Additional Solution Concepts

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Outline

- Minimax Regret
- Iterated Regret Minimization
- Rationalizability
- ϵ-Nash

Minimax Regret

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  - Minimize worst-case losses;
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  - What would you play as the row-player if you were to minimize your regret in the future?

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  Always T for row-player(98 vs. ϵ).

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- malicious col-player:
  Row-player follows Maxmin;
  Resulting in (B, R) action profile.

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- $k_1 > 1, 0 < k_2 < 1$
  (T, R)!!!(which is not NE)
  Justification: players meeting for the first time!

Rationalizability

- It is common knowledge that players are:
  - Perfectly rational, so they are aware of:
    - Their opponent's rationality;
    - Their opponent's knowledge of their rationality;
    - Their opponent's knowledge of their knowledge of opponent's rationality . . . ;
- What strategies a rational player play?
  - Strategies that are best-responses to his beliefs about the opponent;
  - Beliefs are not necessarily correct!(Just reasonable, as opposed to correct beliefs in NE)

Rationalizable Strategies

- Always exist;
- In 2-player games:
  - Remaining strategies after iterated removal of strictly dominated strategies;
- In N-player games:
  - Remaining strategies after iterated removal of never best-responding strategies;

Rationalizable Strategies: Example

- Matching Pennies

	H	T
H	$1, -1$	$-1, 1$
T	$-1, 1$	$1, -1$

  - Players: Row & Col
  - Row plays H, believing that Col plays H;
  - Col plays H is a Rationalizable belief (Col could believe Row plays T);
  - Row plays T is a Rationalizable belief (Row could believe Col plays T);
  - ...
  - So, all pure strategies are rationalizable!

Rationalizable Strategies: Example

- Sometimes results in weak predictions;
- Battle of the Sexes

	B	F
B	$2, 1$	$0, 0$
F	$0, 0$	$1, 2$

  - Even prediction (F, B) is likely to occur!
    - Row plays F, expecting Col to play F;
    - Col plays B, expecting Col to play B;

ϵ-Nash equilibrium

Definition (ϵ-Nash)

Strategy profile $s = (s_1, \ldots, s_n)$ is an ϵ-Nash equilibrium if given $\epsilon > 0$:
$\forall i, s_i' \neq s_i, u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i}) - \epsilon$

- It always exists;
- Makes sense when players are indifferent to sufficiently small gains;
- Has some drawbacks:
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- Makes sense when players are indifferent to sufficiently small gains;
- Has some drawbacks:
  - Sometimes this indifference is unilateral;
  - Example:

	L	R
U	$1, 1$	$1, 0$
D	$1 + \frac{\epsilon}{2}, 1$	$1 + \frac{\epsilon}{2}, 500$

- Row might be indifferent to switching to NE;
- But for the Column, it is a huge difference in payoff!

Fun Puzzle

- Play this game in groups of n, where n is either 2 or 3;
- Together, assign unique numbers from 1 to n to each group member(depending on the size of your group);
- You task is to guess a number from the set $\{0, 1, 2, \ldots, n\}$, but not now! After reading the following rules:
- Don't tell your guess to the other players!
- The payoff u_i of your guess will be: $u_i = (m - i - 1)s_i$, where:
  - i: Your unique number;
  - s_i: Your guess;
  - m: The average of guesses of your group;
  - n: Highest possible guess!(the size of the group);
  - You want to maximize your payoff;

Now guess your number and commit to it.

Fun Puzzle: Answer

- Those who have chosen 0, have taken the rationalizable strategy.
  - Remember the payoff of player i was: $u_i = (m - i - 1)s_i$;
  - Player n reasons as follow: $m \leq n$, so $(m - n - 1) < 0$ that is $u_i \leq 0$;
  - So, he should choose $s_i = 0$, else his utility will be negative!
  - Other players know player n is rational and chooses 0. So, the same reasoning applies for all members!

Conclusion

- Introduced some other solution concepts;
  - Minimax regret
  - Iterated regret minimization
  - Rationalizability
  - ϵ-Nash equilibrium
  - We will see some more in extensive-form games
- Sometimes, the choice of which depends on players beliefs;
- Weaker predictions than Nash equilibrium,
- In reality, sometimes these weak outcomes happen!
- Sometimes players make other choices because they believe their opponent will deviate as well!

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Thank You

Definition (Regret)

The regret of player i for playing action a_i assuming action profile a_{-i} is played by other players:

$$[\max_{a'_i \in A_i} u_i(a'_i, a_{-i})] - u_i(a_i, a_{-i}).$$

Definition (Max Regret)

The maximum regret of player i for playing action a_i:

$$\max_{a_{-i} \in A_{-i}} ([\max_{a'_i \in A_i} u_i(a'_i, a_{-i})] - u_i(a_i, a_{-i})).$$

Definition (Minimax Regret)

The action that yields to smallest maximum regret for player i

$$\arg\min_{a_i \in A_i} [\max_{a_{-i} \in A_{-i}} ([\max_{a'_i \in A_i} u_i(a'_i, a_{-i})] - u_i(a_i, a_{-i}))].$$