for describing both a problem domain and the agent in sequential decision problems. Programs can often incorporate assumptions about the structure of a problem domain to represent the space of policies in a more targeted manner, using a much smaller number of variables than would be needed in a more general formulation. By combining probabilistic programming with black-box variational inference we obtain a generalized variant of well-established policy gradient techniques that allow us to define and learn policies with arbitrary levels of algorithmic sophistication in moderately high-dimensional parameter spaces. Fundamentally, policy programs represent some form of assumptions about what contextual information is most relevant to a decision, whereas the policy parameters represent domain knowledge that generalizes across episodes. This suggests future work to explore how latent variable models may be used to represent past experiences in a manner that can be related to the current information state.

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A Case Studies

All case studies are implemented in Anglican, a probabilistic programming language that is closely integrated into the Clojure language. In Anglican, the macro `defquery` is used to define a probabilistic model. Programs may make use of user-written Clojure functions (defined with `defn`) as well as user-written Anglican functions (defined with `defm`). The difference between the two is that in Anglican functions may make use of the model special forms `sample`, `observe`, and `predict`, which interrupt execution and require action by the inference backend. In Clojure functions, `sample` is a primitive procedure that generates a random value, `observe` returns a log probability, and `predict` is not available.

We note that the interrupt type (`learn`, β, f, η) defined in the main text is implemented in Anglican as a normal sample interrupt (`sample`, α, f, ϕ) with an annotation on the distribution f . A sample from a distribution on model parameters θ for which parameter learning is to be performed, is defined via

```
(sample (learn address dist))
```

A sample for a latent variable z for which no learning is performed is written as normal

```
(sample dist)
```

Full documentation for Anglican can be found at

<http://www.robots.ox.ac.uk/~fwood/anglican>

A.1 Canadian Traveler Problem

```
(ns ctp.model
  (:require [ctp.dijkstra :refer [shortest-path]])
  (:use [anglican runtime emit learn]))

(def inf (/ 1.0 0.0))

(defn argmax
  "returns the index of the max entry in a collection of [k v] pairs"
  [thing]
  (first (apply max-key second thing)))

(defn adjacent
  "returns a vector of nodes indices that are adjacent to u"
  [graph u]
  (mapv first (get graph u)))

(defn distance
  "returns the distance between u and v in a graph,
  or nil when nodes are not adjacent"
  [graph u v]
  (some (fn [[w d o]]
          (when (= v w) d))
        (get graph u)))

(defn open-prob
  "returns the probability that a edge u v is open,
  or nil when edge probabilities are not specified"
  [graph u v]
  (some (fn [[w d o]]
          (when (= v w) o))
        (get graph u)))

(defn sample-weather
  "samples a sub-graph based on edge open/blocked probabilities"
  [graph base-prob]
  (loop [graph graph
        u 0
        sub-graph []
        weather {}]
    (if-let [cs (first graph)]
      (let [vs (map first cs)
            es (map (partial conj #{u}) vs)
            ws (map (fn [v]
                     (get weather
                               #{u v}
                               (sample (flip (* base-prob
                                             (or (open-prob graph u v)
                                                  1.0))))))
                    vs)]
        (recur (rest graph)
               (inc u)
               (conj sub-graph
                     (vec (keep (fn [[c w]]
```

```

        (when w c))
        (map vector cs ws))))
    (merge weather (zipmap es ws))))
  sub-graph)))

(defdist factor* [] []
  (sample [this] nil)
  (observe [this value] value))

(with-primitive-procedures [factor*]
  (defm factor [log-weight]
    (observe (factor*) log-weight)))

(with-primitive-procedures [argmax]
  (defm make-random-policy []
    (fn policy [u vs]
      (sample
        (categorical
          (zipmap vs (repeat (count vs) 1.)))))))

(defm make-edge-policy []
  (let [Q (mem (fn [u v]
                (sample
                  (learn [u v] (gamma 1. 1.))))))]
    (fn policy [u vs]
      (argmax
        (zipmap vs (map (fn [v] (Q u v)) vs))))))

(defm node-policy []
  (let [utility (mem (fn [u]
                      (sample
                        (learn u
                          (gamma 1. 1.))))))]
    (fn policy [u vs]
      (argmax (map (fn [v]
                    [v (utility u)])
                  vs))))))

(with-primitive-procedures [adjacent distance]
  (defm dfs-agent
    [graph start target policy]
    (loop [path [start]
          counts {}
          dist 0.0]
      (let [u (peek path)]
        (if (= u target)
          [path dist counts]
          (let [unvisited
                (filter
                  (fn [v] (not (get counts #{u v})))
                  (adjacent graph u))]
              (if (empty? unvisited)
                (if (empty? (pop path))
                  [nil dist counts]
                  (let [v (peek (pop path))]
                    (recur (pop path)
                          (assoc counts #{u v} 2)
                          (+ dist (distance graph u v))))))
                (let [v (policy u unvisited)]
                  (recur (conj path v)
                        (assoc counts #{u v} 1)
                        (+ dist
                          (distance graph u v))))))))))

(with-primitive-procedures [shortest-path]
  (defm clairvoyant-agent
    "simulates travel along the graph by a clairvoyant agent, that
    knows which edges exist in advance, and can therefore follow the
    shortest path"
    [graph start target _]
    (let [[path dist] (shortest-path graph start target)
          counts (when (seq path)
                   (into {} (map (fn [u v] [#{u v} 1])
                                 (butlast path)
                                 (rest path))))]
      [path dist counts]))

  (defm random-agent
    "simulates travel along graph selecting edges at random"
    [graph start target _]
    (dfs-agent graph start target (random-policy))))

(with-primitive-procedures [sample-weather]

