Game Theory Week 1

Game Theory Course:
Jackson, Leyton-Brown & Shoham

A Flipped Classroom Course

Before Tuesday class: Watch the week's videos, on Coursera or locally at UBC. Hand in the previous week's assignment electronically.

Tuesday class: A lecture with high-level review of concepts from the week's videos. Enrichment lectures about concepts not covered online. Discussion, interactive activities.

Thursday class: A "lab" focusing on group work. We'll review the solutions to the previous week's assignment. Then we'll give you the next assignment (usually 1 or 2 questions) and you'll work in groups. Kevin and Dave/James will be there to offer help, hints, and advice about how to improve answers.

This begins now!

Before Tuesday's class, watch the first week of videos:

`https://www.coursera.org/course/gametheory`

`http://www.cs.ubc.ca/~cs532l/`

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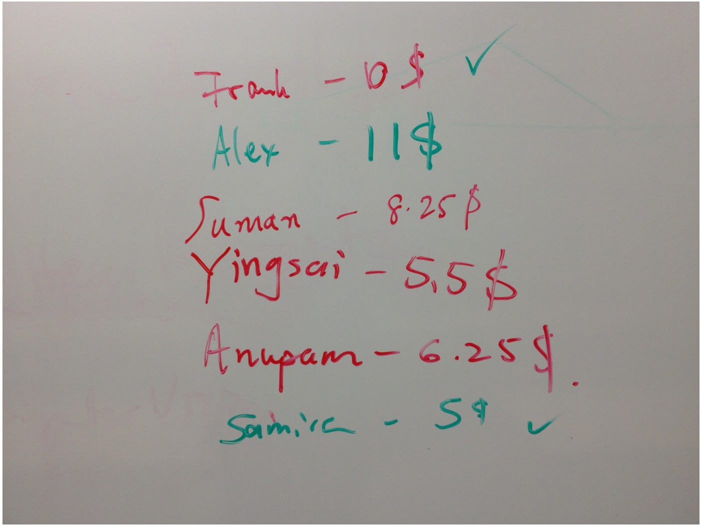

Auction Results

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TCP Backoff Game

- Should you send your packets using correctly-implemented TCP (which has a "backoff" mechanism) or using a defective implementation (which doesn't)?
  - both use a correct implementation: both get 1 ms delay
  - one correct, one defective: 4 ms for correct, 0 ms for defective
  - both defective: both get a 3 ms delay.

- Some questions to discuss after playing:
  - What action should a player of the game take?
  - Would all users behave the same in this scenario?
  - What global behavior patterns should a system designer expect?
  - For what changes to the numbers would behavior be the same?
  - What effect would communication have?
  - Repetitions? (finite? infinite?)
  - Does it matter if I believe that my opponent is rational?

Defining Games - The Normal Form

- Finite, n-person normal form game: $\langle N, A, u \rangle$:
  - Players: $N = \{1, \ldots, n\}$ is a finite set of n , indexed by i
  - Action set for player i A_i
    - $a = (a_1, \ldots, a_n) \in A = A_1 \times \ldots \times A_n$ is an action profile
  - Utility function or Payoff function for player i: $u_i : A \mapsto \mathbb{R}$
    - $u = (u_1, \ldots, u_n)$, is a profile of utility functions
- Writing a 2-player game as a matrix:
  - "row" player is player 1, "column" player is player 2
  - rows correspond to actions $a_1 \in A_1$, columns correspond to actions $a_2 \in A_2$
  - cells listing utility or payoff values for each player: the row player first, then the column

More General Form

Prisoner's dilemma is any game

	C	D
C	a, a	b, c
D	c, b	d, d

with $c > a > d > b$.

Matching Pennies

One player wants to match; the other wants to mismatch.

	Heads	Tails
Heads	$1, -1$	$-1, 1$
Tails	$-1, 1$	$1, -1$

Coordination Game

Which side of the road should you drive on?

	Left	Right
Left	1, 1	0, 0
Right	0, 0	1, 1

General Games: Battle of the Sexes

The most interesting games combine elements of cooperation *and* competition.

	B	F
B	2, 1	0, 0
F	0, 0	1, 2

Keynes Beauty Contest Game: The Stylized Version

- Each player names an integer between 1 and 100.
- The player who names the integer closest to two thirds of the *average* integer wins a prize, the other players get nothing.
- Ties are broken uniformly at random.

Best Response

- If you knew what everyone else was going to do, it would be easy to pick your own action
- Let $a_{-i} = \langle a_1, \ldots, a_{i-1}, a_{i+1}, \ldots, a_n \rangle$.
  - now $a = (a_{-i}, a_i)$

Definition (Best response)

$a_i^* \in BR(a_{-i})$ iff $\forall a_i \in A_i,\ u_i(a_i^*, a_{-i}) \geq u_i(a_i, a_{-i})$.

Nash Equilibrium

- Really, no agent knows what the others will do.
- What can we say about which actions will occur?

- Idea: look for stable action profiles.

Definition (Nash Equilibrium)

$a = \langle a_1, \ldots, a_n \rangle$ is a ("pure strategy") Nash equilibrium iff $\forall i,\ a_i \in BR(a_{-i})$.

Nash Equilibria of Example Games

	C	D
C	$-1, -1$	$-4, 0$
D	$0, -4$	$-3, -3$

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Domination

- Let s_i and s'_i be two strategies for player i, and let S_{-i} be is the set of all possible strategy profiles for the other players
  - What's a "strategy"?
  - For now, just choosing an action ("pure strategy")

Definition

s_i **strictly dominates** s'_i if $\forall s_{-i} \in S_{-i}, u_i(s_i, s_{-i}) > u_i(s'_i, s_{-i})$

Definition

s_i **very weakly dominates** s'_i if $\forall s_{-i} \in S_{-i}, u_i(s_i, s_{-i}) \geq u_i(s'_i, s_{-i})$

Pareto Optimality

- When one outcome o is at least as good for every agent as another outcome o', and there is some agent who strictly prefers o to o':
  - it seems reasonable to say that o is better than o'
  - we say that o Pareto-dominates o'.

Definition (Pareto Optimality)

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- can a game have more than one Pareto-optimal outcome?
- does every game have at least one Pareto-optimal outcome?

Pareto Optimal Outcomes in Example Games

	Left	Right
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The paradox of Prisoner's dilemma:

the (DS) Nash equilibrium is the only non-Pareto-optimal outcome!