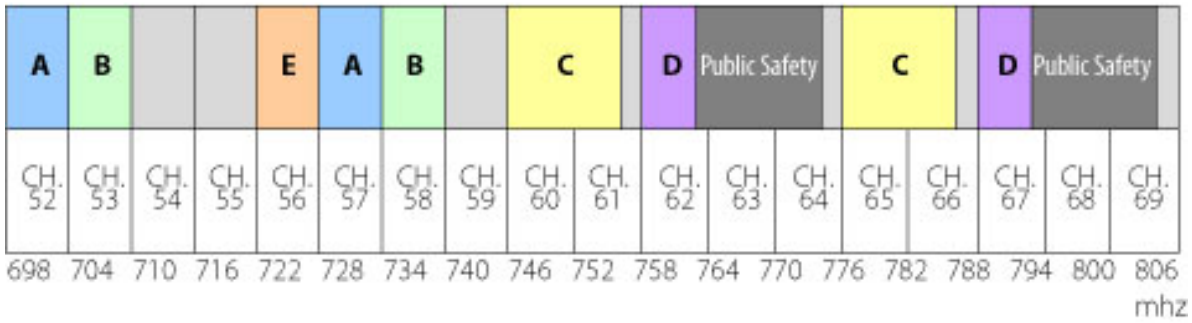
Reallocating Spectrum : The Incentive Auction

Presented by Alexandre Fréchette

Outline

Spectrum Auctions

Governments **auction off** public electromagnetic spectrum.



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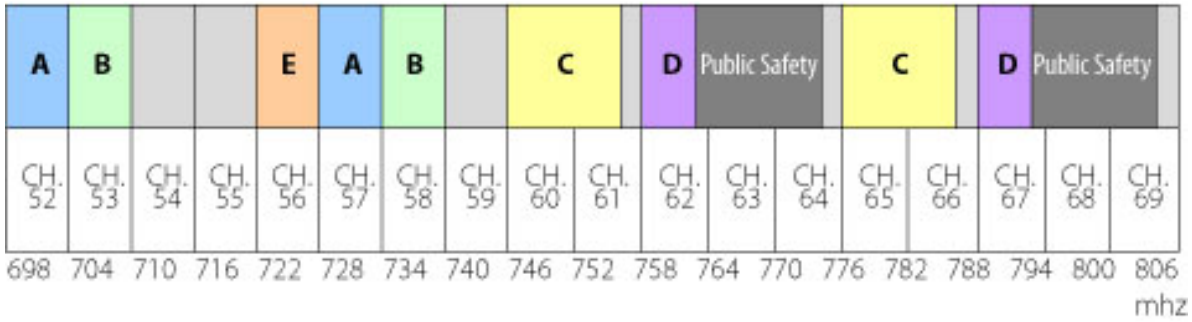

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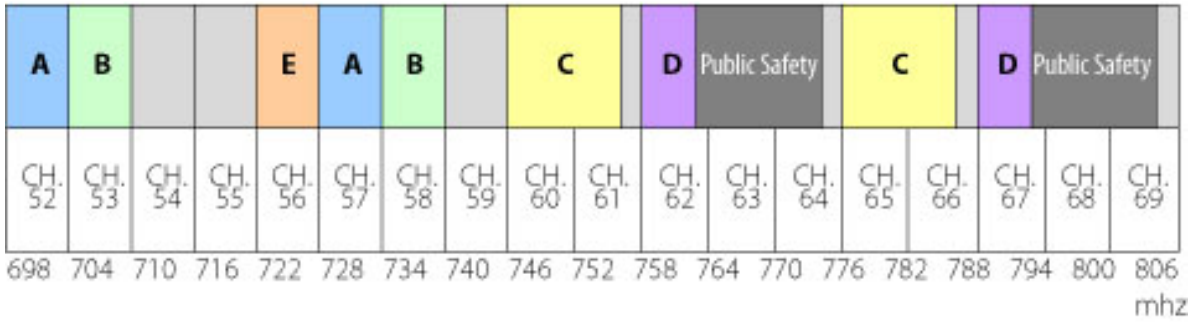

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Spectrum Auctions - *Examples*

