Game Theory intro

ISCI 330 Lecture 3

January 16, 2007

Lecture Overview

1. Recap: Utility Theory
2. Game Theory
3. Example Matrix Games

Self-interested agents

- What does it mean to say that an agent is self-interested?
  - not that they want to harm other agents
  - not that they only care about things that benefit them
  - that the agent has its own description of states of the world that it likes, and that its actions are motivated by this description
- Utility theory:
  - quantifies degree of preference across alternatives
  - understand the impact of uncertainty on these preferences
  - utility function: a mapping from states of the world to real numbers, indicating the agent's level of happiness with that state of the world
  - Decision-theoretic rationality: take actions to maximize expected utility.

Preferences and utility functions

Theorem (von Neumann and Morgenstern, 1944; informal version)

If an agent's preference relation satisfies the axioms Completeness, Transitivity, Substitutability, Decomposability, Monotonicity and Continuity then there exists a function $u : O \rightarrow [0, 1]$ *with the properties that:*

1. $u(o_1) \geq u(o_2)$ *iff the agent prefers* o_1 *to* o_2*; and*
2. *when faced about uncertainty about which outcomes he will receive, the agent always prefers outcomes with larger expected values of* u *to outcomes with smaller expected values of* u*.*

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Non-Cooperative Game Theory

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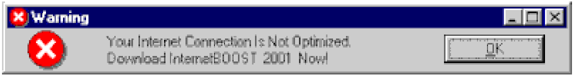
  - mathematical study of interaction between rational, self-interested agents
- Why is it called non-cooperative?
  - while it's most interested in situations where agents' interests conflict, it's not restricted to these settings
  - the key is that the individual is the basic modeling unit, and that individuals pursue their own interests
    - cooperative/coalitional game theory has teams as the central unit, rather than agents

TCP Backoff Game

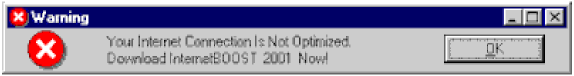
TCP Backoff Game



Should you send your packets using correctly-implemented TCP (which has a "backoff" mechanism) or using a defective implementation (which doesn't)?

- Consider this situation as a two-player game:
  - both use a correct implementation: both get 1 ms delay
  - one correct, one defective: 4 ms delay for correct, 0 ms for defective
  - both defective: both get a 3 ms delay.

TCP Backoff Game



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- Play this game with someone near you, repeating five times.

TCP Backoff Game

- Consider this situation as a two-player game:
  - both use a correct implementation: both get 1 ms delay
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- Questions:
  - What action should a player of the game take?
  - Would all users behave the same in this scenario?
  - What global patterns of behaviour should the system designer expect?
  - Under what changes to the delay numbers would behavior be the same?
  - What effect would communication have?
  - Repetitions? (finite? infinite?)
  - Does it matter if I believe that my opponent is rational?

Defining Games

- Finite, n-person game: $\langle N, A, u \rangle$:
  - N is a finite set of n players, indexed by i
  - $A = \langle A_1, \ldots, A_n \rangle$ is a tuple of action sets for each player i
    - $a \in A$ is an action profile
  - $u = \langle u_1, \ldots, u_n \rangle$, a utility function for each player, where $u_i : A \mapsto \mathbb{R}$

- Writing a 2-player game as a matrix:
  - row player is player 1, column player is player 2
  - rows are actions $a \in A_1$, columns are $a' \in A_2$
  - cells are outcomes, written as a tuple of utility values for each player

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Games in Matrix Form

Here's the TCP Backoff Game written as a matrix ("normal form").

	C	D
C	$-1, -1$	$-4, 0$
D	$0, -4$	$-3, -3$

More General Form

Prisoner's dilemma is any game

$$\begin{array}{c|c|c|}
 & C & D \\ \hline
C & a, a & b, c \\ \hline
D & c, b & d, d \\ \hline
\end{array}$$

with $c > a > d > b$.

Games of Pure Competition

Players have exactly opposed interests

- There must be precisely two players (otherwise they can't have exactly opposed interests)
- For all action profiles $a \in A$, $u_1(a) + u_2(a) = c$ for some constant c
  - Special case: zero sum
- Thus, we only need to store a utility function for one player
  - in a sense, it's a one-player game

Matching Pennies

One player wants to match; the other wants to mismatch.

	Heads	Tails
Heads	1	-1
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Rock-Paper-Scissors

Generalized matching pennies.

	Rock	Paper	Scissors
Rock	0	-1	1
Paper	1	0	-1
Scissors	-1	1	0

...Believe it or not, there's an annual international competition for this game!