Extensive Form Games and Subgame Perfection

ISCI 330 Lecture 12

February 15, 2007

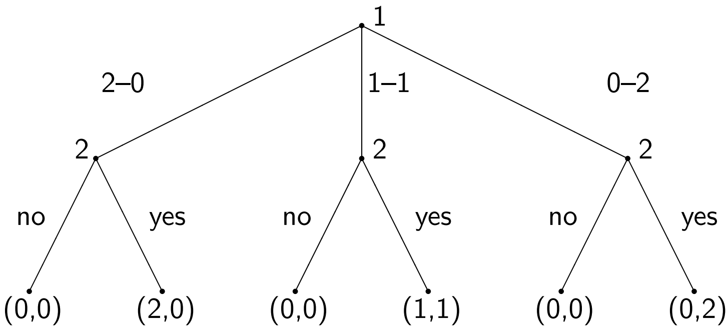
Lecture Overview

Recap

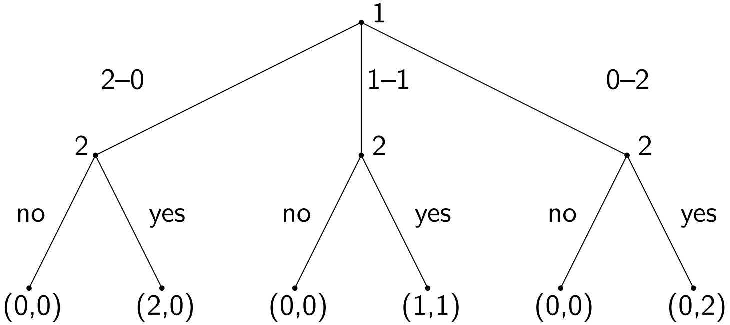
Perfect-Information Extensive-Form Games

Subgame Perfection

Example: the sharing game



Example: the sharing game



Play as a fun game, dividing 100 dollar coins. (Play each partner only once.)

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Pure Strategies

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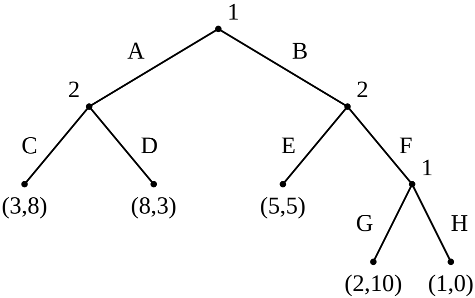
Pure Strategies

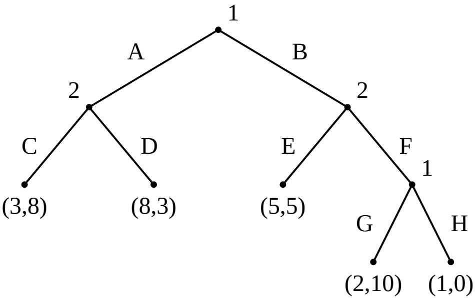
- In the sharing game (splitting 2 coins) how many pure strategies does each player have?
  - player 1: 3; player 2: 8
- Overall, a pure strategy for a player in a perfect-information game is a complete specification of which deterministic action to take at every node belonging to that player.
- Can think of a strategy as a complete set of instructions for a proxy who will play for the player in their absence

Pure Strategies Example



What are the pure strategies for player 2?

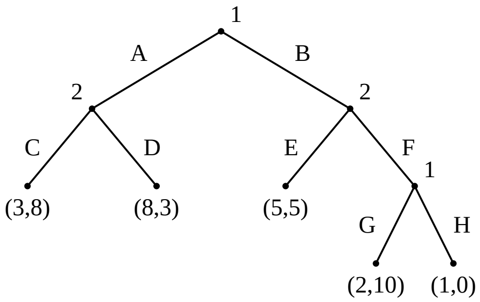
Pure Strategies Example



What are the pure strategies for player 2?

- $S_2 = \{(C, E); (C, F); (D, E); (D, F)\}$

Pure Strategies Example



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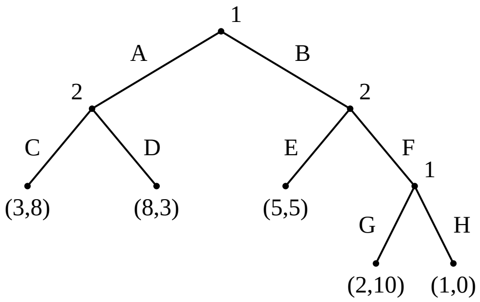
Pure Strategies Example



What are the pure strategies for player 2?

- $S_2 = \{(C, E); (C, F); (D, E); (D, F)\}$

What are the pure strategies for player 1?

- $S_1 = \{(B, G); (B, H), (A, G), (A, H)\}$
- This is true even though, conditional on taking A, the choice between G and H will never have to be made

Nash Equilibria

Given our new definition of pure strategy, we are able to reuse our old definitions of:

- mixed strategies
- best response
- Nash equilibrium

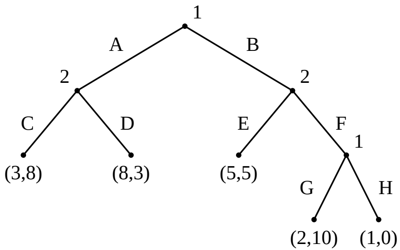
Theorem

Every perfect information game in extensive form has a PSNE

This is easy to see, since the players move sequentially.

