# Analyzing Games

#### ISCI 330 Lecture 4

#### January 18, 2007

Analyzing Games

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#### Lecture Overview



2 Two more examples

3 Pareto Optimality

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Analyzing Games

## Defining Games

- Finite, *n*-person game:  $\langle N, A, u \rangle$ :
  - $\bullet~N$  is a finite set of n players, indexed by i
  - $A=\langle A_1,\ldots,A_n\rangle$  is a tuple of action sets for each player i
    - $a \in A$  is an action profile
  - $u = \langle u_1, \dots, u_n \rangle$ , a utility function for each player, where  $u_i : A \mapsto \mathbb{R}$
- Writing a 2-player game as a matrix:
  - row player is player 1, column player is player 2
  - rows are actions  $a \in A_1$ , columns are  $a' \in A_2$
  - cells are outcomes, written as a tuple of utility values for each player

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#### Prisoner's dilemma

#### Prisoner's dilemma is any game

 $\begin{array}{c|c} C & D \\ \\ C & a, a & b, c \\ \\ D & c, b & d, d \end{array}$ 

with c > a > d > b.

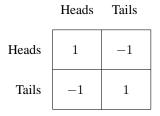


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## Matching Pennies

A zero-sum game: players have exactly opposed interests. One player wants to match; the other wants to mismatch.



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#### Coordination Game

A cooperative game: players have exactly the same interests. Which side of the road should you drive on?

	Left	Right
Left	1	0
Right	0	1

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## Games of Cooperation

Players have exactly the same interests.

• no conflict: all players want the same things

• 
$$\forall a \in A, \forall i, j, u_i(a) = u_j(a)$$

- we often write such games with a single payoff per cell
- why are such games "noncooperative"?

## Coordination Game

#### Which side of the road should you drive on?

Left Right

Left	1	0
Right	0	1

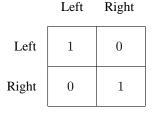
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#### Coordination Game

#### Which side of the road should you drive on?

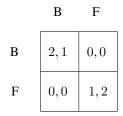


Play this game with someone near you, repeating five times.

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## General Games: Battle of the Sexes

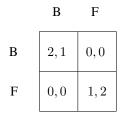
The most interesting games combine elements of cooperation *and* competition.



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## General Games: Battle of the Sexes

The most interesting games combine elements of cooperation *and* competition.



Play this game with someone near you, repeating five times.

#### Lecture Overview



2 Two more examples





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# Analyzing Games

- We've defined some canonical games, and thought about how to play them. Now let's examine the games from the outside
- From the point of view of an outside observer, can some outcomes of a game be said to be better than others?

# Analyzing Games

- We've defined some canonical games, and thought about how to play them. Now let's examine the games from the outside
- From the point of view of an outside observer, can some outcomes of a game be said to be better than others?
  - we have no way of saying that one agent's interests are more important than another's
  - intuition: imagine trying to find the revenue-maximizing outcome when you don't know what currency has been used to express each agent's payoff
- Are there situations where we can still prefer one outcome to another?

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- Idea: sometimes, one outcome o is at least as good for every agent as another outcome o', and there is some agent who strictly prefers o to o'
  - in this case, it seems reasonable to say that o is better than o'
  - we say that *o* Pareto-dominates *o*'.

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- An outcome  $o^*$  is Pareto-optimal if there is no other outcome that Pareto-dominates it.
  - can a game have more than one Pareto-optimal outcome?
  - does every game have at least one Pareto-optimal outcome?

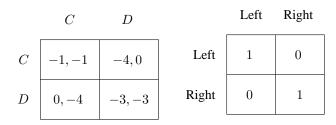
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C	-1, -1	-4,0
D	0, -4	-3, -3

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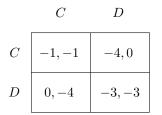
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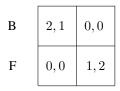


Left	1	0
Right	0	1

Left

Right

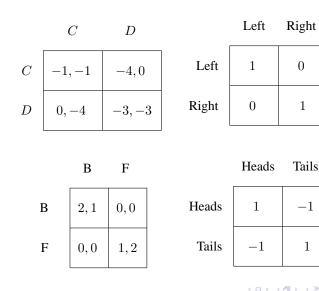
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