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- Since July 1994, the Federal Communications Commission (FCC) has conducted 87 spectrum auctions, which raised over $60 billion for the U.S. Treasury.
- A UK auction in 2000 generated €38.3 billion in revenue.
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A Game - *Reenacting the Turkish Auction of 2000* [1]

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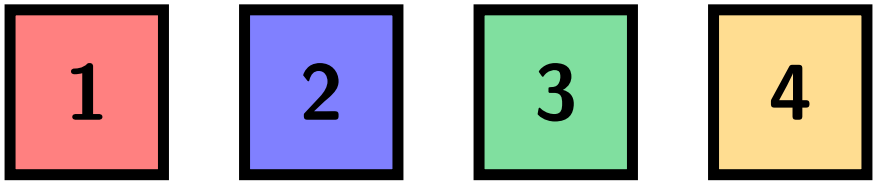

1. You have your **private value/budget**.
2. Auction off **one block at a time**, using **first price auction**.
3. **Reserve/starting price** of block i is final price of block $i-1$.
4. Your utility is the **fraction of sold blocks you own**.

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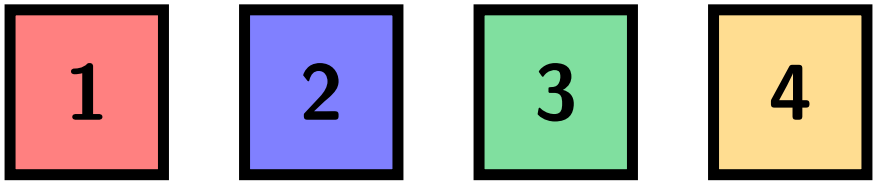

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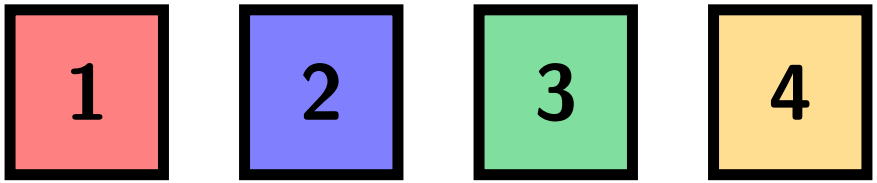

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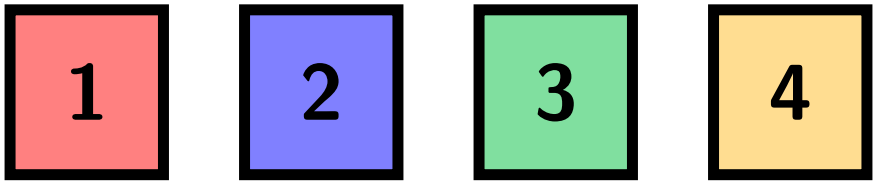

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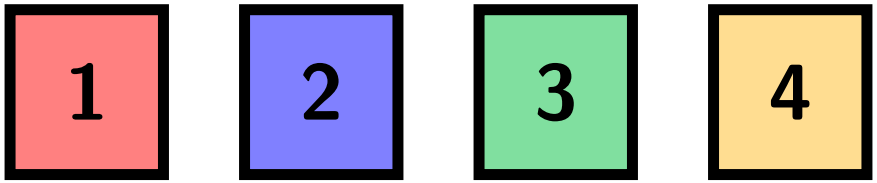
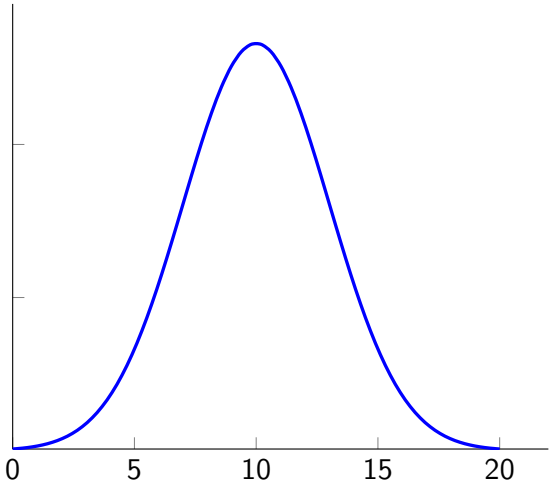


Value distribution - normal with mean 10 and standard deviation 3.