```

```
(defquery ctp
  [problem base-prob make-policy]
  (let [graph (get problem :graph)
        start (get problem :s)
        target (get problem :t)
        sub-graph (sample-weather graph base-prob)
        [path dist counts]
          (dfs-agent sub-graph s t (make-policy))]
    (factor (- (or dist inf)))
    (predict :path path)
    (predict :distance dist)
    (predict :counts counts))))
```

A.2 RockSample POMDP

```
(ns rockwalk.model
  "RockSample POMDP solving using Probabilistic Inference"

  (:require [clojure.tools.cli :as cli])
  (:use [anglican runtime emit learn
         [state :only [get-predicts]]
         [core :only [doquery]]]
        [rockwalk data]))

(defdist factor [] []
  (sample [this] nil)
  (observe [this value] value))

(def +factor+ (factor))

;; We implement an offline algorithm for the RockSample POMDP.
;; At every step, the agent selects the closest stone,
;; senses it, and either goes for the stone or discards it.
;; When there are no stones left, the agent heads straight
;; to the right edge.

;; A problem instance is defined by
;; - the field size (n n);
;; - locations and values of rocks (good/bad).
;; - location of the agent.

(defrecord state [n rocks x y])
;; where 'rocks' is a hash map
;; [^Integer x ^Integer y] -> ^Boolean good

;; The agent always starts at the middle of the left edge.
;; The goal state is beyond the right edge.

(defn goal?
  "true if the state is a goal state"
  [state]
  (= (:x state) (:n state)))

;; The sensor returns a noisy observation of rock value. The
;; accuracy decreases exponentially with the distance. At zero
;; distance, the sensor always returns the correct value. At the
;; half-efficiency distance (hed), the correct value is returned
;; with probability 0.75. At infinity, the correct and the
;; incorrect values are equally probable.

(defn accuracy
  "computes the probability of returning the correct rock value
  by the sensor"
  [state x y hed]
  (let [d (let [dx (- (:x state) x)
                dy (- (:y state) y)]
            (Math/sqrt (+ (* dx dx) (* dy dy))))]
    (efficiency (Math/pow 0.5 (/ d hed))]
      (* 0.5 (+ efficiency 1.))))

;; The robots moves rectilinearly, the manhattan distance is
;; chosen to choose a rock and compute the reward.

(defn distance
  "distance between the current and the next location"
  [state x y]
  (+ (Math/abs (- (:x state) x))
     (Math/abs (- (:y state) y))))

;; In the original formulation of RockSample the moves are free,
;; so the optimum policy is to go to every rock, know its value
;; with certainty, and sample if good. This problem formulation
```

```

;; still works for assessing value iteration algorithms because
;; of implicitly assumed reward discounting. However, a sound
;; problem formulation would either state the discounting factor
;; explicitly, limit the number of moves, or incur a cost on
;; every move.
;;
;; Here, we modify the problem formulation, so that in addition
;; to the rewards for sampling a good rock and for reaching the
;; right edge, there is a penalty for every move. All moves in
;; our space of policies are legal, and the robot never samples
;; a bad rock.

(def +sample-reward+ 10.)
(def +move-reward+ -1.)
(def +goal-reward+ 10.)

(defn goto
  "goes to a target location;
  samples the rock in that target location
  if the rock is there and good;
  returns the updated state and the reward"
  [state [x y :as loc]]
  [(assoc state
    [:x x :y y
     :rocks (dissoc (:rocks state) loc)]
    (+ (* (distance state x y) +move-reward+)
      (if ([:rocks state] loc)
          +sample-reward+
          0.0)))]

;; A rock can be removed without going to the new location,
;; just because it is not deemed worth the attention.

(defn discard
  "discards the rock, returns the updated state"
  [state loc]
  (assoc state :rocks (dissoc (:rocks state) loc)))

;; At every step, the next rock is the closest rock
;; in the left-most column containing rocks

(defn next-rock
  "Returns the coordinates of the next rock"
  [state]
  (loop [nloc nil
        locs (keys (:rocks state))]
    (if (seq locs)
      (let [[x y :as loc] & locs
            [nx ny] nloc]
        (if (or (nil? nloc)
                (< x nx)
                (and (= x nx)
                     (< (distance state x y)
                        (distance state nx ny))))
          (recur loc locs)
          (recur nloc locs)))
      nloc)))

;; Now we can define the query that learns the thresholds.
(with-primitive-procedures [goal? accuracy distance
                             next-rock goto discard]

  (defquery rockwalk
    "rockwalk policy learning"
    [instance hed scale]
    (let [rocks (into
                {}
                (map (fn [[loc _]]
                      [loc
                       (sample
                        [:rock loc] (flip 0.5))])
                    (:rocks instance)))]
      (loop [state (assoc instance
                          :rocks rocks)
            visited []
            reward 0]
        (if (goal? state)
          (do
            ;; (observe +factor+ (* reward scale))
            (predict :visited visited)
            (predict :reward reward)
            (let [loc (next-rock state)]
              (if (nil? loc)
                ;; no rocks left, go to the goal

