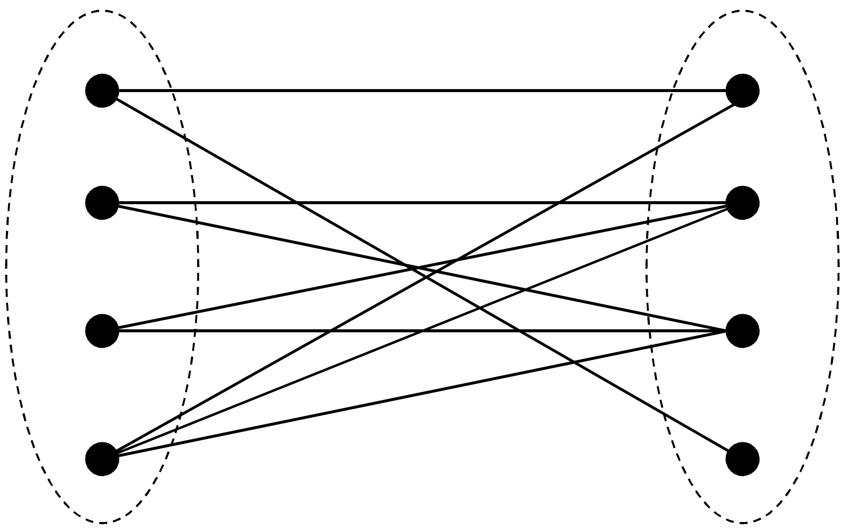
Bipartite Matching Mechanisms



Shahab Bahrami

What you will learn in this talk

Part 1:

What is “Matching” in a graph ?

Why you need a “Matching”?

What are some“ Matching” with specific characteristics?

Part 2:

How you can design a “Matching mechanism”?

What are the properties of a “(Good !) matching mechanism”!

Part 1

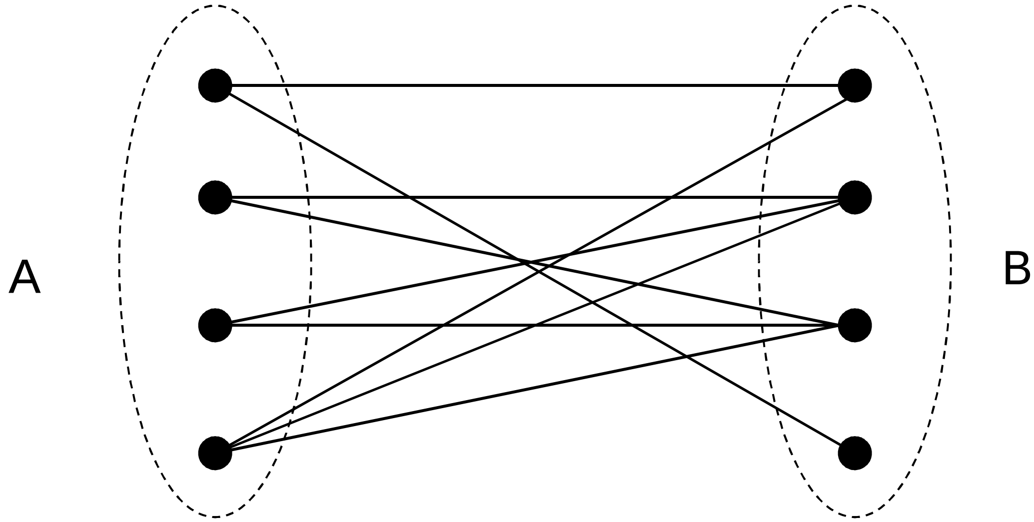
Part 1:

-What is “Matching” in a graph ?

-Why we need a “Matching”?

Bipartite Matching

A graph is bipartite if its nodes set can be partitioned into two subsets A and B so that each edge has one endpoint in A and the other endpoint in B.

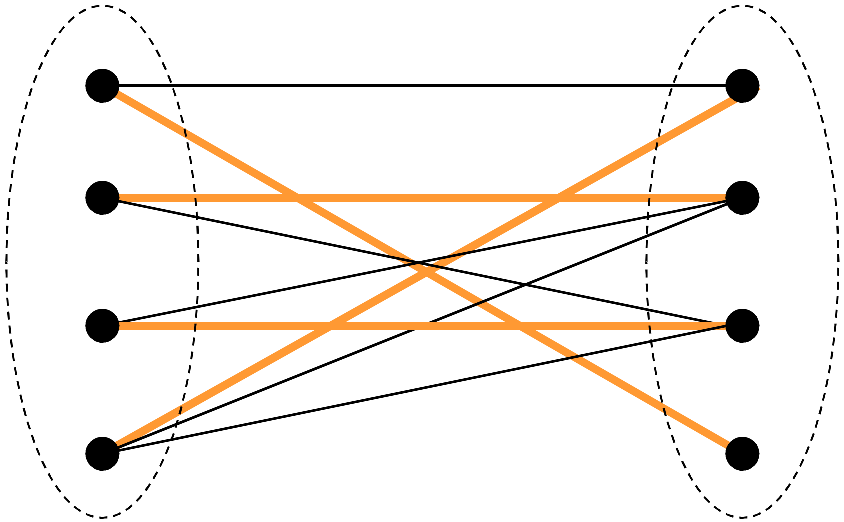


A matching M is a subset of edges so that every nodes has degree at most one in M.

