# Analyzing Games: Nash Equilibria

ISCI 330 Lecture 5

January 23, 2006

#### Lecture Overview

Recap

- 2 Best Response and Nash Equilibriun
- Mixed Strategies

• Idea: sometimes, one outcome o is at least as good for every agent as another outcome o', and there some agent who strictly prefers o to o'

Best Response and Nash Equilibrium

- in this case, it seems reasonable to say that o is better than o'
- we say that o Pareto-dominates o'.

• An outcome  $o^*$  is Pareto-optimal if there is no other outcome that Pareto-dominates it.

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### Best Response

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- Let  $a_{-i} = \langle a_1, \dots, a_{i-1}, a_{i+1}, \dots, a_n \rangle$ .
  - $\bullet \ \, \mathsf{now} \,\, a = (a_{-i}, a_i)$

• Best response:  $a_i^* \in BR(a_{-i})$  iff  $\forall a_i \in A_i, \ u_i(a_i^*, a_{-i}) \ge u_i(a_i, a_{-i})$ 

## Nash Equilibrium

- Now let's return to the setting where no agent knows anything about what the others will do
- What can we say about which actions will occur?

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- What can we say about which actions will occur?

- Idea: look for stable action profiles.
- $a = \langle a_1, \dots, a_n \rangle$  is a Nash equilibrium iff  $\forall i, a_i \in BR(a_{-i})$ .

C	D

$$C = \begin{bmatrix} -1, -1 & -4, 0 \\ 0, -4 & -3, -3 \end{bmatrix}$$

	C	D		Left	Right
C	-1, -1	-4,0	Left	1	0
D	0, -4	-3, -3	Right	0	1

	C	D
C	-1, -1	-4,0
D	0, -4	-3, -3

	Left	Right
Left	1	0
Right	0	1

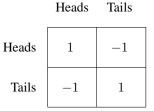
 $\begin{array}{c|cccc} & B & F \\ & & \\ B & & 2,1 & 0,0 \\ & & & \\ F & & 0,0 & 1,2 \end{array}$ 

	C	D
C	-1, -1	-4,0
D	0, -4	-3, -3

	Left	Right
Left	1	0
Right	0	1

В

F



Right

I oft

### Nash Equilibria of Example Games

	(	C	D		Leit	Kigiit
C	-1	,-1	-4, 0	Left	1	0
D	0,	-4	-3, -3	Right	0	1
		В	F		Heads	Tails
]	В	2,1	0,0	Heads	1	-1

The paradox of Prisoner's dilemma: the Nash equilibrium is the only non-Pareto-optimal outcome!

Tails

F

0, 0

1, 2

Mixed Strategies

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### Mixed Strategies

- It would be a pretty bad idea to play any deterministic strategy in matching pennies
- Idea: confuse the opponent by playing randomly
- Define a strategy  $s_i$  for agent i as any probability distribution over the actions  $A_i$ .
  - pure strategy: only one action is played with positive probability
  - mixed strategy: more than one action is played with positive probability
    - these actions are called the support of the mixed strategy
- Let the set of all strategies for i be  $S_i$
- Let the set of all strategy profiles be  $S = S_1 \times ... \times S_n$ .



# Utility under Mixed Strategies

- What is your payoff if all the players follow mixed strategy profile  $s \in S$ ?
  - We can't just read this number from the game matrix anymore: we won't always end up in the same cell

# Utility under Mixed Strategies

- What is your payoff if all the players follow mixed strategy profile  $s \in S$ ?
  - We can't just read this number from the game matrix anymore: we won't always end up in the same cell
- Instead, use the idea of expected utility from decision theory:

$$u_i(s) = \sum_{a \in A} u_i(a) Pr(a|s)$$

$$Pr(a|s) = \prod_{j \in N} s_j(a_j)$$

### Best Response and Nash Equilibrium

Our definitions of best response and Nash equilibrium generalize from actions to strategies.

- Best response:
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- Best response:
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- Nash equilibrium:
  - $s = \langle s_1, \dots, s_n \rangle$  is a Nash equilibrium iff  $\forall i, s_i \in BR(s_{-i})$
- Every finite game has a Nash equilibrium! [Nash, 1950]
  - e.g., matching pennies: both players play heads/tails 50%/50%