```

```
(let [goal [(:n state) (:y state)]
      [state r] (goto state goal)]
  (observe +factor+ (* r scale))
  (recur state
    visited
    (+ reward r)))
(let [;; sample sensor reading for next rock
      good (sample
            [:sense [(:x state) (:y state)]])
        (flip
          (let [[x y] loc]
            (if (get (:rocks state) loc)
                (accuracy state x y hed)
                (- 1. (accuracy state x y hed))))))
      ;; decide whether to visit the rock
      ;; (this is the policy choice)
      visit (sample (learn
                    [:policy [(:x state) (:y state)] loc good]
                    (flip 0.5)))]
  (if visit
    ;; goto to rock, gain reward if rock is good
    (let [[state r] (goto state loc)]
      (observe +factor+ (* r scale))
      (recur state
        (conj visited loc)
        (+ reward r)))
    ;; remove rock from list of visitable rocks
    (let [state (discard state loc)]
      (recur state
        visited
        reward))))))
```

A.3 Guess Who

A.3.1 Model Auxilliary Functions

```
(ns guess-who.model
  (:require [guess-who
            [data :refer [+entities+ +questions+]]]
            [anglican
            [math :refer [max-entries]]]
            [runtime :refer [log exp log-sum-exp]]))

;; GUESS WHO
;;
;; entities {id {attr value}}
;; questions [[attr value]]
;; info {[attr value] [num-true num-false]}
;; belief [[id prob]]

(def +reliability+ 0.9)

(defn update-info
  "incorporates a question response into the information state"
  [info question response]
  (let [[a b] (get info question [0 0])]
    (assoc info
      question (if response
                 [(inc a) b]
                 [a (inc b)]))))

(defn log-likelihood
  "returns log p(info | entity), the joint log probability
  of responses given the attribute values of an entity"
  [info entity]
  (reduce
    +
    0.0
    (map (fn [[attr value] [a b]]
           (let [;; get probability of true response
                 pi (if (= (entity attr) value)
                       +reliability+
                       (- 1 +reliability+))]
             ;; binomial probabilities of responses
             (+ (* (log pi) a) (* (log (- 1 pi)) b))))
         info)))

(defn posterior-belief
  "returns p(id | info) the normalized posterior belief"
  [info]
  (let [log-ws (map (partial log-likelihood info)
                   (vals +entities+))]
```