Maximum Matching

The bipartite matching problem:

Find a matching with the maximum number of edges.



A perfect matching is a matching in which every vertex is matched.

The perfect matching problem: Is there a perfect matching?

Practice…

- Find the maximum matching of the graph in your papers

FindAMaximumMatchingInABipartiteGraph.cdf

Intuition

Kidneys and patients.

Dancing in a party.

Marriage problem.

Students an advisors.

Maximum matching tries to maximize the number of pairs!

Summary of part 1

Now you know

-What is a bipartite matching...

-Difference between the maximum and perfect matching...

Part 2
Matching Mechanism

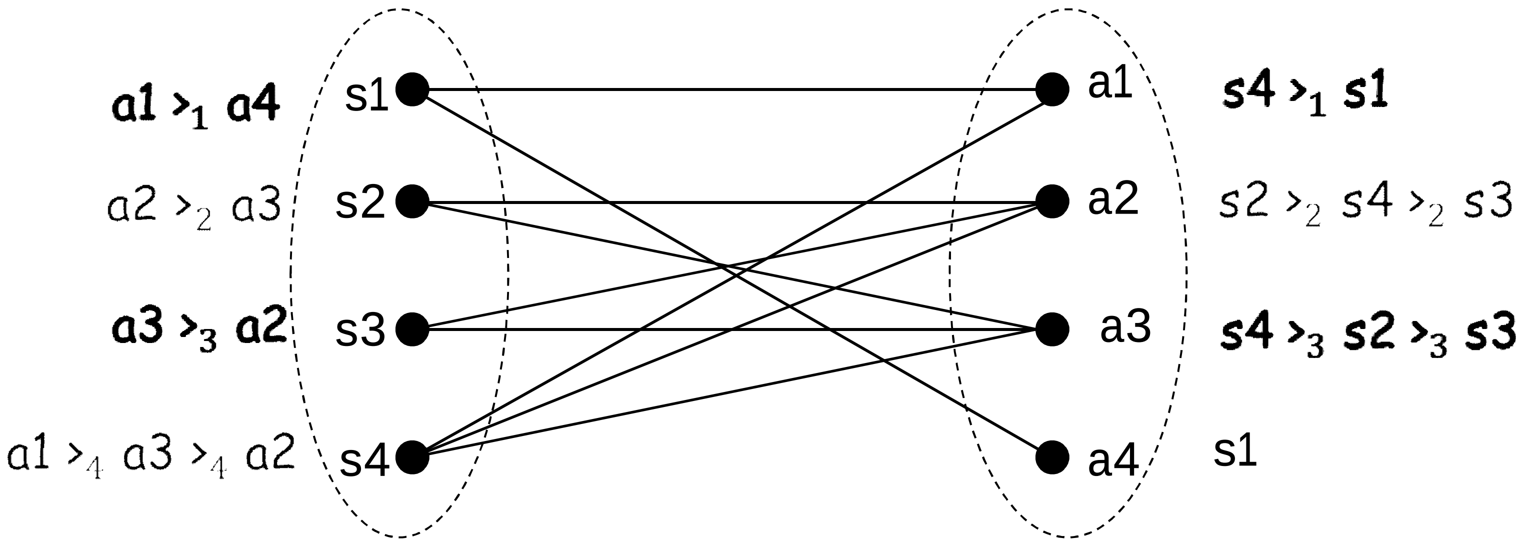
Part 2:

How can you design a “Matching mechanism”?

What are the properties of a “ Rational Matching Mechanism”!

Matching Mechanism

Bipartite matching is used to design mechanism in which money can not be transferred between the agents



A social choice function is a decision about which student should be assigned to which advisors.

Some definitions

Individual rationality: A matching M is IR if no agent i prefers to remain unmatched than to be matched to M(i).

... Matching is only from the edges of the graph!!!

Unblocked: A matching M is Unblocked if there exist no pair (s, a) such that
- s is not matched with a

  but
- The student prefers a to his current advisor.
- The advisor prefers s to his current student.

Stable matching is both IR and Unblocked.

Gale and Shapley theorem

A stable matching always exists.

Deferred acceptance (DA) algorithm... Student-application version.