High-Stakes Does Not Guarantee High Quality

CNW Group/Mobilicity

Spectrum auction design is a **complex** problem, and has serious consequences when done inadequately [2].

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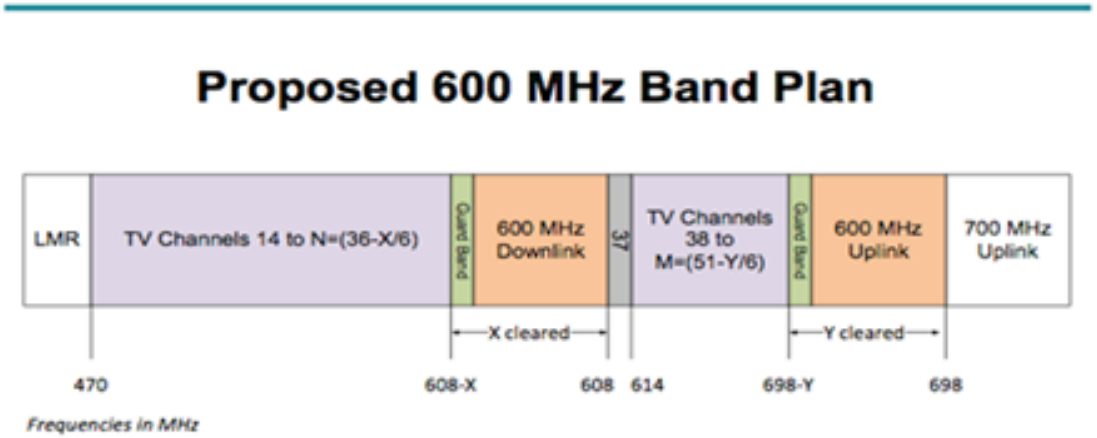
Incentive Auction

The FCC wants to **provide more spectrum** for mobile companies to fuel the next generation of products by buying some off broadcast television companies, and selling it to telecoms.



http://www.hlspectrumreview.com/2012/10/articles/auctions/

united-states-rulemaking-for-incentive-auction-of-broadcaster-spectrum/

Freeing Up Spectrum

Assume some TV stations agree to **go off air**.

Repack the remaining (on-air) stations on a smaller range of channels **without causing interferences**.

A **graph colouring problem** on a graph with a few thousand vertices and hundred of thousands of edges.

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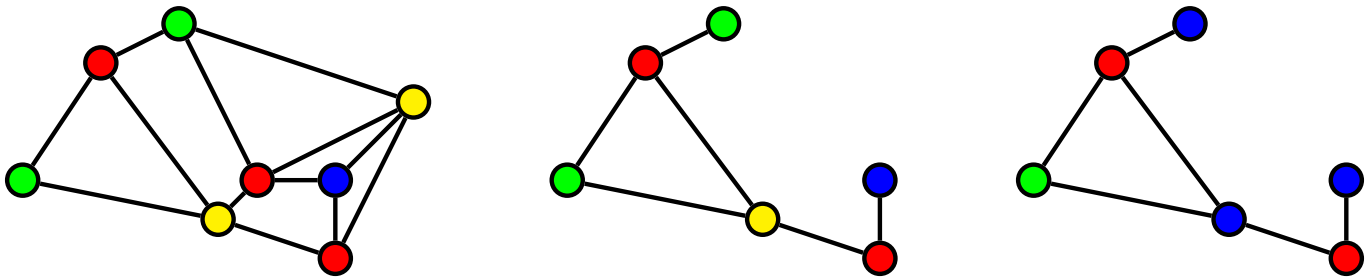