id	beard	ear-rings	eye-color	gender	glasses	hair-color	hair-length	hair-type	hat	mustache	mouth-size	nose-size	red-checks
alex	false	false	brown	male	false	black	short	straight	false	true	large	small	false
alfred	false	false	blue	male	false	ginger	long	straight	false	true	small	small	false
anita	false	false	blue	female	false	blonde	long	straight	false	false	small	small	true
anne	false	true	brown	female	false	black	short	curly	false	false	small	large	false
bernard	false	false	brown	male	false	brown	short	straight	true	false	small	large	false
bill	true	false	brown	male	false	ginger	bald	straight	false	false	small	small	true
charles	false	false	brown	male	false	blonde	short	straight	false	true	large	small	false
claire	false	false	brown	female	true	ginger	short	straight	true	false	small	small	false
david	true	false	brown	male	false	blonde	short	straight	false	false	large	small	false
eric	false	false	brown	male	false	blonde	short	straight	true	false	large	small	false
frans	false	false	brown	male	false	ginger	short	curly	false	false	small	small	false
george	false	false	brown	male	false	white	short	straight	true	false	large	small	false
herman	false	false	brown	male	false	ginger	short	curly	false	false	small	large	false
joe	false	false	brown	male	true	blonde	short	curly	false	false	small	small	false
maria	false	true	brown	female	false	brown	long	straight	true	false	small	small	false
max	false	false	brown	male	false	black	short	curly	false	true	large	large	false
paul	false	false	brown	male	true	white	short	straight	false	false	small	small	false
peter	false	false	blue	male	false	white	short	straight	false	false	large	large	false
phillip	true	false	brown	male	false	black	short	curly	false	false	large	small	true
richard	true	false	brown	male	false	brown	bald	straight	false	true	small	small	false
robert	false	false	blue	male	false	brown	short	straight	false	false	small	large	true
sam	false	false	brown	male	true	white	bald	straight	false	false	small	small	false
susan	false	false	brown	female	false	white	long	straight	false	false	large	small	true
tom	false	false	blue	male	true	black	bald	straight	false	false	small	small	false

Table 1: Ontology for the Guess Who domain, consisting of 24 individuals, characterized by 11 binary attributes and two multi-class attributes.


```

    log-sum-w (reduce log-sum-exp log-ws)]
  (map vector
    (keys +entities+)
    (map #(exp (- % log-sum-w))
      log-ws)))

(defn response-probability
  "returns p(response | belief), the marginal question response
  probability given current belief"
  [belief question]
  (let [;; calculate prior probability that question is true
        ;; (given belief)
        [attr value] question
        [a b] (reduce
              (fn [[a b] [id prob]]
                (if (= (get-in +entities+ [id attr]) value)
                    [(+ a prob) b]
                    [a (+ b prob)]))
              [0.0 0.0]
              belief)
        pi (/ a (+ a b))]
    ;; probability of 'true' response given belief
    (+ (* +reliability+ pi)
      (* (- 1 +reliability+) (- 1 pi))))))

(defn relative-utility [initial-belief final-belief]
  "returns the expected change in reward of a final belief state
  relative to an initial belief state"
  (let [initial-candidates (max-entries initial-belief)
        final-belief (into {} final-belief)]
    (- (reduce max (vals final-belief))
      (/ (reduce + (map final-belief initial-candidates))
        (count initial-candidates))))))

```

A.3.2 Value of Information

```

(ns guess-who.heuristics
  "non-sampling based heuristics for the expected value of an action"
  (:use guess-who.model
    [anglican.math :only [max-entries]]
    [guess-who.data :only [+entities+ +questions+]]))

(defn myopic-voi
  "calculates a myopic estimate of the value of information"
  [info question]
  (let [belief (posterior-belief info)
        theta (response-probability belief question)
        u-true (relative-utility belief
                                (posterior-belief
                                 (update-info info
                                   question
                                   true)))
        u-false (relative-utility belief
                                (posterior-belief
                                 (update-info info
                                   question
                                   false)))]
    (+ (* theta u-true)
      (* (- 1 theta) u-false))))

(defn recursive-voi
  "calculates a recursive estimate of the value of information"
  [info depth question]
  (let [belief (posterior-belief info)
        candidates (max-entries belief)
        theta (response-probability belief question)
        vois (map (fn [response]
                    (let [belief (posterior-belief info)
                          new-info (update-info info question response)
                          new-belief (posterior-belief new-info)]
                      (+ (relative-utility belief new-belief)
                        (if (<= depth 1)
                            0.
                            (reduce max
                                (map (fn [next-question]
                                      (recursive-voi new-info
                                                    (dec depth)
                                                    next-question))
                                    +questions+))))))
                    [true false])]
    (reduce +
      (map * [theta (- 1.0 theta)] vois))))