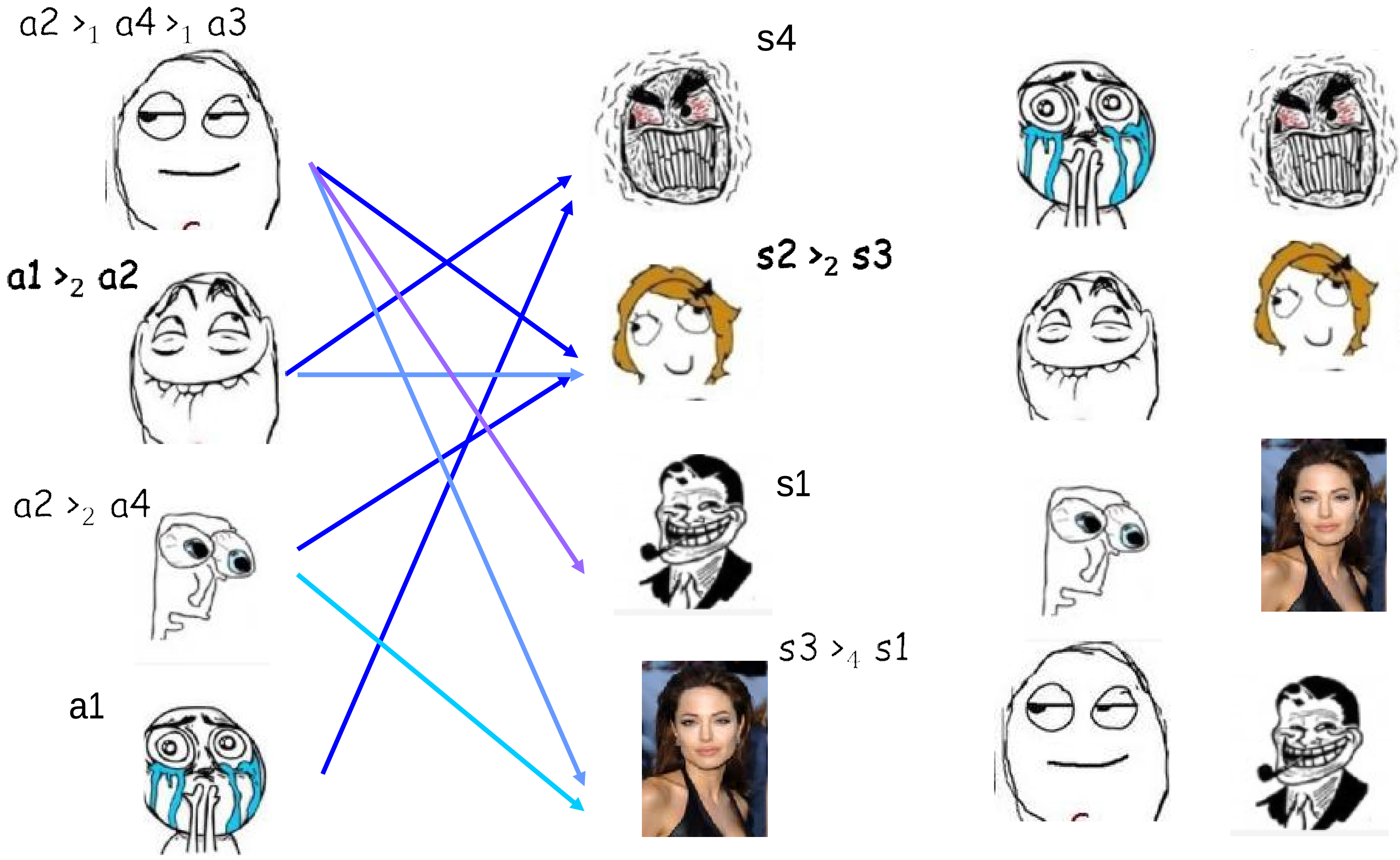
Step 1: each student applies to his most preferred advisor.
Repeat

Step 2: each advisor keeps her most preferred acceptable application an rejects the rest.

Step 3: each student who was rejected at the previous step applies to his next acceptable choice.

Until no students applied in the last step

DA algorithm example



Optimality

Student/advisors preference over matchings:

Student *i* prefers matching M to M′ if M(i)>M′(i).

Student Optimal stable matching: every students likes it at least as well as other stable matchings.

Claim:
There exist only one Student Optimal stable matching.
Student application version of DA gives that matching too.

Designing a mechanism

Question:
Agents' preferences are private information,
Can we find a mechanism that ensures a stable matching?

Theorem:

No mechanism implements stable matching in dominant strategies or in ex post equilibrium.

Relaxing the assumption that all agents are strategic.
For example, advisors tells the truth.

Student application version of DA is strategy proofs.

Summary

- Bipartite matching is an useful tool to model several activities.

- We studied the matching mechanisms.

- We see that we have always unique optimal stable matching.

- By relaxing some assumptions, we saw there exists strategy proof mechanism.

References

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Thank you!

Any Question?