Induced Normal Form

- In fact, the connection to the normal form is even tighter
  - we can "convert" an extensive-form game into normal form



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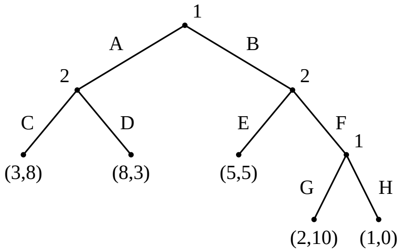

	CE	CF	DE	DF
AG	$3,8$	$3,8$	$8,3$	$8,3$
AH	$3,8$	$3,8$	$8,3$	$8,3$
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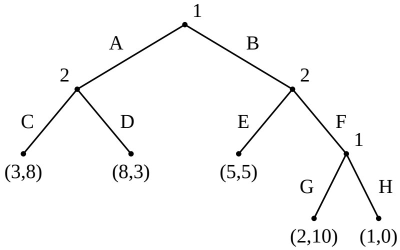

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- this illustrates the lack of compactness of the normal form
  - games aren't always this small
  - even here we write down 16 payoff pairs instead of 5

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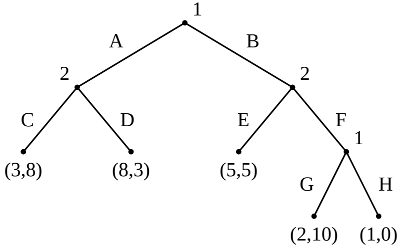

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- while we can write any extensive-form game as a NF, we can't do the reverse.
  - e.g., matching pennies cannot be written as a perfect-information extensive form game

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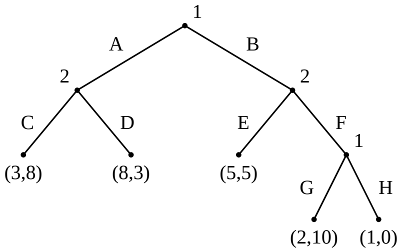

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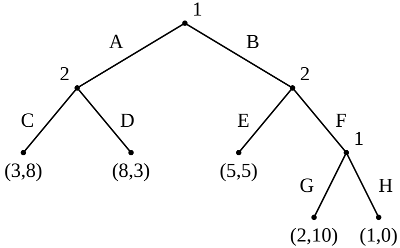

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  - $(A,G),(C,F)$
  - $(A,H),(C,F)$
  - $(B,H),(C,E)$

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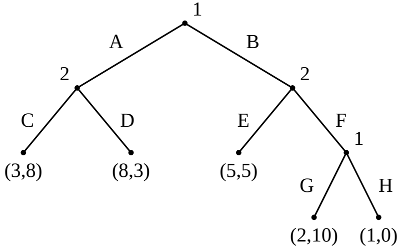

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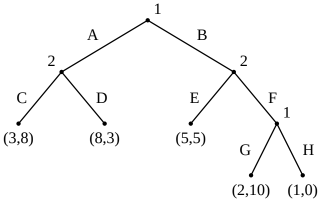

- There's something intuitively wrong with the equilibrium $(B, H), (C, E)$
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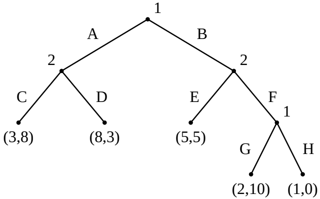
    - After all, G dominates H for him

Subgame Perfection



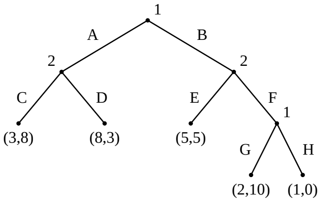
- There's something intuitively wrong with the equilibrium $(B, H), (C, E)$
  - Why would player 1 ever choose to play H if he got to the second choice node?
    - After all, G dominates H for him
  - He does it to threaten player 2, to prevent him from choosing F, and so gets 5
    - However, this seems like a non-credible threat
    - If player 1 reached his second decision node, would he really follow through and play H?

Formal Definition

- Define subgame of G rooted at h:
  - the restriction of G to the descendents of H.
- Define set of subgames of G:
  - subgames of G rooted at nodes in G

- s is a subgame perfect equilibrium of G iff for any subgame G' of G, the restriction of s to G' is a Nash equilibrium of G'
- Notes:
  - since G is its own subgame, every SPE is a NE.
  - this definition rules out "non-credible threats"

Back to the Example



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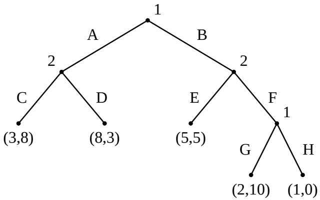

- Which equilibria from the example are subgame perfect?
  - $(A, G), (C, F)$ is subgame perfect
  - (B, H) is an non-credible threat, so $(B, H), (C, E)$ is not subgame perfect
  - (A, H) is also non-credible, even though H is "off-path"