```

A.3.3 Policies

```
(ns guess-who.policy
  "Policy implementations for Guess Who"
  (:require [clojure.core.matrix :as mat
            :refer [add sub div mul mmul array]]
            [guess-who
             [data :refer [+questions+ +entities+]]
             [model :refer [posterior-belief]]
             [heuristics :refer [myopic-voi recursive-voi]]]
            [anglican
             [math :refer [max-entries max-index sigmoid]]])
  (:use [anglican runtime emit
        [state :only [get-predicts]]
        [core :only [doquery]]]))

(mat/set-current-implementation :vectorz)

(defprotocol Policy
  (select [self info]))

;; select questions at random
(defrecord UniformPolicy
  []
  Policy
  (select
   [self _]
   (rand-nth +questions+)))

;; select questions according to highest myopic value of information
(defrecord MyopicVoiPolicy
  []
  Policy
  (select
   [self info]
   (let [value-estimate (map vector
                             +questions+
                             (map (partial myopic-voi info)
                                  +questions+))
         best-questions (max-entries value-estimate)]
     (rand-nth best-questions))))

;; multiplies the belief vector with a weight matrix and
;; selects the question according to the max entry of the
;; resulting vector
(defrecord LinearBeliefPolicy
  [weights]
  Policy
  (select
   [self info]
   (let [belief (mapv second (posterior-belief info))
         values (mmul weights belief)]
     (get +questions+
          (sample (discrete (mat/to-nested-vectors values)))))))

;; like linear policy but also discounts weights of previously
;; asked questions
(defrecord DiscountedBeliefPolicy
  [weights gamma]
  Policy
  (select
   [self info]
   (let [qcounts (map (fn [q] (apply + (info q))) +questions+)
         discounts (mapv #(Math/pow gamma %) qcounts)
         belief (mapv second (posterior-belief info))
         values (mul discounts (mmul weights belief))]
     (get +questions+
          (max-index values))))))

;; applies logistic regression transform to vectorized
;; information state, and returns the max index of the
;; resulting vector
(defrecord LogisticInfoPolicy
  [weights bias]
  Policy
  (select
   [self info]
   (let [info-vector (mapcat (fn [q]
                              (get info q [0.0 0.0]))
                             +questions+)
         values (sigmoid (add (mmul weights
                                   info-vector))
                          bias))]]))
```

```

        bias)))
    (get +questions+
      (sample (discrete (mat/to-nested-vectors values)))))))))

(defn make-policy [policy-type parameters]
  (case policy-type
    :uniform (->UniformPolicy)
    :myopic-voi (->MyopicVoiPolicy)
    :linear-belief (apply ->LinearBeliefPolicy parameters)
    :discounted-belief (apply ->DiscountedBeliefPolicy parameters)
    :logistic-info (apply ->LogisticInfoPolicy parameters)
    (throw (Exception. "policy-type must be one of [:uniform :myopic-voi :linear-belief :discounted-belief :logistic-info]"))))

```

A.3.4 Episode Simulation

```

(ns guess-who.trial
  "Guess Who POMDP solving with BBVB policy search"
  (:require [clojure.core.matrix :as mat :refer [array]]
            [guess-who
             [data :refer [+questions+ +entities+]]
             [heuristics :refer [myopic-voi recursive-voi]]
             [model :refer [+reliability+ posterior-belief
                           response-probability relative-utility
                           update-info]]
             [policy :refer [all]]]
            [anglican
             [trap :refer [value-cont]]
             [state :refer [initial-state]]
             [math :refer [max-entries max-index]]
             [bbvb :refer [get-learned]]
             bbem])
    (:use [anglican runtime emit learn
           [state :only [get-predicts]]
           [core :only [doquery]]]))

(defn value-state-cont
  "returns both value and state"
  [v s]
  [v s])

(with-primitive-procedures
 [update-info posterior-belief response-probability
  relative-utility max-index array make-policy select]

 (defm sample-policy
  "samples policy parameters and returns a policy instance of specified type"
  [policy-type]
  (let [sample-weights (fn [] (array
                              (map (fn [q]
                                     (map (fn [e]
                                            (sample (learn
                                                    [q e]
                                                    (gamma 100.0 100.0))))
                                           (keys +entities+)))
                                     +questions+)))]

      params (case policy-type
                :linear-belief
                (let [weights (sample-weights)
                      [weights]]

                  :discounted-belief
                  (let [weights (sample-weights)
                        gamma (sample (learn :gamma (beta 1 1)))]
                    [weights gamma])

                  :logistic-info
                  (let [weights (array
                              (map (fn [q1]
                                     (flatten
                                      (map (fn [q2]
                                             [(sample (learn [:weights q1 q2 true]
                                                             (gamma 10.0 100.0))
                                                             (sample (learn [:weights q1 q2 false]
                                                             (gamma 10.0 100.0)))]
                                             +questions+)))]
                                     +questions+)))]

                    bias (array
                          (map (fn [q]
                                 (sample (learn [:bias q]
                                                (normal 1.0 0.2))))
                                +questions+)))]

                +questions+)))]

