Homework #1

ISCI 330 Game Theory

1) **(6 Points)** Please answer the questions in a) and b) briefly (e.g., one sentence for each answer part).
   a) Name 1 reasons we might consider game theory appropriate for an integrated science course?

   b) How does the philosopher Mario Bunge's notion of "stuff-free science" apply to game theory…
      i) in terms of agents?

      ii) in terms of payoffs?

2) **(14 Points)** Consider the real number payoffs a, b, c, and d in a symmetric game labelled Game 1 below,

a, a	b, c
c, b	d, d

Game 1

and the modified utility amounts in games 2, 3, and 4, where x is a positive real number:

a+x, a+x	b+x , c+x
c+x, b+x	d+x, d+x

Game 2

ax, ax	bx, cx
cx, bx	dx, dx

Game 3

a^x, a^x	b^x, c^x
c^x, b^x	d^x, d^x

Game 4

For each of the solutions below write down which of the modified games (2, 3, and 4) are guaranteed to have the same solutions as Game 1:
   a) Pareto optimal cells

   b) Nash equilibrium cells

   c) Mixed strategy Nash equilibrium probabilities

   d) Prove your answer in c) with respect to Game 2 (show your work on the back of this page)

3) **(20 Points)** This question reviews some classic 2-player non-zero-sum (NZS) games where the two alternative strategies are labelled cooperate (C) and defect (D), and preferences are represented with ordinal payoffs: $1^{st} > 2^{nd} > 3^{rd} > 4^{th}$. In each game matrices below, write a capital **P** in all cells that are Pareto optimal solutions and a capital **N** in all cells that are a Nash equilibrium solution. The common names of these games are given below each matrices (some games have multiple names). The first one (a) is done for you.

a)

	C	**D**
C	$1^{st},1^{st}$ **P N**	$4^{th},3^{rd}$
D	$3^{rd},4^{th}$	$2^{nd},2^{nd}$ **N**

b)

	C	**D**
C	$1^{st},1^{st}$	$3^{rd},2^{nd}$
D	$2^{nd},3^{rd}$	$4^{th},4^{th}$

Spite

c)

	C	**D**
C	$2^{nd},2^{nd}$	$4^{th},1^{st}$
D	$1^{st},4^{th}$	$3^{rd},3^{rd}$

Prisoner's Dilemma

d)

	C	**D**
C	$2^{nd},2^{nd}$	$3^{rd},1^{st}$
D	$1^{st},3^{rd}$	$4^{th},4^{th}$

Chicken, Hawk-Dove, Snowdrift, Brinkman-ship

e)

	C	**D**
C	$3^{rd},3^{rd}$	$2^{nd},1^{st}$
D	$1^{st},2^{nd}$	$4^{th},4^{th}$

Leader-Follower, Reluctant Hero

f)

	C	**D**
C	$1^{st},1^{st}$	$4^{th},2^{nd}$
D	$2^{nd},4^{th}$	$3^{rd},3^{rd}$

Stag hunt, Assurance, Coordination

g) A dominant strategy is one in which the same strategy is the logical choice regardless of what the other player does. Write a capital D in all cells above that represent a dominant strategy for both players.

5) **(20 Points)** Consider the battle of the sexes game from class where player 1 prefers to do activity B with their partner and player 2 prefers to do activity F with their partner. For this payoff matrix (shown below) we calculated in class a Nash Equilibrium where player 1 chooses B 2/3 of the time and F 1/3 of the time; whereas player 2 chooses B 1/3 of the time and F 2/3 of the time: (2/3, 1/3), (1/3, 2/3). **<u>In your answers below, show your calculations</u>.**

	B	F
B	2,1	0,0
F	0,0	1,2

a) If the players are rational and are correct in assuming the other player is also rational, the players will choose the activities with the above frequencies. In this case, in what proportion of their outings is the couple together (doing the same activity)?

b) In what proportion of outings is player 1 doing B by himself/herself while player 2 does F?

c) In what proportion of outings is player 2 doing B by himself/herself while player 1 does F?

d) Sometimes for convenience, we refer to units of utility as "utils". Assuming the same mixed strategies above, when they go out, what is the average expected payoff for player 1 in utils during each outing? For player 2?

e) Consider the case where player 2 knows that player 1 is selfish and stubborn and will *always* choose activity B. Given this extra information, what is player 2's best strategy?

f) Under the scenario in e) what is the average expected payoff for player 1 in utils during each outing? For player 2?

g) So what assumption about the original game (used in a – d) above justifies the claim that the mixed Nash equilibrium strategy is a rational outcome (even though it results in the players spending most of their outings alone and each getting an average utility that is less than what each player gets in the scenario in e))?

6) **(15 Points)** Now consider a modified battle of the sexes game given by the following payoff matrix where both players prefer doing B alone to doing F alone, but they each still has their different preferred activity when they go out together. **<u>In your answers below, show your calculations</u>.**

	B	F
B	4,2	1,0
F	0,1	2,4

a) Find the mixed Nash equilibrium solution for this game and show your work. Express this as (q, 1-q), (p, 1-p) where q is the frequency with which player 1 plays B and p is the frequency with which player 2 plays B.

b) Find the expected payoff for each player.

c) On what proportion of outings is the couple doing activity B together?

d) On what proportion of outings is the couple doing activity F together?

Academic Honesty Form

For this assignment, it is acceptable to collaborate with other students provided that you write up your solutions independently. The only reference materials that you can use are the course notes and textbook, and the reference textbooks listed on the course web page. In particular, getting help from students or course materials from previous years is not acceptable.

List any people you collaborated with:

List any non-course materials you referred to:

Signature:

Fill in this page and include it with your assignment submission.