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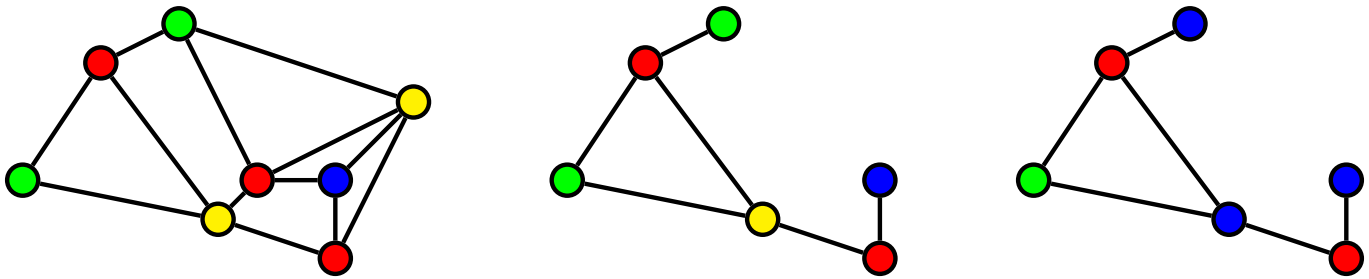

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Nick Arnosti, Auctionomics

Incentive Auction [4]

Three components acting in unison:

(1) **Reverse auction** to buy spectrum off TV stations, and meet a clearing target.

(2) **Forward (ascending prices) auction** to sell cleared spectrum to mobile companies;
A slight adaptation of the successful clock auction previously used.

(3) Coordination mechanism to direct reallocation goals;
Adjusts clearing target based on global efficiency.

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Reverse Auction

Which stations should we compensate, and **how much** should we give them? *"That's easy, just use VCG!"*

Let N be the **set of stations**, let $\mathcal{F} \subseteq 2^N$ be the collection of subsets of stations that can **feasibly be repacked**.
Then given bids $\hat{b} \in \mathbb{R}^N$,

$$\chi^{\text{VCG}}(\hat{b}) = \arg\min_{S \in \mathcal{F}} \sum_{i \in S} \hat{b}_i.$$

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Deferred Acceptance Auction

Alternate mechanism for the reverse auction [4, 3].

"Deferred-acceptance (DA) auctions choose allocations by an **iterative process of rejecting the least attractive bid.**"

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Allocation Rule

At each step t of the DA auction, we have a set $A_t \subseteq N$ of **active bidders**.

The DA auction is specified by **scoring functions** for any active set $A \subseteq N$ and station $i \in A$

$$s_i^A : B_i \times B^{N \setminus A} \to \mathbb{R}^+,$$

where B_i is the "bid space" of station i. Scoring functions must be **non-decreasing in their first argument**.

Then at each step the DA auction **removes from its active set the bidders with highest non-zero bid scores**, and otherwise returns $\chi^{\mathrm{DA}}(\hat{b}_t) = A_t$ if all scores are zero.

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Simplest DA Algorithm for the Reverse Auction

For the reverse auction, A can be thought of as the "packable" stations. Then a simple scoring function give **non-zero score only to repackable bidders**:

$$s_i^A(\hat{b}) = \begin{cases} 0 & \text{if } N \setminus A \cup \{i\} \notin \mathcal{F} \\ \hat{b}_i & \text{otherwise} \end{cases}$$

This only requires to **check feasibility**, instead of actually optimizing over feasible sets of stations. Still **NP**-complete, but empirically easier.

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Payment Rules

Pay-as-bid / first-price payments:

$$\wp_i^{\mathrm{FP}}(\hat{b}) = \begin{cases} \hat{b}_i & \text{if } i \in A \\ 0 & \text{otherwise} \end{cases}$$

Threshold price - highest bid without changing outcome:

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Dominant Strategy Incentive Compatibility

Proposition

A threshold price, deferred acceptance auction $(N, B, \chi^{\mathsf{DA}}, \wp^{\mathsf{TP}})$ *is dominant strategy incentive compatible.*

Flexibility with Scoring Rules

Added resilience through **various scoring rules**:

- use **imperfect feasibility checking**,
- impose **auction budget constraints**, or
- bound **efficiency** with interference graph specific scoring rules.

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Conclusion

The **nature of the reverse auction** and **intractability of standard techniques** required the design of novel deferred acceptance auction.

This is an ongoing FCC auction design project, with still much to address:

- Generalize to **more than two outcomes**;
  e.g. possibility of going to lower quality spectrum instead of just off-air.
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