```

```

        [weights bias])
        nil])
    (make-policy policy-type params)))

(defquery simulate-episode
  "simulates an episode of guess who, sampling responses
  based on the current belief"
  [policy-type depth inverse-temp initial-info]
  (let [initial-info (or initial-info {})
        initial-belief (posterior-belief initial-info)
        inverse-temp (or inverse-temp 1.0)
        policy (sample-policy policy-type)]
    (loop [questions []
          info initial-info
          belief initial-belief
          reward 0.0]
      (if (>= (count questions) depth)
        (do
         (predict :policy policy)
         (predict :questions questions)
         (predict :reward reward)
         (predict :info info))
        (let [;; select question according to sampled policy
              question (select policy info)
              ;; simulate response according to marginal
              ;; probability given current belief
              response (sample
                        (flip (response-probability belief
                                question)))

              ;; update information and belief state
              new-info (update-info info question response)
              new-belief (posterior-belief new-info)
              ;; update reward
              new-reward (relative-utility initial-belief new-belief)]
          ;; factor according to change in reward
          (observe (flip (exp (* inverse-temp (- new-reward reward)))) true)
          ;; continue to next question
          (recur (conj questions question)
                 new-info
                 new-belief
                 new-reward)))))))

(defn learn-policies
  "learns a policy using BBEM inference. returns empirical
  distribution of policies from last iteration"
  [policy-type depth number-of-steps
   & {:keys [initial-proposals
             number-of-particles
             base-stepsize
             adagrad
             robbins-monro]
      :or {number-of-particles 100
          base-stepsize 1.0
          adagrad 0.9
          robbins-monro 0.9}}]
  (let [samples (->> (doquery :bbem
                             simulate-episode
                             [policy-type depth]
                             :initial-proposals initial-proposals
                             :number-of-particles number-of-particles
                             :base-stepsize base-stepsize
                             :adagrad adagrad)
                (drop (* number-of-steps number-of-particles))
                (take number-of-particles))
        proposals (get-learned (first samples))
        policies (map (comp :policy get-predicts)
                      samples)]
    [policies proposals]))

(defn test-episode
  "plays a guess who episode with fixed policy
  and returns the final reward"
  [true-id policy number-of-questions]
  (loop [number-of-questions number-of-questions
        info {}]
    (if (zero? number-of-questions)
      (let [belief (posterior-belief info)
            guess-id (rand-nth (max-entries belief))]
        (if (= guess-id true-id)
          1.
          0.))
      (let [question (select policy info)
            [attr value] question
```

```

        response (if (sample (flip +reliability+))
                    (= (get-in +entities+ [true-id attr]) value)
                    (not= (get-in +entities+ [true-id attr]) value)))
    (recur (dec number-of-questions)
          (update-info info question response))))))

(defn test-sweep
  "runs a test episode for each entity multiple
  times and returns the total reward"
  [policy number-of-questions number-of-sweeps]
  (reduce
    (fn [reward id]
      (+ reward
         (test-episode id
                       policy
                       number-of-questions)))
      0.0
    (reduce concat
            (repeat number-of-sweeps
                    (keys +entities+)))))

(defn trial
  "learns a policy and runs a number of test sweeps. returns
  the rewards for all test episodes and the learned policy."
  [policy-type number-of-questions
   number-of-particles number-of-steps number-of-test-sweeps
   & {:keys [base-stepsize
            adagrad
            robbins-monro
            initial-proposals]
      :or {base-stepsize 1.0
          adagrad 0.9
          robbins-monro 0.9}}]
  (let [[policies proposals] (learn-policies policy-type number-of-questions number-of-steps
                                             :number-of-particles number-of-particles
                                             :base-stepsize base-stepsize
                                             :adagrad adagrad
                                             :robbins-monro robbins-monro)
        rewards (doall (map #(test-sweep % number-of-questions number-of-test-sweeps)
                             policies))]
    [rewards policies proposals]))

```